

Hadamard Matrix Conjecture

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Abstract

The Hadamard Matrix Conjecture has been an open problem for decades. In this paper, we attempt to solve the problem of the existence of Hadamard Matrices. The solution is that the rank of a square matrix must be divisible by 4. We use AT Math to solve this problem.

Keywords: Hadamard Matrix Conjecture.

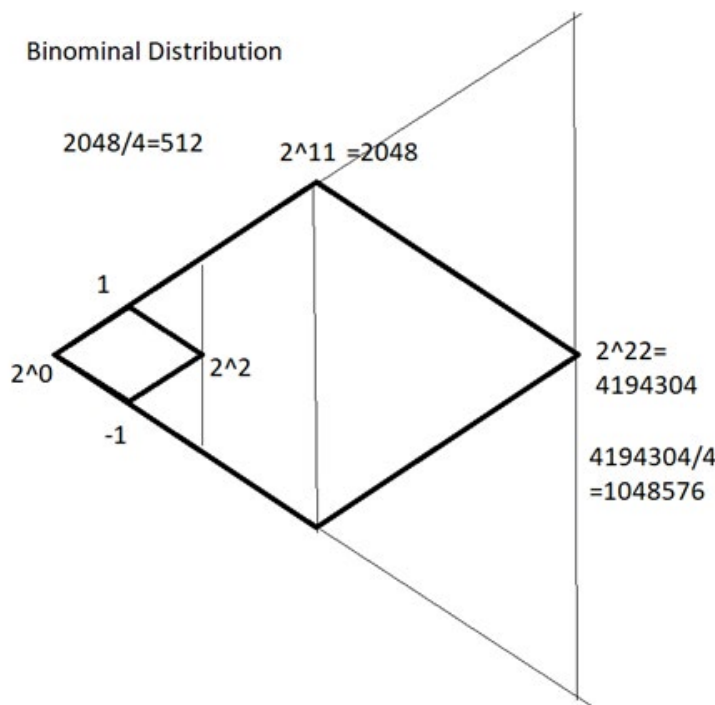


Figure 1: The Binominal Distribution

Introduction

In this paper, we solve the existence of Hadamard Matrices.

$$\begin{aligned}
 H_1 &= [1] \\
 H_2 &\stackrel{\text{def}}{=} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \sin 45^\circ & \cos 45^\circ \\ \cos 45^\circ & -\sin 45^\circ \end{bmatrix} \\
 |D| &= -\sin^2 45^\circ - \cos^2 45^\circ \Rightarrow \text{Euler's Trigonometric Identity} \\
 &= -1/2 - 1/2 \\
 &= -1 \\
 H_n &= \begin{bmatrix} |1 & 1|, |1 & 1|, \dots \end{bmatrix} = 2^{n-2}, n \geq 4 \\
 &\begin{bmatrix} |1 & -1|, |1 & -1| \\ \dots \end{bmatrix}
 \end{aligned}$$

This is the binominal distribution the largest matrix necessary to encompass the entire universe is 2^{22} .

$$\begin{aligned}
 &2^{22}/4 \\
 &= 2^{22}/2^2 \\
 &\text{Binominals } n \geq 4, \text{ are always divisible by } 2^2 \\
 \hat{u} &= [1, 2, 4] \\
 \hat{v} &= [1, 5, 11] \\
 \hat{u} \cdot \hat{v} &= |\hat{u}| |\hat{v}| \cos \theta \\
 1+5+11 &= 17 \\
 2+10+22 &= 34 \\
 4+20+44 &= 68 \\
 \dots &2^n
 \end{aligned}$$

$$|u||v| \cos \theta = 2^n \cos \theta = 0 \text{ for } \hat{u} \perp \hat{v}$$

$$2^n \neq 0$$

$$n \ln 2 \neq 0$$

$$\cos \theta = 0$$

$$\theta = 90^\circ \Rightarrow \hat{u} \perp \hat{v}$$

$$(\sqrt{2})^n = 2^{n/2}$$

$$\ln 2^{n/2} = n/2 \ln 2$$

$$= n(-0.693)/2$$

$$= -n(M/t)$$

$$= -2n$$

$$= \int \sin \theta - \int \cos \theta \text{ from } 1 \rightarrow \pi/4$$

$$= [\cos 1 - \cos(\pi/4)] - [\sin 1 - \sin(\pi/4)]$$

$$= [0.5403 - 0.7071] - [0.84147 - 0.7071]$$

$$= -0.1668 - 0.13437$$

$$= 0.30117$$

$$-2n = 0.30117$$

$$n = -0.150585$$

$$= 1/-6.641 \approx 1/-G = 1/E = t$$

$$n = t$$

$$t^2 - t - 1 = E = 1$$

$$t = 2, -1$$

$$t^2 - t - 1 = \mathbb{Z}$$

$$\mathbb{Z}' = 2t - 1$$

$$\mathbb{Z}'' = 2$$

$$M = \ln t$$

$$t = e^M = \mathbb{Z} = \mathbb{Z}' = \mathbb{Z}'' = 2$$

$$\ln e^M = \ln 2$$

$$M = \ln 2$$

$$M = \ln t$$

$$t = 2$$

$$2^2 - 2 - 1 = 1 = E$$

$$n = t = 2$$

$$(\sqrt{2})^n = 2^{n/2}$$

$$\ln 2^{n/2} = n/2 \ln 2$$

$$\ln 2 = \ln 2$$

True!

$$t^2 - t - 1 = E = 1/t$$

$$3t^2/2 - 2t - 1 = 1$$

$$3t^2 - 4t - 2 = 2$$

$$t^2 - 4/3t - 4/3 = 0$$

$$t = 2, 1/3$$

$$t = 1/E = 1/3 \quad E = 3$$

$$t^2 - t - 1 = 3$$

$$t^2 - t - 4 = 0$$

$$t = [-1 \pm \sqrt{17}]/2 \neq \mathbb{Z}$$

So $\mathbb{Z} = 1$ when $t = 2$

$$t \equiv [1 \pm \mathbb{Z}]/2 \text{ where } \mathbb{Z} = \sqrt{[b^2 - 4ac]}$$

$$\text{Let } t = 2$$

$$2 = [1 \pm \mathbb{Z}]/2$$

$$4 = [1 \pm \mathbb{Z}]$$

$$\pm \mathbb{Z} = 3 = t$$

$$\mathbb{Z} = \pm 3$$

$$\text{Let } t = 3$$

$$3 = [1 \pm \mathbb{Z}]/2$$

$$6 = [1 \pm \mathbb{Z}]$$

$$\mathbb{Z} = \pm 5$$

$$\text{Let } t = 4$$

$$4(2) = [1 \pm \mathbb{Z}]$$

$$\mathbb{Z} = \pm 7$$

$$\text{Let } t = 5$$

$$5(2) = [1 \pm \mathbb{Z}]$$

$$\mathbb{Z} = \pm 9$$

$$\mathbb{Z} = \pm \{3, 5, 7, 9, \dots\}$$

$$\text{where } \mathbb{Z} = \sqrt{[b^2 - 4ac]}$$

$$\text{For } -4/3 = s$$

$$\mathbb{Z} = 8/3 = SF$$

Aside

$$SF = F = 1/E$$

$$t = 1/E$$

$$[s \pm SF]/t = \mathbb{Z}$$

$$= v \pm SF/t = \mathbb{Z}$$

$$= 1/\sqrt{2} \pm t = \mathbb{Z}$$

$$\mathbb{Z} = 1/\sqrt{2} \pm t = \mathbb{Z}' = \mathbb{Z}'' = e^M$$

$$1/\sqrt{2} = e^M$$

$$\ln(1/\sqrt{2}) = M = \ln t$$

$$t = 1/\sqrt{2} = \sin 45^\circ = \cos 45^\circ$$

$$t = \sin t = \cos t$$

$$y = y' = t$$

$$E = dE/dt$$

$$t^2 - t - 1 = 2t - 1$$

$$t = 3; E = 5$$

$$t = \sin \theta = \cos \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1^2 \text{ Euler}$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$1 = 1/\sin^2 \theta - \cot^2 \theta$$

$$E = E^2 - \cot^2 \theta$$

$$E^2 - E - \cot^2 \theta = 0$$

$$\text{Let } \cot^2 \theta = 1$$

$$\theta = 45^\circ$$

$$E^2 - E - 1 = 0$$

$$x^2 - x - 1 = 0 \Rightarrow \text{GMP}$$

$$E = t$$

$$E^2 - E - 1 = 0$$

$$\mathbb{Z} = \sqrt{[b^2 - 4ac]}$$

$$= \sqrt{[(-1)^2 - 4(1)(-1)]}$$

$=\sqrt{1+4}$
 $=\sqrt{5} \neq \text{Integer}$
 $=\text{hypotenuse of the GM Triangle with legs 1 \& 2}$

$$\mathbb{Z}^2 = (\sqrt{5})^2 = 5 = \text{Integer} = E \Rightarrow t=3 \quad y=y'$$

$$\mathbb{Z} = \mathbb{Z}' = \mathbb{Z}'' = e^M = 5$$

$$M = \text{Ln } 5 = \text{Ln } t$$

$$t=5$$

$$t=1,2,3,5=n$$

$$(\sqrt{2})^n = 2^{n/2}$$

$$(\sqrt{2})^n = 2^{n/2}$$

$$\sqrt{2} = E_{\max} = \sin 45^\circ + \cos 45^\circ = 1/\sqrt{2} + 1/\sqrt{2} = 2/\sqrt{2} = \sqrt{2}$$

$$t=2;$$

$$2.828 = \bar{P} = Mv = 4(1/\sqrt{2}) \neq \text{Integer}$$

$$2^2 = 4 = M$$

$$2^{5/2} = 2^{2.5} = 5.6568 \neq \text{Integer}$$

The integers are $t=1,2,4$.

$$E^2 - E - \cot \theta = 0$$

For the 1-2- $\sqrt{5}$ triangle:

$$E = -(-1) \pm \sqrt{(-1)^2 - 4(1)(-2/1)}/2$$

$$= [1 \pm \sqrt{9}]/2$$

$$= 2; -1$$

For the 3-4-5 triangle:

$$E = (-1) \pm \sqrt{(-1)^2 - 4(1)(-3/4)}/2$$

$$= (-1) \pm \sqrt{1+3}/2$$

$$= (-1) \pm 1$$

$$= 0; -2$$

$$E^2 - E - \cot \theta = 0$$

Pythagoras

$$a^2 + b^2 = c^2 = \mathbb{Z} = \pm \sqrt{[b^2 - 4ac]}$$

$$\mathbb{Z}^2 = \pm [b^2 - 4ac]$$

$$\mathbb{Z}^2 = \pm [(-1)^2 - 4(1)(\cot \theta)]$$

$$\mathbb{Z}^2 = \pm [1 - 4 \cot \theta]$$

$$\mathbb{Z} = \pm \sqrt{[1 - 4a/b]}$$

$$\mathbb{Z}^2 = 1 - 4a/b$$

$$\mathbb{Z} \times \mathbb{Z} = \mathbb{Z}$$

$$\mathbb{Z} = 1 - 4a/b$$

$$\mathbb{Z} - 1 = \mathbb{Z} = -4a/b$$

$$\mathbb{Z}/4 = -a/b = -\cot \theta$$

$$\mathbb{Z}/4 = \mathbb{Z} = -\cot \theta$$

$$\mathbb{Z} = -\cos \theta / \sin \theta$$

$$y = y' = -\sin \theta = \cos \theta$$

$$\mathbb{Z} = 1$$

$$\theta = 45^\circ = \pi/4$$

$$\mathbb{Z} = \{\mathbb{Z}^2, \mathbb{Z} - 1, \mathbb{Z}/4\}$$

$$\mathbb{Z}/4 \Rightarrow \mathbb{Z} \geq 4 = n$$

$$2^{n/2} = 2^2 = 4$$

The Hadamard must be divisible by 1,2,4,8,16,32 or 1,2,4\

There are no Hadamard Matrices for: 668, 716, 892, 1132, 1244, 1388, 1436, 1676, 1772, 1916, 1948, and 1964 because they are not divisible consecutively by 4.

$$\mathbb{Z} = \mathbb{Z}^2 = \mathbb{Z} - 1 = \mathbb{Z}/4$$

$$\sqrt{\mathbb{Z}} = \sqrt{[\mathbb{Z} - 1]} = \sqrt{[\mathbb{Z}/4]}$$

$$\mathbb{Z} - 1 = \mathbb{Z}/4$$

$$4(\mathbb{Z} - 1) = \mathbb{Z}$$

$$4\mathbb{Z} - \mathbb{Z} = 1$$

$$3\mathbb{Z} = 1$$

$$\mathbb{Z} = 1/3 = 1/t = E$$

$$E = \mathbb{Z}$$

$$t^2 - t - 1 = E = \mathbb{Z}$$

$$\mathbb{Z}' = \mathbb{Z}'' = 2$$

$$t^2 - t - 1 = 2$$

$$t^2 - t - 3 = 0$$

$$t^2 - t - t = 0$$

$$t^2 - 2t = 0$$

$$t(t - 2) = 0$$

$$t = 0; t = 2$$

QED

Conclusion

Hadamard Matrices must be divisible by 4.

References

Wikipedia.