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Green's Function - Reciprocity Theorem - Mutual Energy Theorem - Huygens' Principle - Poynting's Theorem - Mutual Energy Flow Theorem - Law of Conservation of Energy - Photon

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Abstract

Maxwell's electromagnetic theory is a single-source electromagnetic theory. In contrast, the reciprocity theorem involves two sources. The electromagnetic theory with two sources is fundamentally different from that with a single source. This paper elucidates the step-by-step development of the two-source electromagnetic theory. It clarifies the relationships between Green's function, the reciprocity theorem, Poynting's theorem, the mutual energy theorem, Huygens' principle, the mutual energy flow theorem, the mutual energy flow law, and the law of conservation of energy. It also attempts to reconstruct the historical development sequence of these theorems and laws. Shortly after Maxwell proposed his electromagnetic theory, Poynting's theorem was introduced in 1884, followed by Lorentz's reciprocity theorem in 1896. In 1954, Rumsey proposed the principle of action and reaction. Welch proposed the time-domain reciprocity theorem in 1960. In 1987, the author proposed the mutual energy theorem. The mutual energy theorem is the Fourier transform of Welch's time-domain reciprocity theorem. However, the author emphasizes that it is an energy theorem, not merely a reciprocity theorem. The author's formulation of the mutual energy theorem was questioned by most scientists in the electromagnetic field at the time. The main point of contention was that the so-called mutual energy theorem was not truly an energy theorem but still a reciprocity theorem. To address this, the author further proposed the mutual energy flow theorem in 2017, considering it the law of conservation of energy in electromagnetic field theory, and introduced the concept of self-energy flow reverse collapse. In 2020, the author made a first revision to the mutual energy flow, adding a normalization factor of 1/2. After 2022, the author replaced the concept of self-energy flow reverse collapse with the concept of self-energy flow as reactive power. The idea that self-energy flow is reactive power contradicts Maxwell's electromagnetic theory, leading the author to revise Maxwell's theory, primarily concerning the magnetic field. The author argues that the magnetic field in Maxwell's electromagnetic theory is not the true magnetic field but rather the average magnetic field along a loop. Maxwell's theory provides this average magnetic field, and obtaining the true magnetic field requires a phase correction to this average magnetic field.

Keywords: Maxwell's Equations, Poynting's Theorem, Reciprocity Theorem, Mutual Energy Theorem, Green's Function, Silver-Müller Radiation Condition, Law of Conservation of Energy, Photon, Wave Collapse, Action and Reaction, Retarded Wave, Advanced Wave, Retarded Potential, Advanced Potential, Reactive Power, Magnetic Field

1. Introduction

As we know, Maxwell proposed his equations in 1861-2 [1,2]. Subsequently, Poynting, a student of Maxwell, proposed Poynting's theorem in 1884 [3].

$$-\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=\int_{V}(\boldsymbol{E}\cdot\boldsymbol{J}+\boldsymbol{E}\cdot\frac{\partial}{\partial t}\boldsymbol{D}+\boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B})dV. \quad (1)$$

Lorentz proposed the Lorentz reciprocity theorem in 1896

$$-\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_1(\omega)\times\boldsymbol{H}_2(\omega)-\boldsymbol{E}_2(\omega)\times\boldsymbol{H}_1(\omega))\cdot\hat{n}d\Gamma = \int_V(\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)-\boldsymbol{E}_2(\omega)\cdot\boldsymbol{J}_1(\omega))dV \quad (2)$$

The Silver-Müller radiation condition ensures that when the surface Γ is a sphere with an infinite radius,

$$\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_1(\omega)\times\boldsymbol{H}_2(\omega)-\boldsymbol{E}_2(\omega)\times\boldsymbol{H}_1(\omega))\cdot\hat{n}d\Gamma=0 \quad (3)$$

Thus, we have

$$\int_V \boldsymbol{E}_2(\omega)\cdot\boldsymbol{J}_1(\omega)dV=\int_V \boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)dV. \quad (4)$$

Rumsey developed the reciprocity law into the action-reaction principle in 1954 [4]

$$<1,2>=<2,1>, \quad (5)$$

where $<1,2>=\int_V \boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)dV,\ <1,2>=\int_V \boldsymbol{E}_2(\omega)\cdot\boldsymbol{J}_1(\omega)dV.$

Welch proposed the time-domain reciprocity theorem in 1960

$$-\int_{t=-\infty}^{\infty}dt\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_1(t)\times\boldsymbol{H}_2(t)+\boldsymbol{E}_2(t)\times\boldsymbol{H}_1(t))\cdot\hat{n}d\Gamma$$

$$=\int_{t=-\infty}^{\infty}dt\int_V(\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)+\boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t))dV$$

The author proposed the mutual energy theorem in 1987

$$-\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_1(\omega)\times\boldsymbol{H}_2^*(\omega)+\boldsymbol{E}_2^*(\omega)\times\boldsymbol{H}_1(\omega))\cdot\hat{n}d\Gamma$$

$$=\int_V(\boldsymbol{E}_2^*(\omega)\cdot\boldsymbol{J}_1(\omega)+\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2^*(\omega))dV$$

At the end of 1987, de Hoop proposed his reciprocity theorem

$$-\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_1(t+\tau)\times\boldsymbol{H}_2(t)+\boldsymbol{E}_2(t)\times\boldsymbol{H}_1(t+\tau))\cdot\hat{n}d\Gamma$$

$$=\int_{t=-\infty}^{\infty}dt\int_V(\boldsymbol{E}_1(t+\tau)\cdot\boldsymbol{J}_2(t)+\int_{t=-\infty}^{\infty}dt\int_V\boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t+\tau))dV$$

The mutual energy theorem is the Fourier transform of de Hoop's reciprocity theorem. Welch's time-domain reciprocity theorem is de Hoop's reciprocity theorem with $\tau = 0$. Therefore, Welch's reciprocity theorem, the mutual energy theorem, and de Hoop's reciprocity theorem are essentially the same theorem. However, whether this theorem is an energy theorem is controversial. The author believes it is an energy theorem. But most scientists in electromagnetic field theory at the time (and even now) questioned whether the mutual energy theorem is an energy theorem. An important reason is that the mutual energy theorem involves advanced waves, which are not recognized by electromagnetic field theorists as physically objective existence. If advanced waves are not physically objective, then these theorems cannot be called energy theorems.

However, the author insists that the mutual energy theorem is an energy theorem. The author proposed the mutual energy flow theorem in 2017 [5]. The mutual energy flow theorem is an extension of the mutual energy theorem, introducing the concept of self-energy flow reverse collapse. Thus, self-energy flow does not transfer energy. Since self-energy flow does not transfer energy, mutual energy flow should be considered the only energy flow, making the mutual energy flow theorem the law of conservation of energy. To demonstrate that the mutual energy flow theorem is indeed an energy theorem, the author completed a proof of the mutual energy theorem from Poynting's theorem. Thus, it seems the mutual energy theorem can be considered an energy theorem.

In 2020, the author discovered that the mutual energy flow must be scaled by $\frac{1}{2}$ [6]. Although it is a simple factor of $\frac{1}{2}$, it implies a bug in Maxwell's electromagnetic theory. Maxwell's equations have been revised accordingly.

After 2022, the author found that reverse collapse can be replaced by the concept of self-energy flow as reactive power [7]. Self-energy flow not transferring energy means that only mutual energy flow transfers energy. Therefore, mutual energy flow is the only energy flow, and the mutual energy flow theorem is the law of conservation of energy.

This paper introduces the vector Green's function and uses it to simultaneously prove the Lorentz reciprocity theorem and the mutual energy theorem, further clarifying the relationship between the two. This also further clarifies the relationship between Poynting's theorem and the mutual energy theorem. Additionally, it elucidates a developmental process: first came Poynting's theorem, then the Lorentz reciprocity theorem, followed by Welch's reciprocity theorem, then the mutual energy and mutual energy flow theorems. From the mutual energy theorem, Huygens' principle was developed, along with the mutual energy flow theorem. Then, from the mutual energy flow theorem, the mutual energy flow law and the law of conservation of energy were developed, leading to the law that radiation does not overflow the universe. Finally, a bug in Maxwell's electromagnetic theory was discovered, and a reasonable revision was made. The revised theory is effective not only for electromagnetic waves but also for photons, as photons are essentially mutual energy flow.

2 Vector Green's Function

Consider the mathematical vector formula

$$\nabla \cdot (\boldsymbol{F}_1 \times \boldsymbol{F}_2{}') = \nabla \times \boldsymbol{F}_1 \cdot \boldsymbol{F}_2{}' - \boldsymbol{F}_1 \cdot \nabla \times \boldsymbol{F}_2{}' \tag{6}$$

Let

$$\boldsymbol{F}_2{}' = \nabla \times \boldsymbol{F}_2 \tag{7}$$

We obtain,

$$\nabla \cdot (\boldsymbol{F}_1 \times (\nabla \times \boldsymbol{F}_2)) = \nabla \times \boldsymbol{F}_1 \cdot (\nabla \times \boldsymbol{F}_2) - \boldsymbol{F}_1 \cdot \nabla \times (\nabla \times \boldsymbol{F}_2) \tag{8}$$

This is the first Green's identity. Similarly, we have

$$\nabla \cdot (\boldsymbol{F}_2 \times (\nabla \times \boldsymbol{F}_1)) = \nabla \times \boldsymbol{F}_2 \cdot (\nabla \times \boldsymbol{F}_1) - \boldsymbol{F}_2 \cdot \nabla \times (\nabla \times \boldsymbol{F}_1) \tag{9}$$

Subtracting (9) from (8) yields

$$\begin{aligned} &\nabla \cdot (\boldsymbol{F}_1 \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \times \nabla \times \boldsymbol{F}_1) \\ &= -(\boldsymbol{F}_1 \cdot \nabla \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \cdot \nabla \times \nabla \times \boldsymbol{F}_1) \end{aligned} \tag{10}$$

2.1 Vector Second Green's Identity

Integrating the above over volume yields,

$$\begin{aligned} &-\oiint_\Gamma (\boldsymbol{F}_1 \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \times \nabla \times \boldsymbol{F}_1) \cdot \hat{n} d\Gamma \\ &= \int_V (\boldsymbol{F}_1 \cdot \nabla \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \cdot \nabla \times \nabla \times \boldsymbol{F}_1) dV \end{aligned} \tag{11}$$

This is the second vector Green's theorem.

2.2 Frequency-Domain Maxwell's Equations

The two curl equations of Maxwell's equations are

$$\begin{cases} \nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}(\mu_0 \boldsymbol{H}) \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}(\epsilon_0 \boldsymbol{E}) \end{cases} \tag{12}$$

Considering the time factor exp(jωt), we have

$$\frac{\partial}{\partial t} \rightarrow j\omega \quad (13)$$

Thus, the frequency-domain Maxwell's equations are,

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + j\omega\epsilon_0 \boldsymbol{E} \end{cases} \quad (14)$$

2.3 Reciprocity Theorem

Consider two current elements J_1, J_2 in region V. Their generated fields are $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]^T$, $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]^T$, where T denotes matrix transpose. See Figure 1

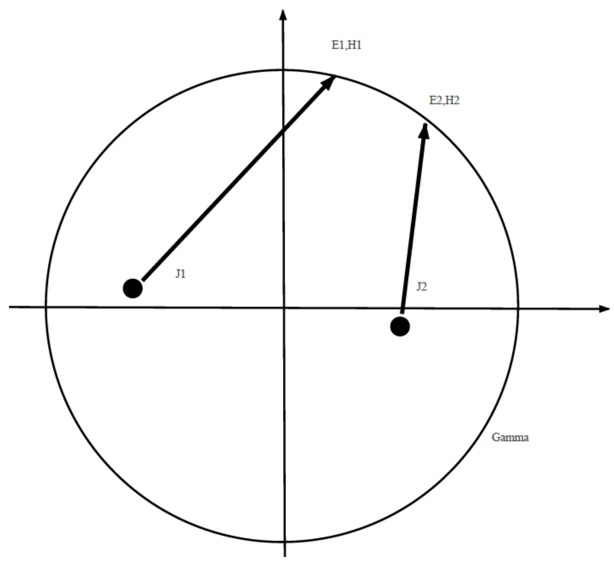

Figure 1: Inside Boundary Γ, there are Two Sources $\boldsymbol{J}_1$**,** $\boldsymbol{J}_2$**.** $\boldsymbol{J}_1$ is the Source, Generating the retarded Wave $\boldsymbol{E}_1$**,** $\boldsymbol{H}_1$**.** $\boldsymbol{J}_2$ is the Sink, Generating the Advanced Wave $\boldsymbol{E}_2$**,**$\boldsymbol{H}_2$.

Consider,

$$\boldsymbol{F}_1 \leftarrow \boldsymbol{E}_1, \quad \boldsymbol{F}_2 \leftarrow \boldsymbol{E}_2 \quad (15)$$

The symbol "⟵" denotes a substitution relationship. The above means substituting $\boldsymbol{E}_1$ for $\boldsymbol{F}_1$ and $\boldsymbol{E}_2$ for $\boldsymbol{F}_2$, yielding

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times \nabla \times \boldsymbol{E}_2 - \boldsymbol{E}_2 \times \nabla \times \boldsymbol{E}_1) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot \nabla \times \nabla \times \boldsymbol{E}_2 - \boldsymbol{E}_2 \cdot \nabla \times \nabla \times \boldsymbol{E}_1) dV \quad (16)$$

Considering Maxwell's equations (14), we get,

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times (-j\omega\mu_0 \boldsymbol{H}_2) - \boldsymbol{E}_2 \times (-j\omega\mu_0 \boldsymbol{H}_1)) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot \nabla \times (-j\omega\mu_0 \boldsymbol{H}_2) - \boldsymbol{E}_2 \cdot \nabla \times ((-j\omega\mu_0 \boldsymbol{H}_1))) dV \quad (17)$$

Or

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot (\boldsymbol{J}_2 + j\omega\epsilon_0\boldsymbol{E}_2) - \boldsymbol{E}_2 \cdot (\boldsymbol{J}_1 + j\omega\epsilon_0\boldsymbol{E}_1))dV \quad (18)$$

Or

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 - \boldsymbol{E}_2 \cdot \boldsymbol{J}_1)dV \quad (19)$$

This is the Lorentz reciprocity theorem [8]. The surface integral on the right can be zero if Γ is a sphere with an infinite radius, the two currents $\boldsymbol{J}_1$,$\boldsymbol{J}_2$ are near the origin, and the Silver-Müller radiation condition is used, i.e.,

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = 0 \quad (20)$$

Thus,

$$\int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 - \boldsymbol{E}_2 \cdot \boldsymbol{J}_1)dV = 0 \quad (21)$$

i.e.,

$$\int_{V_1} (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1)dV = \int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2)dV \quad (22)$$

where $\boldsymbol{V}_1$ contains only current $\boldsymbol{J}_1$ and $\boldsymbol{V}_2$ contains only current $\boldsymbol{J}_2$. This is another form of the reciprocity theorem. Rumsey expressed this as

$$< 2,1 >=< 1,2 > \quad (23)$$

where

$$< 2,1 >= \int_{V_1} (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1)dV \quad (24)$$

is the effect of electric field $\boldsymbol{E}_2$ on current $\boldsymbol{J}_1$.

$$< 1,2 >= \int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2)dV \quad (25)$$

where <1,2> is the effect of electric field $\boldsymbol{E}_1$ on current $\boldsymbol{J}_2$. Thus, equation (23) is Rumsey's action-reaction principle [4].

2.4 Mutual Energy Theorem

Consider

$$\boldsymbol{F}_1 \leftarrow \boldsymbol{E}_1, \quad \boldsymbol{F}_2 \leftarrow \boldsymbol{E}_2^* \quad (26)$$

$\boldsymbol{E}_2$* is the complex conjugate of $\boldsymbol{E}_2$. The above means substituting $\boldsymbol{E}_1$ for *F1* and $\boldsymbol{E}_2$* for $\boldsymbol{F}_2$

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times \nabla \times \boldsymbol{E}_2^* - \boldsymbol{E}_2^* \times \nabla \times \boldsymbol{E}_1) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot \nabla \times \nabla \times \boldsymbol{E}_2^* - \boldsymbol{E}_2^* \cdot \nabla \times \nabla \times \boldsymbol{E}_1)dV \quad (27)$$

Considering Maxwell's equations (14), we get,

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times (-j\omega\mu_0\boldsymbol{H}_2)^* - \boldsymbol{E}_2^* \times (-j\omega\mu_0\boldsymbol{H}_1)) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot \nabla \times (-j\omega\mu_0\boldsymbol{H}_2)^* - \boldsymbol{E}_2^* \cdot \nabla \times (-j\omega\mu_0\boldsymbol{H}_1))dV \quad (28)$$

Or

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times (-j\omega\mu_0\boldsymbol{H}_2^*) + \boldsymbol{E}_2^* \times (-j\omega\mu_0\boldsymbol{H}_1)) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot \nabla \times (-j\omega\mu_0\boldsymbol{H}_2^*)\boldsymbol{E}_2^* \cdot \nabla \times (-j\omega\mu_0\boldsymbol{H}_1))dV \quad (29)$$

Or

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

Or

$$= \int_V (\boldsymbol{E}_1 \cdot (\boldsymbol{J}_2^* + j\omega\epsilon_0\boldsymbol{E}_2)^* + \boldsymbol{E}_2^* \cdot (\boldsymbol{J}_1 + j\omega\epsilon_0\boldsymbol{E}_1))dV \quad (30)$$

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=\int_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*+\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV \tag{31}$$

The above is the mutual energy theorem, which was proposed by the author in 1987 [5,9,10]. The author called it the mutual energy theorem because it is considered an energy theorem. Of course, the mutual energy theorem can also be used as a reciprocity theorem. Similarly, using the Silver-Müller radiation condition, we can obtain

$$\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=0 \tag{32}$$

This assumes Γ is a sphere with an infinite radius, and the two currents $\boldsymbol{J}_1,\boldsymbol{J}_2$ are near the origin. Thus,

$$\int_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*+\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=0 \tag{33}$$

Or

$$-\int_{V_1}(\boldsymbol{E}_2^*(\omega)\cdot\boldsymbol{J}_1(\omega))dV=\int_{V_2}(\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2^*(\omega))dV \tag{34}$$

This is another form of the mutual energy theorem. The author believes it is more appropriate to write this as an action-reaction principle because action and reaction should be equal in magnitude and opposite in sign.

$$-<2,1>=<1,2> \tag{35}$$

where

$$<2,1>=\int_{V_1}(\boldsymbol{E}_2^*(\omega)\cdot\boldsymbol{J}_1(\omega))dV \tag{36}$$

is the effect of electric field $\boldsymbol{E}_2$ on current $\boldsymbol{J}_1$.

$$<1,2>=\int_{V_2}(\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2^*(\omega))dV \tag{37}$$

where $<1,2>$ is the effect of electric field $\boldsymbol{E}_1$ on current $\boldsymbol{J}_2$. Thus, equation (35) is the action-reaction principle.

2.5 Conjugate Transformation

Perform the following transformation on an electromagnetic field:

$$\boldsymbol{E}^\dagger,\boldsymbol{H}^\dagger,\boldsymbol{J}^\dagger=\boldsymbol{E}^*,-\boldsymbol{H}^*,-\boldsymbol{J}^* \tag{38}$$

Use the symbol "$\boldsymbol{E}^\dagger$" to denote the conjugate transformation of the electromagnetic field. Under this transformation, quantities that originally satisfied Maxwell's equations still satisfy them. The following verification shows that applying the conjugate transformation to all variables in Maxwell's equations (14) yields:

$$\begin{cases}\nabla\times\boldsymbol{E}^\dagger=-j\omega\mu_0\boldsymbol{H}^\dagger\\ \nabla\times\boldsymbol{H}^\dagger=\boldsymbol{J}^\dagger+j\omega\epsilon_0\boldsymbol{E}^\dagger\end{cases} \tag{39}$$

Or

$$\begin{cases}\nabla\times\boldsymbol{E}^*=-j\omega\mu_0(-\boldsymbol{H}^*)\\ \nabla\times(-\boldsymbol{H}^*)=(-\boldsymbol{J}^*)+j\omega\epsilon_0\boldsymbol{E}^*\end{cases} \tag{40}$$

Taking the complex conjugate of equation (14) gives:

$$\begin{cases}\nabla\times\boldsymbol{E}=-j\omega\mu_0\boldsymbol{H}\\ \nabla\times(-\boldsymbol{H})=(-\boldsymbol{J})-j\omega\epsilon_0\boldsymbol{E}\end{cases} \tag{41}$$

The above becomes the frequency-domain Maxwell's equations (14). It can be seen that the electromagnetic field after the conjugate transformation still satisfies Maxwell's equations.

Additionally, the author discovered that the conjugate transformation converts a retarded wave into an advanced wave, and vice versa. For example,

$$\boldsymbol{A}^{(r)}=\frac{\mu_0}{4\pi}\int\frac{\boldsymbol{J}}{r}\exp(-jkr)dV \tag{42}$$

The superscript (r) denotes retarded. Below, we use the superscript (a) to denote advanced. Thus,

$$\boldsymbol{E}^{(r)} = -j\omega \boldsymbol{A}^{(r)} \tag{43}$$

$$\boldsymbol{B}^{(r)} = \nabla \times \boldsymbol{A}^{(r)} \tag{44}$$

Applying the electromagnetic field conjugate transformation to $\boldsymbol{E}^{(r)}$,

$$(\boldsymbol{E}^{(r)})^{\dagger} = (\boldsymbol{E}^{(r)})^{*} = (-j\omega \boldsymbol{A}^{(r)})^{*}$$

$$= j\omega \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}^{*}}{r} \exp(+jkr) dV$$

$$= -j\omega \frac{\mu_0}{4\pi} \int \frac{(-\boldsymbol{J}^{*})}{r} \exp(+jkr) dV$$

$$= -j\omega \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}^{\dagger}}{r} \exp(+jkr) dV \tag{45}$$

The above shows that $(\boldsymbol{E}^{(r)})^{\dagger}$ becomes the advanced wave electric field generated by current $\boldsymbol{J}^{\dagger}$. Here, exp(+jkr) is the advanced factor. Applying the electromagnetic field conjugate transformation to $\boldsymbol{B}$(r),

$$(\boldsymbol{B}^{(r)})^{\dagger} = -(\nabla \times \boldsymbol{A}^{(r)})^{*}$$

$$= -\nabla \times (\frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}}{r} \exp(-jkr) dV)^{*}$$

$$= \nabla \times (\frac{\mu_0}{4\pi} \int \frac{-\boldsymbol{J}^{*}}{r} \exp(+jkr) dV)$$

$$= \nabla \times (\frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}^{\dagger}}{r} \exp(+jkr) dV) \tag{46}$$

The above shows that $(\boldsymbol{B}^{(r)})^{\dagger}$ becomes the advanced wave magnetic field generated by current $\boldsymbol{J}^{\dagger}$. Here, exp($+jkr$) is the advanced factor. Note that $\boldsymbol{B}^{(r)}$ is the magnetic field as defined by Maxwell's electromagnetic theory. (Actually, the author has realized that the magnetic field defined by Maxwell is not the true magnetic field but rather the average magnetic field along a loop [7]. However, here the author still uses $\boldsymbol{B}$ to denote the magnetic field calculated according to Maxwell's equations.)

2.6 Relationship Between the Mutual Energy Flow Theorem and the Lorentz Reciprocity Theorem

Apply the conjugate transformation to all quantities with subscript 2 in the Lorentz reciprocity theorem (19):

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^{\dagger} - \boldsymbol{E}_2^{\dagger} \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^{\dagger} - \boldsymbol{E}_2^{\dagger} \cdot \boldsymbol{J}_1) dV \tag{47}$$

Or

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times (-\boldsymbol{H}_2^{*}) - \boldsymbol{E}_2^{*} \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot (-\boldsymbol{J}_2^{*}) - \boldsymbol{E}_2^{*} \cdot \boldsymbol{J}_1) dV \tag{48}$$

Or

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^{*} + \boldsymbol{E}_2^{*} \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^{*} + \boldsymbol{E}_2^{*} \cdot \boldsymbol{J}_1) dV \tag{49}$$

The above is the mutual energy theorem[5,9,10]. It can be seen that after applying the electromagnetic conjugate transformation to the quantities with subscript 2 in the reciprocity theorem, the reciprocity theorem transforms into the mutual energy theorem. Conversely, applying the conjugate transformation to subscript 2 of the mutual energy theorem yields the reciprocity theorem.

We know that in the reciprocity theorem, both quantities are retarded waves. Therefore, in the mutual energy theorem, one subscript, e.g., subscript 1, denotes a retarded wave, while subscript 2 denotes an advanced wave. Thus, the mutual energy theorem is often used to study systems where a retarded wave and an advanced wave are synchronized. The retarded wave is generated by a transmitting antenna, and the advanced wave is generated by a receiving antenna. Thus, the mutual energy theorem studies systems composed of one transmitting and one receiving antenna. It can also study transformer systems, where the primary coil generates a retarded wave and the secondary coil generates an advanced wave.

However, strictly speaking, a current generates half retarded and half advanced waves. But sometimes the advanced wave does not play a role, only the retarded wave does. In this case, we say the current is a source. Sometimes the retarded wave does not play a role, only the advanced wave does; we then say the current is a sink.

Why do retarded and advanced waves sometimes play a role and sometimes not? This is because both retarded and advanced waves are actually reactive power waves; these waves themselves do not transfer energy. Only when a retarded wave and an advanced wave are synchronized do they form mutual energy flow, and these waves then play a role. This point will be further explained later in this paper.

2.7 Mutual Energy Flow Theorem

We know that

$$\oiint_{\Gamma}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = 0 \tag{50}$$

where $\xi_1=[\boldsymbol{E}_1,\boldsymbol{H}_1]^T$ is the retarded wave, $\xi_2=[\boldsymbol{E}_2,\boldsymbol{H}_2]^T$ is the advanced wave, and Γ is a sphere enclosing $\boldsymbol{J}_1,\boldsymbol{J}_2$. Considering the sphere's radius $R = \infty$, the Silver-Müller radiation condition ensures the above is zero. The above being zero can also be reasoned as follows: the retarded and advanced waves reach the surface Γ at different time, one at some past time and the other at some future time. Therefore, they do not reach Γ simultaneously, so the products $\boldsymbol{E}_1 \times \boldsymbol{H}_2^*$ and $\boldsymbol{E}_2^* \times \boldsymbol{H}_1$ are zero, making the surface integral zero. Combining this with (49) gives:

$$\int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* + \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = 0 \tag{51}$$

Or

$$-\int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*) dV \tag{52}$$

In the mutual energy theorem

$$-\oiint_{\Gamma}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* + \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{53}$$

assume $\boldsymbol{J}_2$ is outside region V, so within region $\boldsymbol{V}_1$, $\boldsymbol{J}_2= 0$. See Fig. 2. Thus, we have

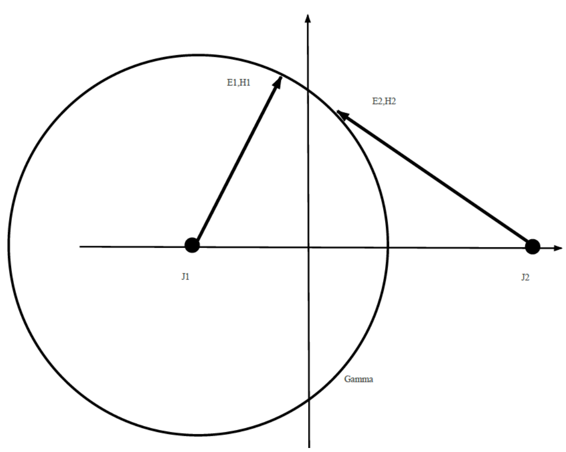

Figure 2: Inside boundary Γ, there is one source $\boldsymbol{J}_1$. $\boldsymbol{J}_1$ is the source generating the retarded wave $\boldsymbol{E}_1,\boldsymbol{H}_1$. $\boldsymbol{J}_2$ is the sink generating the advanced wave. $\boldsymbol{J}_2$ is outside surface Γ.

$$-\oiint_{\Gamma_1}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{54}$$

Or

$$\oiint_{\Gamma_1}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = -\int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{55}$$

Considering (55, 52)

$$-\int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \oiint_{\Gamma_1}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*) dV \tag{56}$$

The above is the mutual energy flow theorem. The concept of mutual energy flow was proposed by the author in 2017. However, the above formula was an intermediate step in the author's derivation of Huygens' principle, already proposed in 1989 [9]. It was not until 2017 that the author named the above formula the mutual energy flow theorem [11]. In 1989, although there was a vague notion that electromagnetic energy might be transferred by $\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$, there was not enough time to fully develop this understanding. The surface Γ in the above can be generalized to any closed surface enclosing current $\boldsymbol{J}_1$, or enclosing $\boldsymbol{J}_2$, or any infinite open surface, as long as this surface can separate currents $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. Thus, the above can be further written as

$$-\int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}_{1\to 2} d\Gamma = \int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*) dV \tag{57}$$

where $\hat{n}_{1\to 2}$) is the normal pointing from 1 to 2. Γ is a surface separating currents $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$.

2.8 Huygens' Principle

In 1989, the author proposed Huygens' principle [9]

$$\boldsymbol{J}_2 = \hat{z}\delta(\boldsymbol{x} - \boldsymbol{x}') \tag{58}$$

$$-\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \hat{z}\delta(\boldsymbol{x} - \boldsymbol{x}')) dV \tag{59}$$

Or

$$-\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = E_{1z}(\boldsymbol{x}) \tag{60}$$

Huygens' principle tells us that the electric field should originally be obtained from current $\boldsymbol{J}_1$ according to:

$$E_{1z}(\boldsymbol{x}) = -\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{61}$$

But if we measure the electric and magnetic fields on surface Γ

$$\boldsymbol{E}_1|_\Gamma, \quad \boldsymbol{H}_1|_\Gamma \tag{62}$$

then the electric field at a distant point can be directly calculated from these measured fields:

$$E_{1z}(\boldsymbol{x}) = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{63}$$

This is the so-called near-field measurement. Above, we used Huygens' principle to find $\boldsymbol{E}_{1z}(x)$. Similarly, we can find $\boldsymbol{E}_{1x}(x)$, $\boldsymbol{E}_{1y}(x)$. Huygens' principle is a mathematical formula. In this case, we use the mutual energy theorem as a mathematical formula; it functions as a reciprocity theorem. Since the source of $\xi_2=[\boldsymbol{E}_2,\boldsymbol{H}_2]^T$ is a δ function, ξ_2 can be regarded as the Green's function

$$\xi_2 = [G_E, G_H]$$

$$E_{1z}(\boldsymbol{x}) = \oiint_\Gamma (\boldsymbol{E}_1 \times G_H^* + G_E^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{64}$$

The above is Huygens' principle. Similarly, $\boldsymbol{E}_{1x}(x)$, $\boldsymbol{E}_{1y}(x)$ can be obtained, hence yielding $\boldsymbol{E}_1$.

3 Law of Conservation of Energy

3.1 The Mutual Energy Theorem is a Sub-theorem of Poynting's Theorem

Here, we re-prove the mutual energy theorem to show that it is a sub-theorem of Poynting's theorem, thus confirming that the mutual energy theorem is indeed an energy theorem. Poynting's theorem is:

$$-\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \boldsymbol{E} \cdot \boldsymbol{J} + \boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D} + \boldsymbol{B} \cdot \frac{\partial}{\partial t}\boldsymbol{B} \tag{65}$$

Integrating the above over time, and considering

$$\int_{t=-\infty}^{\infty} (\boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D} + \boldsymbol{B} \cdot \frac{\partial}{\partial t}\boldsymbol{B}) dt = \int_{t=-\infty}^{\infty} \frac{\partial}{\partial t} U dt$$

$$= U(\infty) - U(-\infty) = 0 \tag{66}$$

where

$$U = \frac{1}{2}(\boldsymbol{E} \cdot \boldsymbol{D} + \boldsymbol{H} \cdot \boldsymbol{B}) \tag{67}$$

Integrating equation (65) over volume and time gives:

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = \int_{t=-\infty}^{\infty} dt \int \boldsymbol{E} \cdot \boldsymbol{J} dV \quad (68)$$

The above is the time-integrated Poynting's theorem. Consider superposition:

$$\boldsymbol{E} = \sum_{i=1}^{2} \boldsymbol{E}_i, \quad \boldsymbol{H} = \sum_{i=1}^{2} \boldsymbol{H}_i, \quad \boldsymbol{J} = \sum_{i=1}^{2} \boldsymbol{J}_i \quad (69)$$

(68) can be rewritten as:

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} \sum_{i=1}^{2} \sum_{j=1}^{2} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma = \int_{t=-\infty}^{\infty} dt \int \sum_{i=1}^{2} \sum_{j=1}^{2} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV \quad (70)$$

From (68), we directly obtain:

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = \int_{t=-\infty}^{\infty} dt \int \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV \quad (71)$$

Subtracting (71) for i=1 and (71) for i=2 from (70):

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma = \int_{t=-\infty}^{\infty} dt \int \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV \quad (72)$$

Or

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t)) dV \quad (73)$$

The above is Welch's time-domain reciprocity theorem [12]. This can be further generalized to:

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t+\tau) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t+\tau)) \cdot \hat{n} d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}_1(t+\tau) \cdot \boldsymbol{J}_2(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t+\tau)) dV \quad (74)$$

The above is de Hoop's cross-correlation reciprocity theorem [13]. Applying the Fourier transform to the above gives:

$$-\oiint_{\Gamma} (\boldsymbol{E}_1(\omega) \times \boldsymbol{H}_2^*(\omega) + \boldsymbol{E}_2^*(\omega) \times \boldsymbol{H}_1(\omega)) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1(\omega) \cdot \boldsymbol{J}_2^*(\omega) + \boldsymbol{E}_2^*(\omega) \cdot \boldsymbol{J}_1(\omega) dV \quad (75)$$

The above is the mutual energy theorem [5]. Thus, the mutual energy theorem can be derived from Poynting's theorem, so it should be regarded as an energy theorem. Additionally, the mutual energy theorem is the Fourier transform of de Hoop's reciprocity theorem, so it is also a type of reciprocity theorem. Therefore, the mutual energy theorem should be both a reciprocity theorem and an energy theorem.

Assume $\boldsymbol{J}_1$,$\boldsymbol{J}_2$ are near the origin, and Γ is a spherical surface with an infinite radius. The retarded wave reaches the surface Γ at some future time, and the advanced wave reaches it at some past time. Therefore, the retarded and advanced waves do not reach Γ simultaneously. Thus, we have:

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma = 0 \quad (76)$$

So from (73), we get:

$$\int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t)) dV = 0 \quad (77)$$

Or

$$-\int_{t=-\infty}^{\infty} dt \int_{V_1} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) = \int_{t=-\infty}^{\infty} dt \int_{V_2} (\boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t)) dV \quad (78)$$

Or

$$\int_{t=-\infty}^{\infty} dt \int_V \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \boldsymbol{E}_i(t) \cdot \boldsymbol{J}_j(t) dV = 0 \quad (79)$$

The above can be generalized to:

$$\int_{t=-\infty}^{\infty} dt \int_V \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_i(t) \cdot \boldsymbol{J}_j(t) dV = 0 \quad (80)$$

It is worth mentioning that the above is still an energy theorem and cannot be called the law of conservation of energy because, according to Maxwell's electromagnetic theory, part of the energy in space is also propagated by self-energy, i.e., equation (71). However, when the author saw equation (80), he immediately realized that this equation is the law of conservation of energy. The author's view is that if $\boldsymbol{N}$ is very large, including all current elements $\boldsymbol{J}_i$ in the universe, then energy exchange should only occur among these N current elements.

On the other hand, if the above is the law of conservation of energy, and if there are only N current elements in the universe, where $\boldsymbol{N}$ is still finite, e.g., $N = 2$ for example, the above should still be the law of conservation of energy. But if the above holds as the law of conservation of energy, then self-energy flow should not transfer energy, i.e., for equation (71), we should have:

$$-\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma}(\boldsymbol{E}_i \times \boldsymbol{H}_i)\cdot \hat{n} d\Gamma = \int_{t=-\infty}^{\infty} dt \int \boldsymbol{E}_i \cdot \boldsymbol{J}_i dV = 0 \quad (81)$$

However, Maxwell's electromagnetic theory conflicts with the above. In Maxwell's theory, for any transmitting antenna, we have:

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma}(\boldsymbol{E}_i \times \boldsymbol{H}_i)\cdot \hat{n} d\Gamma \neq 0 \quad (82)$$

The author believes that if the above self-energy flow is not zero, self-energy flow and mutual energy flow would jointly transfer energy, which would require that the self-energy flow ($E_i \times H_i^*$) and the mutual energy flow ($\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}^*_2 \times \boldsymbol{H}_1$) must overlap to form a photon. This would require the self-energy flow to collapse to the destination of the mutual energy flow (i.e., where the mutual energy flow is absorbed). The author finds this unimaginable. To solve this problem, in 2017, the author proposed the concept of self-energy flow reverse collapse [11]. That is, the author assumed that there exists another type of electromagnetic field in nature, currently unknown to us, that causes the self-energy flow to reverse collapse. Reverse collapse is a physical process that satisfies the time-reversed Maxwell's equations. Reverse collapse means that the self-energy flow does not collapse to the destination of the mutual energy flow but collapses back to the source that emitted the self-energy flow.

Additionally, the mutual energy flow theorem (57) can also be generalized to the mutual energy flow law:

$$-\int_{t=-\infty}^{\infty} dt \int_{V_i} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV$$

$$= \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma}(\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i)\cdot \hat{n}_{1\to 2} d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \int_{V_j} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV \quad (83)$$

Equation (80) is the law of conservation of energy. Energy exchange between currents is conducted through mutual energy flow. The above is no longer just the mutual energy flow theorem but the mutual energy flow law, because since 2017, the author has proposed the concept of self-energy flow reverse collapse [11]. With this reverse collapse, mutual energy flow becomes the only energy flow, so the mutual energy flow theorem is called the mutual energy flow law.

3.2 Demonstrating the Mutual Energy Theorem as an Energy Theorem Using a Transformer

In Section 2.4, the author proved the mutual energy theorem using Green's formula, which only shows that it is a theorem but not necessarily an energy theorem. The author also proved that the mutual energy theorem is a sub-theorem of Poynting's theorem, which strongly supports it being an energy theorem. However, this is still not enough; most electromagnetic theorists still cannot accept the mutual energy theorem as an energy theorem. They still believe that the so-called mutual energy theorem is merely a reciprocity theorem, a mathematical theorem. This is because the mutual energy theorem involves advanced waves, which are not recognized by most scientists as physically objective. If advanced waves are not physically objective, then the mutual energy theorem indeed cannot be an energy theorem. Therefore, calling it an energy theorem would be incorrect! The author firmly believes that the mutual energy theorem is an energy theorem. This section further illustrates, using the principle of a transformer, that the mutual energy theorem is indeed an energy theorem, not only an energy theorem but also the law of conservation of energy. Of course, we must also prove that advanced waves are physically objective.

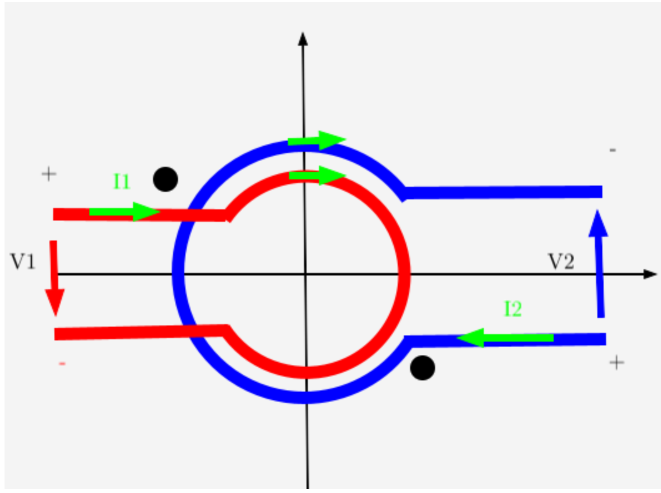

Figure 3: A two-coil Transformer. Coil 1 (red) has input Voltage V_1 and current I_1. Coil 2 (blue) has input voltage V_2 and current I_2. The Identically Marked Terminals of the Two Coils are Indicated by Black Dots

Consider a two-coil transformer as shown in Fig. 3, each with a single turn. From Neumann's energy law [14], the mutual interaction energy between the two coils is:

$$U = \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{I_1 I_2}{r} d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2 \tag{84}$$

If we apply power to the two-coil system via the coils:

$$P = U_1 I_1 + U_2 I_2 \tag{85}$$

P is the power supplied to the system, U_1 is the voltage across coil 1, I_1 is the current in coil 1, U_2 is the voltage across coil 2, and I_2 is the current in coil 2. This power must increase the energy of the transformer system, so:

$$P = \frac{\partial}{\partial t} U \tag{86}$$

Or

$$U_1 I_1 + U_2 I_2 = \frac{\partial}{\partial t} U \tag{87}$$

Here, $U_1 I_1$ is the energy supplied by coil 1 to the system, and $U_2 I_2$ is the energy supplied by coil 2 to the system. This energy causes the transformer system's energy to increase:

$$\frac{\partial}{\partial t} U = \frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{I_1 I_2}{r} d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2$$

$$= I_2 \frac{\partial}{\partial t} I_1 \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{1}{r} d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2 + I_1 \frac{\partial}{\partial t} I_2 \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{1}{r} d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2 \tag{88}$$

So

$$U_1 I_1 + U_2 I_2 = I_2 \frac{\partial}{\partial t} I_1 \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{1}{r} d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2 + I_1 \frac{\partial}{\partial t} I_2 \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{1}{r} d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2 \tag{89}$$

Or

$$U_1 I_1 + U_2 I_2 = I_2 \frac{\partial}{\partial t} \oint_{C_1} \frac{\mu_0}{4\pi} \oint_{C_2} \frac{I_1 d\boldsymbol{l}_1}{r} \cdot d\boldsymbol{l}_2 + I_1 \oint_{C_1} \frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \oint_{C_2} \frac{I_2 d\boldsymbol{l}_2}{r} \cdot d\boldsymbol{l}_1 \tag{90}$$

Or

$$U_1 I_1 + U_2 I_2 = I_2 \oint_{C_2} \frac{\partial}{\partial t} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 + I_1 \oint_{C_1} \frac{\partial}{\partial t} \boldsymbol{A}_2 \cdot d\boldsymbol{l}_1 \tag{91}$$

Comparing terms containing I_1 in the above equation, we get:

$$U_1 = \oint_{C_1} \frac{\partial}{\partial t} \boldsymbol{A}_2 \cdot d\boldsymbol{l}_1 \tag{92}$$

Considering Kirchhoff Circuit voltage Laws

$$\mathcal{E}_{2\to1} = -U_1 \tag{93}$$

$$\mathcal{E}_{2\to1} = -\oint_{C_1} \frac{\partial}{\partial t} \boldsymbol{A}_2 \cdot d\boldsymbol{l}_1 \tag{94}$$

Comparing terms containing I_2, we get:

$$U_2 = \oint_{C_2} \frac{\partial}{\partial t} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{95}$$

Considering Kirchhoff Circuit voltage Laws

$$\mathcal{E}_{1\to2} = -U_2 \tag{96}$$

Additionally, we know that the induced electromotive force (EMF) is:

$$\mathcal{E}_{1\to2} = -\oint_{C_2} \frac{\partial}{\partial t} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{97}$$

$\varepsilon_{1\to2}$ represents the induced electromotive force (EMF) in coil 2 due to the current in coil 1, while $\varepsilon_{2\to1}$ is the induced EMF in coil 1 due to the current in coil 2.

$$U_1 I_1 + U_2 I_2 = -(\mathcal{E}_{2\to1} I_1 + \mathcal{E}_{1\to2} I_2) \tag{98}$$

We know that

$$\int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} U = U(\infty) - U(-\infty) = 0 \tag{99}$$

Above, we assumed

$$U(\infty) = U(-\infty) \tag{100}$$

meaning the system's energy does not change from the beginning to the end. Therefore,

$$\int_{t=-\infty}^{\infty} dt (U_1 I_1 + U_2 I_2) = 0 \tag{101}$$

Transforming the above to the frequency domain gives:

$$U_1^*(\omega) I_1(\omega) + U_2(\omega) I_2^*(\omega) = 0 \tag{102}$$

Note that the complex conjugate can be applied to either term arbitrarily because we are concerned with average power, for which:

$$\mathrm{Re} F^* G = \mathrm{Re} F G^* \tag{103}$$

where Re denotes the takeing the real part, and F, G are arbitrary functions. Rewriting (102) as:

$$\mathcal{E}_{2\to1}^*(\omega) I_1(\omega) + \mathcal{E}_{1\to2}(\omega) I_2^*(\omega) = 0 \tag{104}$$

In the above, we keep the taking of the real part implicit. Alternatively,

$$\oint_{C_1} E_2^*(\omega) \cdot d\boldsymbol{l} I_1(\omega) + \oint_{C_2} \boldsymbol{E}_1(\omega) \cdot dl I_2^*(\omega) = 0 \tag{105}$$

Or

$$-\oint_{C_1} E_2^*(\omega) \cdot I_1(\omega) d\boldsymbol{l} = \oint_{C_2} \boldsymbol{E}_1(\omega) \cdot I_2^*(\omega) dl \tag{106}$$

where

$$\boldsymbol{E}_1 = -\frac{\partial}{\partial t} \boldsymbol{A}_1 = -j\omega \boldsymbol{A}_1 \tag{107}$$

$$\boldsymbol{E}_2 = -\frac{\partial}{\partial t} \boldsymbol{A}_2 = -j\omega \boldsymbol{A}_2 \tag{108}$$

Considering the transformation

$$\oint_C \cdots I dl \to \int_{V_1} \cdots \boldsymbol{J} dV \tag{109}$$

we obtain

$$-\int_{V_1} \boldsymbol{E}_2^*(\omega)\cdot \boldsymbol{J}_1(\omega)dV = \int_{V_2} \boldsymbol{E}_1(\omega)\cdot \boldsymbol{J}_2^*(\omega)dV \qquad (110)$$

The above is the mutual energy theorem [5]. We see that in this problem, the above was derived from the energy conservation of the transformer's primary and secondary coils. Therefore, it is indeed the law of conservation of energy. The left side represents the power provided by the primary coil current overcoming the electric field of the secondary coil. The right side represents the power received by the secondary coil current from the electric field of the primary coil. The equality of the left and right sides indicates an ideal transformer.

3.3 Advanced Waves are Physically Objective

From (104), we know the law of conservation of energy is:

$$\mathcal{E}_{2\to1}^*(\omega)I_1(\omega) + \mathcal{E}_{1\to2}(\omega)I_2^*(\omega) = 0 \qquad (111)$$

Considering

$$\mathcal{E}_{2\to1} = -j\omega M_{2\to1}I_2 \qquad (112)$$

$$\mathcal{E}_{1\to2} = -j\omega M_{1\to2}I_1 \qquad (113)$$

Thus,

$$(-j\omega)^* M_{2\to1}^* I_2^* I_1(\omega) + (-j\omega)M_{1\to2}I_1I_2^*(\omega) = 0 \qquad (114)$$

Simplifying:

$$M_{2\to1}^* = M_{1\to2} \qquad (115)$$

The above is still an expression of the law of conservation of energy. But considering

$$M_{1\to2} = \frac{\mu_0}{4\pi}\oint_{C_2}\oint_{C_1}\frac{d\boldsymbol{l}_1\cdot d\boldsymbol{l}_2}{r} \qquad (116)$$

$$M_{2\to1} = \frac{\mu_0}{4\pi}\oint_{C_1}\oint_{C_2}\frac{d\boldsymbol{l}_2\cdot d\boldsymbol{l}_1}{r} \qquad (117)$$

Since $M_{1\to2}$ and $M_{2\to1}$ are real numbers and the order of integration can be interchanged, we have:

$$M_{1\to2} = M_{2\to1} \qquad (118)$$

From the above and considering $M_{1\to2}, M_{2\to1}$ are real, equation (115) is self-evident. This further verifies that equation (111) is an identity, i.e., the law of conservation of energy.

If we move coil 2 to a location far from coil 1, the retardation effect exp(-jkr) must be considered:

$$M_{1\to2} = \frac{\mu_0}{4\pi}\oint_{C_2}\oint_{C_1}\exp(-jkr)\frac{d\boldsymbol{l}_1\cdot d\boldsymbol{l}_2}{r} \qquad (119)$$

In this case, assuming the law of conservation of energy still holds, which requires equation (115) to hold, further requires:

$$M_{2\to1} = \frac{\mu_0}{4\pi}\oint_{C_1}\oint_{C_2}\exp(+jkr)\frac{d\boldsymbol{l}_2\cdot d\boldsymbol{l}_1}{r} \qquad (120)$$

The inner integrals in equations (119, 120) imply:

$$\boldsymbol{A}_1 = \oint_{C_1}\exp(-jkr)\frac{d\boldsymbol{l}_1}{r} \qquad (121)$$

$$\boldsymbol{A}_2 = \oint_{C_2}\exp(+jkr)\frac{d\boldsymbol{l}_2}{r} \qquad (122)$$

This indicates that if coil 1 produces a retarded wave, coil 2 should produce an advanced wave. This also shows that advanced waves exist! The existence of advanced waves further ensures that the mutual energy flow theorem is not merely a reciprocity theorem but also an energy theorem. It is worth mentioning that the author's method above is very close to Lorenz's method for proposing retarded potentials [15]. Lorenz's retarded potential method is equivalent to Maxwell's displacement current method. Lorenz's retarded potential method was completed in 1867, slightly later than the introdaction of the Maxwell's equations. This method should be easier to understand than Maxwell's displacement current. It also provides the retarded potential general solution to Maxwell's equations (Maxwell could only solve the special case where the radiation source was zero in his equations). Although the author's method is similar to Lorenz's, it goes a step further by proving that advanced waves must exist; otherwise, the law of conservation of energy (111) or (80) would not hold.

Additionally, the author first proposed that the system composed of a transformer's primary and secondary coils is essentially the same as the system composed of a transmitting and receiving antenna; the only difference is the distance between the two coils. For a transformer, the two coils are very close. For an antenna system, the two coils are farther apart. They should obey the same physical laws. Traditional electromagnetic theory completely regards these two systems as different entities, which is entirely wrong. This also shows that the two-source electromagnetic system proposed in this paper has advantages over the traditional one-source electromagnetic theory.

3.4 First Revision of the Mutual Energy Flow

We have previously obtained the mutual energy flow theorem:

$$-\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1)dV = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*)dV \tag{123}$$

However, this mutual energy flow theorem is clearly problematic! The Poynting vectors:

$$\boldsymbol{S}_{11} = \boldsymbol{E}_1 \times \boldsymbol{H}_1^* \tag{124}$$

$$\boldsymbol{S}_{22} = \boldsymbol{E}_2 \times \boldsymbol{H}_2^* \tag{125}$$

The mutual energy flow density is:

$$\boldsymbol{S}_m = \boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1 \tag{126}$$

is certainly problematic. Because both $\boldsymbol{S}_m$ and $\boldsymbol{S}_{11}$ are supposed to transfer electromagnetic energy. The author intends to replace $\boldsymbol{S}_1$ with $\boldsymbol{S}_m$. $\boldsymbol{S}_1$ has only one term, $\boldsymbol{E}_1 \times \boldsymbol{H}_1^*$, while $\boldsymbol{S}_m$ has two terms, $\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1$. Taking the transformer we studied as an example,

$$|\boldsymbol{E}_1| \sim |\boldsymbol{E}_2| \tag{127}$$

$$|\boldsymbol{H}_1| \sim |\boldsymbol{H}_2| \tag{128}$$

Therefore, $|\boldsymbol{S}_m|$ is approximately twice $|\boldsymbol{S}_1|$. Hence, we need to compress the magnitude of the mutual energy flow density to half:

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \tag{129}$$

This is the so-called normalization of the mutual energy flow, completed by the author in 2020 [6]. It took the author about one year to figure out this $\frac{1}{2}$ factor. This $\frac{1}{2}$ factor is necessary. Now consider the impact of this normalization factor on Maxwell's equations. The original mutual energy theorem was:

$$-\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1)dV = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*)dV \tag{130}$$

If we update Maxwell's equations to:

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = 2\boldsymbol{J} + j\omega\epsilon_0 \boldsymbol{E} \end{cases} \tag{131}$$

The above indicates we have made the following transformation in Maxwell's equations:

$$\boldsymbol{J} \to 2\boldsymbol{J} \tag{132}$$

Substituting this into (130) gives:

$$-\int_V (\boldsymbol{E}_2^* \cdot 2\boldsymbol{J}_1)dV = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot 2\boldsymbol{J}_2^*)dV \tag{133}$$

Or

$$-\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1)dV = \frac{1}{2}\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \frac{1}{2}\int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*)dV \tag{134}$$

Of course, (131) can also be rewritten as:

$$\begin{cases}\nabla\times\frac{1}{2}\boldsymbol{E}=-j\omega\mu_0\frac{1}{2}\boldsymbol{H}\\ \nabla\times\frac{1}{2}\boldsymbol{H}=\boldsymbol{J}+j\omega\epsilon_0\frac{1}{2}\boldsymbol{E}\end{cases}\tag{135}$$

This is equivalent to the transformation:

$$\boldsymbol{E},\boldsymbol{H}\rightarrow\frac{1}{2}\boldsymbol{E},\frac{1}{2}\boldsymbol{H}\tag{136}$$

Substituting this into (130) gives:

$$-\int_V(\frac{1}{2}\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=\oiint_\Gamma(\frac{1}{2}\boldsymbol{E}_1\times\frac{1}{2}\boldsymbol{H}_2^*+\frac{1}{2}\boldsymbol{E}_2^*\times\frac{1}{2}\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=\int_V(\frac{1}{2}\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*)dV\tag{137}$$

Or

$$-\int_V(\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=\frac{1}{2}\oiint_\Gamma(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=\int_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*)dV\tag{138}$$

The above is the first revised mutual energy theorem. This revision compresses the mutual energy flow to half its original value. Compressing the mutual energy flow density by half actually requires a modification to Maxwell's equations as in (131). This modification implies that a current generates half retarded and half advanced waves. This is consistent with Wheeler-Feynman's absorber theory [16, 17] and Dirac's self-force problem [18].

3.5 Second Revision of the Mutual Energy Flow

The following two expressions represent the self-energy flow density,

$$\boldsymbol{S}_{11}=\boldsymbol{E}_1\times\boldsymbol{H}_1^*,\quad \boldsymbol{S}_{22}=\boldsymbol{E}_2\times\boldsymbol{H}_2^*\tag{139}$$

The following expression represents the mutual energy flow density,

$$\boldsymbol{S}_m=\frac{1}{2}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\tag{140}$$

The author's aim is to replace the self-energy flow with the mutual energy flow. However, once the mutual energy flow replaces the self-energy flow, the self-energy flow should no longer transfer energy. Otherwise, both the self-energy flow and the mutual energy flow would transfer energy simultaneously, and the total transferred energy would exceed the expected value.

In 2017, the author proposed the idea of the reverse collapse of the self-energy flow [11]. However, this viewpoint also encounters problems. First, it requires the introduction of a time-reversed wave, which complicates the problem. Moreover, the author was unable to convince others, or even himself, that the self-energy flow truly collapses in reverse. Furthermore, if the reverse-collapsed electromagnetic wave also constitutes a mutual energy flow, this mutual energy flow would necessarily cancel the actual mutual energy flow, rendering the system unsolvable. Therefore, the reverse collapse is not a good solution.

At the same time as proposing reverse collapse, the author also considered that perhaps the self-energy flow corresponds to reactive power, in which case it would not transfer energy. That is, the self-energy flow satisfies

$$\begin{cases}\mathrm{Re}\oiint_\Gamma(\boldsymbol{E}_1\times\boldsymbol{H}_1^*)\cdot\hat{n}d\Gamma=0\\ \mathrm{Re}\oiint_\Gamma(\boldsymbol{E}_2\times\boldsymbol{H}_2^*)\cdot\hat{n}d\Gamma=0\end{cases}\tag{141}$$

However, Maxwell's electromagnetic theory does not support the above formulas. According to Maxwell's theory, we have

$$\begin{cases}\mathrm{Re}\oiint_\Gamma(\boldsymbol{E}_1\times\boldsymbol{H}_1^*)\cdot\hat{n}d\Gamma\neq0\\ \mathrm{Re}\oiint_\Gamma(\boldsymbol{E}_2\times\boldsymbol{H}_2^*)\cdot\hat{n}d\Gamma\neq0\end{cases}\tag{142}$$

Therefore, at that stage the author had no choice but to consider the idea of the reverse collapse of the self-energy flow. Around 2022, however, the author began to consider that Maxwell's electromagnetic theory itself might be flawed. The self-energy flow is inherently reactive power, namely, for electromagnetic waves

$$\mathrm{Re}\boldsymbol{E}_1 \times \boldsymbol{H}_1^* = 0, \quad \mathrm{Re}\boldsymbol{E}_2 \times \boldsymbol{H}_2^* = 0 \tag{143}$$

This further requires

$$H_1 = \pm j E_1, \quad H_2 = \pm j E_2 \tag{144}$$

3.6 Correction of the Magnetic Field in Maxwell's Electromagnetic Theory

Around 2022, the author began to study the definition of the magnetic field. According to Maxwell's electromagnetic theory, for electromagnetic waves, the magnetic field and the electric field are in phase. However, from the discussion in the previous section, the author has already realized that the magnetic field and the electric field should maintain a 90° phase difference, that is, with a phase factor of $\pm j$.

Therefore, the author considered whether the definition of the magnetic field in electromagnetic waves and in the quasi-static case are consistent. Under quasi-static conditions, the magnetic field is

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^3} dV \tag{145}$$

In the radiation case, the magnetic field is

$$\boldsymbol{B}^{(r)} = \nabla \times \boldsymbol{A}^{(r)} \tag{146}$$

The frequency-domain retarded potential is

$$\boldsymbol{A}^{(r)} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r} \exp(-jkr) dV \tag{147}$$

Thus,

$$\boldsymbol{B}^{(r)} = \frac{\mu_0}{4\pi}\int_V \nabla\left(\frac{1}{r}\exp(-jkr)\right) \times \boldsymbol{J}\, dV \tag{148}$$

Hence,

$$\boldsymbol{B}^{(r)} = \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \left(\frac{\boldsymbol{r}}{r^3} + jk\hat{r}\frac{1}{r}\right)\exp(-jkr)\, dV \tag{149}$$

The author's viewpoint is that it should satisfy

$$\lim_{kr\to 0} \boldsymbol{B}^{(r)} = \boldsymbol{B} \tag{150}$$

The above equation states that the radiation magnetic field should degenerate into the quasi-static magnetic field where $kr \to 0$. However,

$$\lim_{kr\to 0} \boldsymbol{B}^{(r)} = \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \left(\frac{\boldsymbol{r}}{r^3} + jk\hat{r}\frac{1}{r}\right) dV$$

$$= \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \left(\frac{\boldsymbol{r}}{r^3}\right) dV + jk\frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^2} dV$$

$$= \boldsymbol{B} + jk\frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^2} dV \tag{151}$$

Therefore

$$\lim_{kr\to 0} \boldsymbol{B}^{(r)} \neq \boldsymbol{B} \tag{152}$$

Therefore, the radiation magnetic field $\boldsymbol{B}^{(r)}$ cannot degenerate into the quasi-static magnetic field $\boldsymbol{B}$. Thus, $\boldsymbol{B}$ and $\boldsymbol{B}^{(r)}$ are completely different entities. The author obtained the above formula around 2022, and upon seeing this result, his heart nearly jumped out of his chest. He realized this was an extremely important discovery: $\boldsymbol{B}^{(r)}$ and $\boldsymbol{B}$ are fundamentally not the same!

In addition, the author discovered that

$$\lim_{kr\to 0} \boldsymbol{E}_I^{(r)} = \boldsymbol{E}_I$$

where

$$\boldsymbol{E}_I^{(r)} = -j\omega \boldsymbol{A}^{(r)}, \quad \boldsymbol{E}_I = -j\omega \boldsymbol{A}$$

There are

$$\lim_{kr\to 0} \boldsymbol{E}_S^{(r)} \neq \boldsymbol{E}_S$$

where

$$\boldsymbol{E}_S^{(r)} = -\nabla\phi^{(r)}, \quad \boldsymbol{E}_S = -\nabla\phi$$

Thus, for the electric field, the retarded inductive part $\boldsymbol{E}_I^{(r)}$ can completely degenerate into the quasi-static inductive field $\boldsymbol{E}_I$. However, the retarded static part $\boldsymbol{E}_S^{(r)}$ cannot degenerate into the quasi-static static field. Nevertheless, for electromagnetic waves, we usually neglect $\boldsymbol{E}_S^{(r)}$. Therefore, we can consider

$$\lim_{kr\to 0} \boldsymbol{E}^{(r)} = \boldsymbol{E}$$

That is, we may regard $\boldsymbol{E}^{(r)}$ as degenerating into $\boldsymbol{E}$. With this, we can write Maxwell's equations in the frequency domain as

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H}_{Maxwell} \\ \nabla \times \boldsymbol{H}_{Maxwell} = 2\boldsymbol{J} + j\omega\epsilon_0 \boldsymbol{E} \end{cases} \tag{153}$$

Here, the magnetic field is denoted with the subscript Maxwell, because the author believes that the magnetic field derived from Maxwell's equations is not the true magnetic field. Instead,

$$\boldsymbol{B}_{Maxwell}^{(r)} = \nabla \times \boldsymbol{A}^{(r)}$$

$$= \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \left(\frac{\boldsymbol{r}}{r^3} + jk\hat{r}\frac{1}{r}\right)\exp(-jkr)dV$$

The author speculates that the correct magnetic field should be

$$\boldsymbol{B}^{(r)} = \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \left(\frac{\boldsymbol{r}}{r^3} + (-j)jk\hat{r}\frac{1}{r}\right)\exp(-jkr)dV$$

$$= \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \left(\frac{\boldsymbol{r}}{r^3} + k\hat{r}\frac{1}{r}\right)\exp(-jkr)dV$$

The basis of this speculation is that the author believes the radiated electromagnetic field (the second term of above formula) should maintain the same phase as the static magnetic field (the first term). The above discussion only considered the retarded wave, but in fact the advanced wave should also be discussed. Combining the retarded and advanced waves together yields the results,

$$\boldsymbol{H}_f^{(r)} = (-j)\boldsymbol{H}_{Maxwell\,f}^{(r)} \tag{154}$$

$$\boldsymbol{H}_f^{(a)} = (j)\boldsymbol{H}_{Maxwell\,f}^{(a)} \tag{155}$$

and

$$\boldsymbol{H}_n^{(r)} = \boldsymbol{H}_{Maxwell\ n}^{(r)} \tag{156}$$

$$\boldsymbol{H}_n^{(a)} = \boldsymbol{H}_{Maxwell\ n}^{(a)} \tag{157}$$

The subscript n denotes the near field, f denotes the far field, the superscript (r) denotes the retarded case, and (a) denotes the advanced case. The subscript Maxwell indicates that this field is obtained according to Maxwell's equations, but it is not the correct field. Considering that quantities with subscript 1 are retarded waves, and those with subscript 2 are advanced waves, we have

$$\boldsymbol{H}_{1f} = (-j)\boldsymbol{H}_{1Maxwell\,f} \tag{158}$$

$$\boldsymbol{H}_{2f} = (j)\boldsymbol{H}_{2Maxwell\,f} \tag{159}$$

Since the mutual energy flow only involves the far field, we neglect all near fields,

$$\boldsymbol{H}_{1} = (-j)\boldsymbol{H}_{1Maxwell} \tag{160}$$

$$\boldsymbol{H}_{2} = (j)\boldsymbol{H}_{2Maxwell} \tag{161}$$

Thus, the previously obtained mutual energy flow theorem should be written as:

$$-\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1)dV$$

$$= \frac{1}{2}\oiint_\Gamma(\boldsymbol{E}_1 \times \boldsymbol{H}^*_{Maxwell\ 2} + \boldsymbol{E}_2^* \times \boldsymbol{H}_{Maxwell\ 1}) \cdot \hat{n}d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*)dV \tag{162}$$

The mutual energy flow density obtained according to Maxwell's equations is rewritten as:

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}^*_{Maxwell\ 2} + \boldsymbol{E}_2^* \times \boldsymbol{H}_{Maxwell\ 1}) \tag{163}$$

This mutual energy flow density is still incorrect! Because Maxwell's electromagnetic theory is problematic. The mutual energy flow density derived from Maxwell's electromagnetic theory is also problematic. This mutual energy flow density should be corrected. After correction, it becomes:

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \tag{164}$$

Since the mutual energy flow density is corrected, the author revises the mutual energy flow theorem. The revised mutual energy flow theorem is:

$$-\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1)dV = \frac{1}{2}\oiint_\Gamma(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*)dV \tag{165}$$

The above is no longer just the mutual energy theorem; because self-energy flow does not transfer energy, mutual energy is the only energy transfer. Therefore, this formula is also the law of conservation of energy.

The mutual energy flow density given by (164) is actually the energy flow density of a photon. Therefore, $\boldsymbol{S}_m$ in (164) can be regarded as the photon itself.

Note that formula (165) is already different from formula (138). The electric and magnetic fields in (138) are obtained according to Maxwell's electromagnetic theory. Formula (165) calculates the electromagnetic fields according to the author's electromagnetic theory. (165) is not derived from Maxwell's electromagnetic theory at all. Formula (138) is mainly obtained according to Maxwell's electromagnetic theory, except for the $\frac{1}{2}$ factor. The $\frac{1}{2}$ factor is also the author's contribution.

It is worth mentioning that formula (165) also holds under quasi-static conditions. Therefore, formula (165) can be considered to be obtained in quasi-static electromagnetic theory and then generalized to radiating electromagnetic field theory. Since this formula maintains the same form in both radiating electromagnetic field theory and quasi-static electromagnetic field theory, it can be used as an axiom of electromagnetic field theory.

4 Example of Photon Mutual Energy Flow

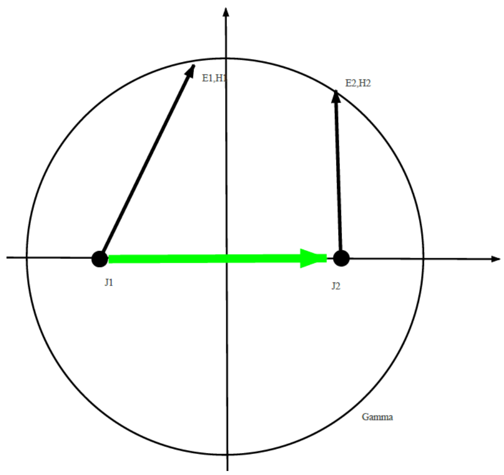

Figure 4: Inside Boundary Γ, there are two Sources $\boldsymbol{J}_1$, $\boldsymbol{J}_2$. $\boldsymbol{J}_1$ is the Source Generating the retarded Wave $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]^T$. $\boldsymbol{J}_2$ is the Sink Generating the advanced wave, $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]^T$. The green arrow is the mutual energy flow. The mutual energy flow is generated at $\boldsymbol{J}_1$ and annihilated at $\boldsymbol{J}_2$.

Assume an electric dipole current element at x=0, y=0, z=0, with dipole moment direction $\hat{z}$, i.e., See Figure 4.

$$\boldsymbol{J}_1 = \boldsymbol{J}_{10}\exp(j\omega t)\delta(x-0)\delta(y-0)\delta(z-0)\Delta l\hat{z} \tag{166}$$

A second dipole is at $x = L$, $y = 0$, $z = 0$,

$$\boldsymbol{J}_2 = \boldsymbol{J}_{20}\exp(j\omega t)\delta(x-L)\delta(y-0)\delta(z-0)\Delta l\hat{z} \tag{167}$$

$\boldsymbol{J}_2$ is not arbitrary; it is influenced by $\boldsymbol{J}_1$. The field generated by $\boldsymbol{J}_1$ is,

$$\boldsymbol{E}_1^{(r)} = -j\omega\boldsymbol{A}_1^{(r)}$$

$$= -j\omega\int_V \frac{\boldsymbol{J}_1}{r}\exp(-jkr)dV$$

$$= -j\omega\int_V \frac{\boldsymbol{J}_{10}\exp(j\omega t)\delta(x-0)\delta(y-0)\delta(z-0)\Delta l\hat{z}}{r}\exp(-jkr)dV$$

$$= -j\omega\boldsymbol{J}_{10}\frac{\exp(j\omega t)\Delta l\hat{z}}{r}\exp(-jkr) \tag{168}$$

The distance attenuation factor $\frac{1}{r}$ in the above can be omitted. This is because the retarded and advanced waves interfere after synchronization. The interference strengthens the electromagnetic wave along the line connecting the source and sink, while weakening the wave amplitude in other directions due to interference. So

$$\boldsymbol{E}_1^{(r)} \to j\omega J_{10}\Delta l(-\hat{z})\exp(-jkx) \tag{169}$$

Thus,

$$\boldsymbol{E}_1 \sim jJ_{10}(-\hat{z})\exp(-jkx) \tag{170}$$

"~" indicates proportionality or having the same phase. Considering ($-\hat{z}$) is the direction of $\boldsymbol{E}_1$. So

$$E_1 = jJ_{10}\exp(-jkx) \tag{171}$$

Considering

$$\boldsymbol{J}_2 \sim \boldsymbol{E}_1|_{x=L} = jJ_{10}(-\hat{z})\exp(-jkL) \tag{172}$$

Comparing with (170)

$$\boldsymbol{E}_2 \sim jJ_{20}(-\hat{z})\exp(-jk(x-L)) \tag{173}$$

We have already ignored the distance attenuation factor $\frac{1}{r}$ because the retarded wave acts as a waveguide for the advanced wave, and the advanced wave acts as a waveguide for the retarded wave. Interference strengthens along the line connecting the source and sink. Considering

$$if \quad x < L, \quad then \quad |x-L| = -(x-L) \tag{174}$$

So

$$\boldsymbol{E}_2 \sim -j\boldsymbol{J}_2\exp(+jk|x-L|) \tag{175}$$

where $\boldsymbol{J}_2 = J_{20}$ ($\hat{z}$), considering (172)

$$\boldsymbol{E}_2 \sim -j(jJ_{10}(-\hat{z})\exp(-jkL))\exp(+jk|x-L|) \tag{176}$$

Hence,

$$\boldsymbol{E}_2 \sim -j(J_{10}j(-\hat{z})\exp(-jkL))\exp(-jk(x-L))$$

$$= -j(J_{10}j(-\hat{z})\exp(-jkx) \tag{177}$$

Considering the direction of $\boldsymbol{E}_2$ is ($-\hat{z}$), so

$$E_2 = J_{10}\exp(-jkx) \tag{178}$$

So we obtain,

$$E_1 = jJ_{10}\exp(-jkx) \tag{179}$$

$$E_2 = J_{10}\exp(-jkx) \tag{180}$$

Considering Faraday's law

$$\nabla \times \boldsymbol{E} = -j\omega\mu_0\boldsymbol{H}_{Maxwell} \tag{181}$$

Or

$$\nabla \times \boldsymbol{E} \sim -j\boldsymbol{H}_{Maxwell} \tag{182}$$

Or

$$\boldsymbol{H}_{Maxwell} \sim j\nabla \times \boldsymbol{E} \tag{183}$$

So

$$\boldsymbol{H}_{Maxwell\ 1} \sim j\nabla \times \boldsymbol{E}_1 \tag{184}$$

So

$$\boldsymbol{H}_{Maxwell\ 1} \sim j\nabla E_1 \times (-\hat{z}) \tag{185}$$

Similarly

$$\boldsymbol{H}_{Maxwell\ 2} \sim j\nabla E_2 \times (-\hat{z}) \tag{186}$$

$\boldsymbol{E}_1$, $\boldsymbol{H}_1$ are retarded waves for $x > 0$, and advanced waves for $x < 0$,

$$\boldsymbol{H}_1 = -j\boldsymbol{H}_{Maxwell\ 1}\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases}$$

$$= -jj\nabla E_1 \times (-\hat{z})\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases}$$

$$= \nabla E_1 \times (-\hat{z}) \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$= \frac{d}{dx} E_1 \hat{x} \times (-\hat{z}) \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$= \frac{d}{dx} E_1 \hat{y} \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases} \quad (187)$$

$\boldsymbol{E}_2$, $\boldsymbol{H}_2$ are retarded waves for $x > L$, and advanced waves for $x \leq L$,

$$\boldsymbol{H}_2 = -j\boldsymbol{H}_{Maxwell\ 2} \begin{cases} -1 & x \leq L \\ 1 & x > L \end{cases}$$

$$= -jj\nabla E_2 \times (-\hat{z}) \begin{cases} -1 & x \leq L \\ 1 & x > L \end{cases}$$

$$= \nabla E_2 \times (-\hat{z}) \begin{cases} -1 & x \leq L \\ 1 & x > L \end{cases}$$

$$= \frac{d}{dx} E_2 \hat{x} \times (-\hat{z}) \begin{cases} -1 & x \leq L \\ 1 & x > L \end{cases}$$

$$= \frac{d}{dx} E_2 \hat{y} \begin{cases} -1 & x \leq L \\ 1 & x > L \end{cases} \quad (188)$$

So

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1)$$

$$= \frac{1}{2}\hat{x}(E_1 H_2^* + E_2^* H_1)$$

$$\sim \frac{1}{2}\hat{x} \begin{cases} j\exp(-jkx)(-\frac{d}{dx}E_2)^* + \exp(-jkx)^*(-\frac{d}{dx}E_1) & x < 0 \\ j\exp(-jkx)(-\frac{d}{dx}E_2)^* + \exp(-jkx)^*(\frac{d}{dx}E_1) & 0 \leq x \leq L \\ j\exp(-jkx)(\frac{d}{dx}E_2)^* + \exp(-jkx)^*(\frac{d}{dx}E_1) & L < x \end{cases}$$

$$= \frac{1}{2}\hat{x} \begin{cases} j\exp(-jkx)(-\frac{d}{dx}\exp(-jkx))^* + \exp(-jkx)^*(-\frac{d}{dx}j\exp(-jkx)) & x < 0 \\ j\exp(-jkx)(-\frac{d}{dx}\exp(-jkx))^* + \exp(-jkx)^*(\frac{d}{dx}j\exp(-jkx)) & 0 \leq x \leq L \\ j\exp(-jkx)(\frac{d}{dx}\exp(-jkx))^* + \exp(-jkx)^*(\frac{d}{dx}j\exp(-jkx)) & L < x \end{cases}$$

$$= \frac{1}{2}\hat{x} \begin{cases} j\exp(-jkx)(jk\exp(-jkx))^* + \exp(-jkx)^*(-(-jk)j\exp(-jkx)) & x < 0 \\ j\exp(-jkx)(jk\exp(-jkx))^* + \exp(-jkx)^*((-jk)j\exp(-jkx)) & 0 \leq x \leq L \\ j\exp(-jkx)(-jk\exp(-jkx))^* + \exp(-jkx)^*((-jk)j\exp(-jkx)) & L < x \end{cases}$$

$$= \frac{1}{2}k\hat{x} \begin{cases} j\exp(-jkx)(j\exp(-jkx))^* + \exp(-jkx)^*(jj\exp(-jkx)) & x < 0 \\ j\exp(-jkx)(j\exp(-jkx))^* + \exp(-jkx)^*(-jj\exp(-jkx)) & 0 \leq x \leq L \\ j\exp(-jkx)(-j\exp(-jkx))^* + \exp(-jkx)^*(-jj\exp(-jkx)) & L < x \end{cases}$$

$$= \frac{1}{2}k\hat{x} \begin{cases} j(j)^* + jj & x < 0 \\ j(j)^* - jj & 0 \leq x \leq L \\ j(-j)^* - jj & L < x \end{cases}$$

$$=\frac{1}{2}k\hat{x}\begin{cases}1-1 & x<0\\1+1 & 0\le x\le L\\-1+1 & L<x\end{cases}$$

$$=k\hat{x}\begin{cases}0 & x<0\\1 & 0\le x\le L\\0 & L<x\end{cases} \tag{189}$$

Hence

$$\hbar\boldsymbol{S}_m\sim\hbar k\hat{x}\begin{cases}0 & x<0\\1 & 0\le x\le L\\0 & L<x\end{cases}=\boldsymbol{p}\begin{cases}0 & x<0\\1 & 0\le x\le L\\0 & L<x\end{cases} \tag{190}$$

$\hbar\boldsymbol{S}_m$ is generated at $x = 0$ and annihilated at $x = L$. $\hbar\boldsymbol{S}_m$ is the photon. The momentum of the $\hbar\boldsymbol{S}_m$ mutual energy flow density is p, see Fig. 5.

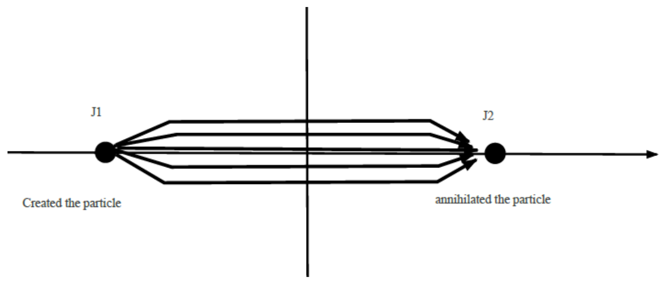

Figure 5: The Mutual Energy Flow is Generated at $\boldsymbol{J}_1$ and Annihilated at the sink $\boldsymbol{J}_2$. The Mutual Energy flow is Actually a Particle

$$\boldsymbol{S}_{11}=\boldsymbol{E}_1\times\boldsymbol{H}_1^*$$

$$=(E_1(-\hat{z}))(\frac{d}{dx}E_1\hat{y}\begin{cases}-1 & x<0\\1 & x\ge 0\end{cases})^*$$

$$=E_1(-jkE_1)^*\hat{x}\begin{cases}-1 & x<0\\1 & x\ge 0\end{cases}$$

$$\sim j\hat{x}\begin{cases}-1 & x<0\\1 & x\ge 0\end{cases} \tag{191}$$

Only generated, not annihilated. $\boldsymbol{S}_{11}$ is reactive power. It does not transfer energy.

$$\boldsymbol{S}_{22}=\boldsymbol{E}_2\times\boldsymbol{H}_2$$

$$=(E_2(-\hat{z}))(\frac{d}{dx}E_2\hat{y}\begin{cases}-1 & x<L\\1 & x\ge L\end{cases})^*$$

$$=E_2(-jkE_2)^*\hat{x}\begin{cases}-1 & x<L\\1 & x\ge L\end{cases}$$

$$\sim j\hat{x}\begin{cases}-1 & x<L\\1 & x\ge L\end{cases} \tag{192}$$

So $\boldsymbol{S}_{22}$ is only annihilated, not generated. $\boldsymbol{S}_{22}$ is reactive power. It does not transfer energy. The energy is transfered only through the mutual energy flow. The mutual energy flow is actually the photon.

5 Conclusion

In electromagnetic field theory, there are Poynting's theorem, Green's function, the reciprocity theorem, the mutual energy theorem, Huygens' principle, the mutual energy flow theorem, the mutual energy flow law, and the law of conservation of energy. This paper attempts to clarify the relationships between these theorems and hows in a single paper. Ultimately, these theorems and hows lead to the concept of the photon.

First came Maxwell's equations (1861-2), followed by Poynting's theorem (1884), the Lorentz reciprocity theorem in 1896 [8], Rumsey's action-reaction principle in 1954, Welch's time-domain reciprocity theorem in 1960 [1-4,8,12]. The author proposed the mutual energy theorem and Huygens' principle in 1987-1989 [5,9,10]. de Hoop's reciprocity theorem was proposed at the end of 1987 [19]. The mutual energy theorem, as a reciprocity theorem, has been independently discovered multiple times, for example, Petrusenko's so-called second Lorentz reciprocity theorem is one of them [20].

The mutual energy flow theorem was proposed by the author in 2017 [11]. Up to this point, all these theorems were still within the framework of Maxwell's equations.

However, contradictions still existed within the framework of Maxwell's equations, leading the author to propose the $\frac{1}{2}$ normalization factor in 2020 [6]. After 2022, the author revised Maxwell's electromagnetic theory, especially corrected the definition of the magnetic fields [7].

This paper particularly emphasizes the relationship between the vector Green's formula and the reciprocity theorem and the mutual energy theorem. It emphasizes the development process of understanding. We can say that Poynting's theorem is an electromagnetic energy theorem with only one source. The reciprocity theorem - mutual energy theorem is an electromagnetic theorem with two sources. For radiating electromagnetic phenomena, people began by studying radiation from a single source, then moved to studying two sources, i.e., the electromagnetic phenomena of a source and a sink, thus including both retarded and advanced waves. Recognizing that the reciprocity theorem is also an energy theorem is a significant progress. Only by introducing the concept of mutual energy could the concept of mutual energy flow be proposed later. Further study of mutual energy flow led to the realization that the mutual energy flow theorem should be the law of conservation of energy, and the idea that radiation should not overflow the universe. This further led to the discovery of a bug in Maxwell's electromagnetic theory. Solving this bug led to a complete resolution of the wave-particle duality problem.

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