Research Article

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Graphical Analysis of Sunspot Time Series 1700-2015

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Abstract
The aim of this research is to perform some graphical analysis for the sunspot time series 1700-2015. This analysis including correlation analysis, regression function analysis and spectral analysis.

Keywords: Sunspots, Autocorrelation, Regression Function, Spectral Density Function.

Introduction
Dark spots on the surface of the sun have been observed for nearly two thousand years. The astrophysical explanation for the formation of sunspots, as well as their periodicity, is still a subject of intensive research. The record of sunspot numbers reveals an intriguing cyclical phenomenon of an approximate 11-year period which has been challenging our intellect ever since Samual Schwake first announced the sunspot cycle in 1843 after spending seventeen years in painstaking observations. These data are regarded as a good indicator of the overall evolution of magnetic oscillation of the sun. They provide an important source of information concerning the fluid motions of the solar dynamo as well.

Solar activity, as indexed by sunspots, are observations describing the number of sunspots observed annually. This series is highly asymmetric and has not proved amenable to linear models even when transformed by any transformation. It has a cycle of length varying from 7 to 14 years.

Several models are fitted to this series in mathematical, statistical and solar physical literatures in order to capture their main features as well as prediction, Priestly (1988), Li (2001) [1].

In this paper we try to stud the main properties of the yearly average sunspot data 1700-2015. This series consists of 316 observations describing the number of sunspots observed annually between 1700 and 2015. The data taken from the Royal Observatory of Belgium, SILSO.

Some Graphical Results
The yearly average sunspot data 1700-2015 data is denoted by {x(n), n=1, 2, …, 316}. The time plot of this series is shown in Figure (1).

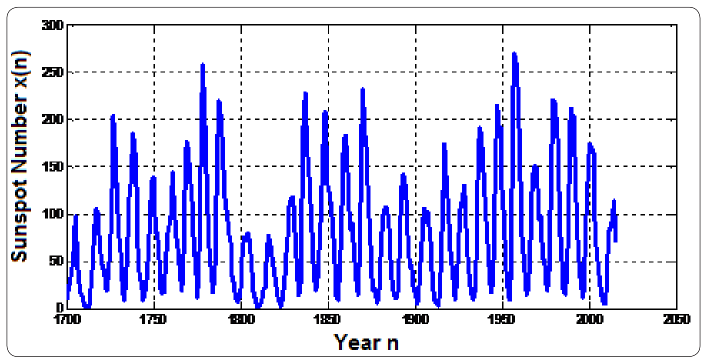

Figure 1: The time plot of sunspot data 1700-2015

It is clear that the data is highly asymmetric. The main feature of this series is a cycle of activity varying in duration between 9 and 14 years, with an average period of approximately 11 years. Obviously, there is 29 cycles from 1700 to 2015.

The series exhibits another feature, namely different gradients of "ascensions" and "descensions", i.e. in each cycle the rise to the maximum has steeper gradient than the fall to the next minimum. The descent periods has longer duration and greater variation than the ascend periods. The average ascend period is 3.7 years with standard deviation of 1.7 years, while the average descent period is 5.2 years with standard deviation of 2.4 years. The coefficients of variation in ascend and descent periods are nearly the same, about 46%.

It is well known that this series is not stationary: i.e. their probabilistic properties change over time. The probability distribution of this series is non-Gaussian. This is clear from the frequency histogram of this series and the three-dimensional histograms of [x(n-1), x(n)] and [x(n-2), x(n)] shown in the next figures.

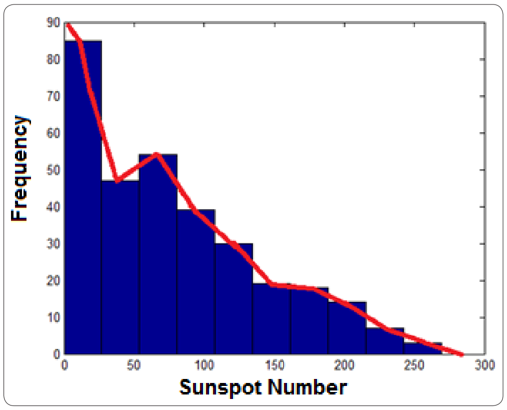

Figure 2: The frequency histogram of sunspot data 1700-2015

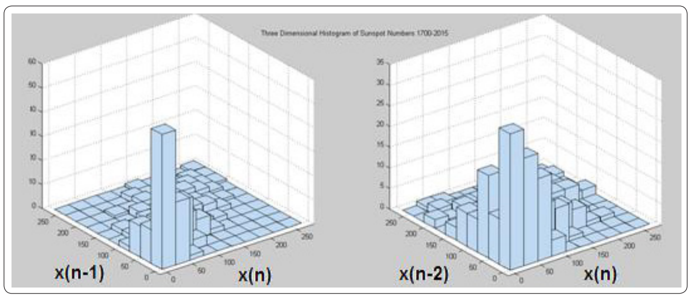

Figure 3: Three-dimensional histograms of sunspot data 1700-2015

It is clear from previous figures that there is dominant pick near the origin, known as the point of concentration, which cases serious problems in the analysis of this series.

Correlation Analysis

Next figure shows the autocorrelation function of this series together with the approximate upper and lower 3-segma limits, i.e. the confidence limits of the 99% confidence interval, the two parallel lines.

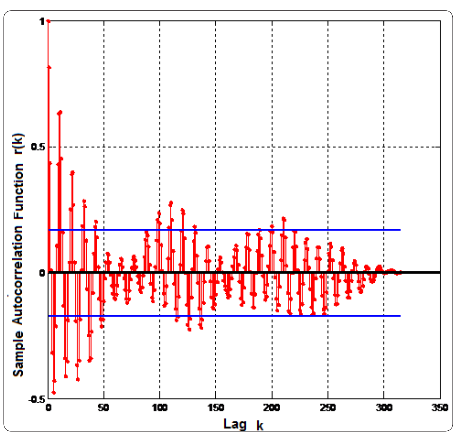

Figure 4: Autocorrelation function of sunspot data 1700-2015

It is clear that this time series is highly serially correlated with long memory. This means that the series needs a mathematical model with some delayed dependency. The following table shows the highest significance autocorrelations that lie outside 6-segma limits which are, i.e. the confidence limits of the 99.9999998% confidence interval.

Table 1: The significance autocorrelations that lie outside 6-segma limits

K	1	11	10	5	12	2	9	6	27	16	22	21
r(k)	0.82	0.64	0.63	-0.47	0.46	0.44	0.43	-0.43	-0.42	-0.41	0.40	0.39

Clearly lags 1, 10, 11are very effective with positive effect on present values of this series.

The periodicity of the autocorrelation function of this series in the last figure indicates that this series is periodic. The highest significance autocorrelations in last table are at lags near 11 and 22, this means that the periodicity of this series is near 11 year and their multiplicity.

Regression Function Analysis

A kernel smoothing approach can be used in order to estimate the functional relationship between the "present" x(n) on the "past" x(n-k), this is known as the regression function of x(n) on x(n-k), and denoted by g(k)=E[x(n)|x(n-k)]; k=1,2,…, n-1, Thanoon (2011) [2].

The following figure shows the estimated regression function of x(n) on x(n-k), k=1,2,3,…, 12, for sunspot data 1700-2015 using Bartlett window.

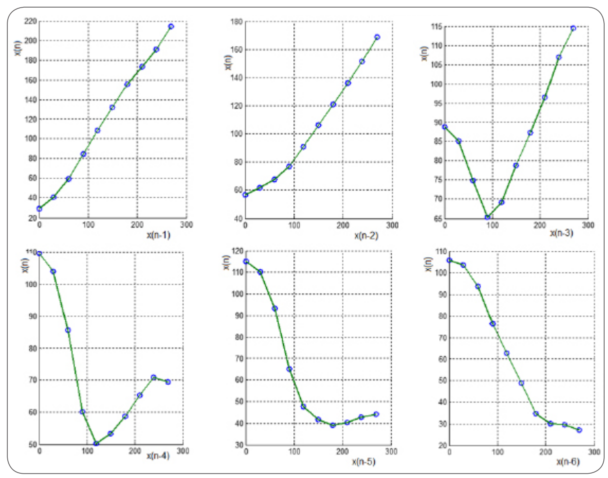

Figure 4a: g(k)=E[x(n)|x(n-k)], k=1,2,…, 6, for sunspot data 1700-2015.

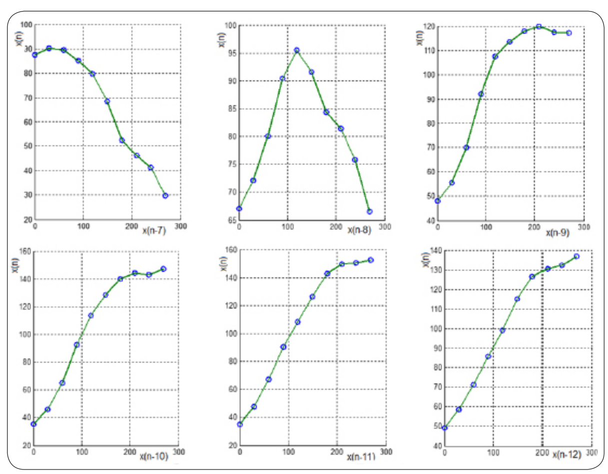

Figure 4b: g(k)=E[x(n)|x(n-k)], k=7,8, … , 12, for sunspot data 1700-2015.

It is clear that the relationship between x(n) and x(n-1) is positive and close to linear, then a change point near x(n-k)=100 appears for k=3, 4, 5 and then the relationship becomes negative at lag k=6. After that the relationship returns slowly lag after lag to become similar to that at lag k=1. This is another indication of the periodicity of this series.

Spectral Analysis

The normalized power of spectral density function of this series, based on a non-parametric method using Bartlett window, see, [e.g. Priestly (1981)], is shown in the following figure [3].

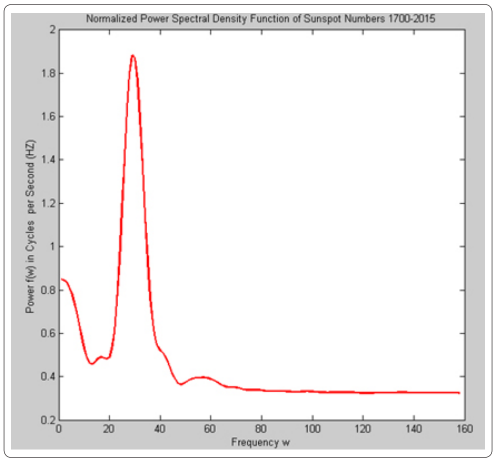

Figure 5: The normalized power spectral density function of sunspot data 1700- 2015.

It is clear that there is a dominant peak at the frequency of f*=29, this means that the period of this time series, according to spectral analysis technique, is:

$$Period = \frac{n}{f^*} = \frac{316}{29} = 10.8966 \ \ year.$$

Note that Olvera (2010) used periodogram analysis to estimate the sunspot cycle period for the modern sunspot cycle, the period is estimated to be 10.4883 years/cycle [4]. It is also important to note that there is no consensus of researchers on structure of the duration of the solar cycle: most researchers think that the period is 11 years, other researchers believe that the period is compound of multiple 11 years: e.g. 22 or 33 years.

Conclusions

This paper is mainly devoted to stud some properties of the yearly average sunspot data 1700-2015 using some graphical tools. This series is highly asymmetric and non-stationary and periodic. The cycle of activity varying in duration between 9 and 14 years, with an average period of approximately 11 years.

The correlation analysis indicated that this time series is highly serially correlated with long memory. lags 1, 10, 11 are very effective with positive effect on present values of this series. The highest significance autocorrelations are at lags near 11 and 22, which mean that the periodicity of this series is near 11 year and their multiplicity.

Another indications of the periodicity of this series are indicated by the regression function and the autocorrelation function.

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