

Goldbach’s Conjecture Proof by Induction

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Abstract

Its present the Goldbach’s Conjecture developing with mathematical induction using the number $\alpha = \frac{p^n \sqrt{\pi^m}}{q^n \sqrt{\pi^m}}$ to express any natural number obtained through sum of prime numbers.

Keywords: Goldbach

1. Introduction

The original phrase of Goldbach’s Conjecture was:

”dass jede Zahl, welche aus zweyen numeris primis zusammengesetzt ist, ein aggregatum so vieler numerorum primorum sey, als man will (die unitatem mit dazu gerechnet), bis auf die congeriem omnium unitatum”

-Every integer that can be written as the sum of two primes can also be written as the sum of as many primes as one wishes, until all terms are units-

Goldbach considering 1 to be prime number wrote a second conjecture:

... eine jede Zahl, die grösser ist als 2, ein aggregatum trium numerorum primorum sey.

-Every integer greater than 2 can be written as the sum of three primes.-

Considering that the conjecture could be established like next:

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \frac{p_3}{q_3} = \alpha$$

the term $\frac{p_i}{q_i}$ is a prime whenever $q_i = p_i$ or $q_i = 1$. The condition established on second Conjecture will be used.

2. Goldbach’s Conjecture with Three Prime Numbers

Let be $\alpha = \frac{p^n \sqrt{\pi^m}}{q^n \sqrt{\pi^m}}$

And lets define the **initial case** when $\alpha \geq 2$

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \frac{p_3}{q_3} = 2$$

Considering second Goldbach’s Conjecture we have:

If

$$p_1 \vee p_2 \vee p_3 = 0 \text{ and the other two} = 1$$

Or

$p_1 \vee p_2 \vee p_3$ any natural number and q_1, q_2 and q_3 the same number.

base case is fulfilled.

Now lets define the **induction step**

When

$\alpha = \frac{p^n \sqrt{\pi^m}}{q^n \sqrt{\pi^m}}$ representing any natural number when

$m = m = 0$ and $n = n = 1$ with $p|_q \in \mathbb{Z}$; and the three left terms prime mbers then we have (case $P(p = k); k \in \mathbb{N}$):

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \frac{p_3}{q_3} = \frac{k^n \sqrt{\pi^m}}{q^n \sqrt{\pi^m}}$$

If

$n = n = 1$ and $m = m = 0$ and $p = k + 1$

Substituting

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \frac{p_3}{q_3} = \frac{(k+1)^n \sqrt{\pi^m}}{q^n \sqrt{\pi^m}}$$

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \frac{p_3}{q_3} = \frac{k+1}{q} = \frac{k}{q} + \frac{1}{q} = k + 1$$

With $q = 1$ the conditions established on Second Goldbach 's Conjecture are satisfied.

3. Goldbach 's Conjecture with n Prime Numbers

With **initial case** being the same

Lets define the **inductive case** $P(p = t)$

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \dots + \frac{p_k}{q_k} = \frac{t^n \sqrt{\pi^m}}{q^m \sqrt{\pi^m}}$$

for $P(p = t + 1)$

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \dots + \frac{p_l}{q_l} = \frac{(t+1)^n \sqrt{\pi^m}}{q^m \sqrt{\pi^m}}$$

with

$n = n = 1$ and $m = m = 0$ and $q = 1$ we have

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \dots + \frac{p_l}{q_l} = t + 1$$

And even more we can add a term on left side to reach $l + 1$ hypothesis; such that

$$\frac{p_1}{q_1} + \frac{p_2}{q_2} + \dots + \frac{p_l}{q_l} + \frac{p_{l+1}}{q_{l+1}} = t + 1$$

assuming true inductive case $P(p = t)$ we have

$$t + \frac{p_{l+1}}{q_{l+1}} = t + 1$$

Like Goldbach assume 1 like a primer number; the term $\frac{p_{l+1}}{q_{l+1}}$ can be equal to 1.

References

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