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Geometric Mean Method Combined With Ant Colony Optimization Algorithm to Solve Multi-Objective Transportation Problems in Fuzzy Environments

Ekanayake E.M.U.S.B.

Department of Physical Sciences, Faculty of Applied Sciences, Rajarata University of Sri Lanka, Mihinthale, Sri Lanka

***Corresponding author**
Ekanayake E.M.U.S.B., Department of Physical Sciences, Faculty of Applied Sciences, Rajarata University of Sri Lanka, Mihinthale, Sri Lanka

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Abstract
The transportation problem (TP) is a well-known subject in the field of optimization and a very prevalent challenge for businesspeople. The goal is to reduce the total transportation cost, of delivering resources from sources to destinations. The literature demonstrates that various approaches have been designed with a single goal in mind, although TPs are not always developed with a bi-goal in mind. Solving transportation difficulties with several objectives is a common task. In this study, a new method for addressing multi-criteria TP using geometric means, along with a novel approach of the Ant Colony Optimization algorithm (ACO) for solving multi-objective TP in a fuzzy environment. Fuzzy numbers have been used to solve real-world problems in various domains, including operations research and optimization. The ACO Algorithm has long been recognized as a viable alternative strategy for solving optimization problems. The purpose of this study is to provide a unique approach for organizing fuzzy numbers as well as enhancements to the ACO algorithm for solving the Multi-Objective TP model. Our method, such as Geometric Mean Ant Colony Optimization Algorithm (GMACOA), outperforms other methods in terms of objective values. Numerical examples are provided to demonstrate the method in comparison to various current methods.

Key Words: Multicriteria distribution problem, Ant Colony Optimization algorithm, Geometric mean, Fuzzy environment.

Introduction

In addition, TP is one of the most important distribution problems in Operations Research (OR). OR has numerous uses in engineering, business, and government systems. Daily, it is also employed to solve problems in the manufacturing and service industries. The TP is a well-known optimization problem in OR that takes a single objective function into account; nevertheless, in real-world applications, two or more criteria are more important than any single criterion. When delivering a homogeneous product from a source to a destination, the decision-maker takes numerous aspects into account, including transportation costs, a fixed price for an open route, product delivery time, deterioration rate of commodities, and so on.

The TP treats many objective functions at the same time to accommodate the criteria [1]. Proposed the transportation problem originally, while Koopmans (1949) studied it in detail in the Optimal Utilization of Transportation System. On the other side, created efficient methods for discovering solutions, and Charnels, devised the stepping stone method. Furthermore, numerous researchers are working on this topic. In this paper, we look at various methods for solving a balanced and unbalanced transportation problem using fuzzy numbers and the ant colony algorithm [2, 3].

Fuzzy set theory has been used in a variety of domains, including OR, management science, and control theory. In real-world scenarios, supply, demand, and unit transportation costs are all uncertain [4]. Used fuzzy programming approaches to handle multi-objective linear programming issues. Several strategies for solving transportation problems in fuzzy environments are proposed in the literature, such as the concept of fuzzy set, which was introduced by in 1965. Bellman and discussed the concept of decision-making in a fuzzy environment [5, 6].

Many writers have researched fuzzy linear programming problem approaches since this pioneering work, including who demonstrated that solutions produced by fuzzy linear programming are always efficient, and among others. A fuzzy transportation problem is one in which the decision parameters are fuzzy integers. Chanas et al. studied various TP situations with interval and fuzzy parameters [7, 8]. The goal of the fuzzy transportation problem is to move some products from various sources to various destinations while incurring the least amount of fuzzy transportation costs and satisfying the fuzzy supply and demand requirements.

Presented a fuzzy compromised programming strategy for MOTP [9]. By giving weights to objectives, the decision maker's preferences are taken into account. Introduced a preference-based fuzzy GPA for solving a MOTP with fuzzy coefficients. They explain the fuzzy goal's membership function. This method converts membership functions into membership goals. The Euclidean distance function is utilized to provide the suitable preference structure of goals. K.B. Provided a review of the various techniques employed in MOTP. This document compiles all possible work on MOTP and provides an overview of several methodologies such as goal programming, fuzzy techniques, and evolutionary algorithms. Proposed a novel way to determine a fair MOTP solution. It is suggested in this strategy to create a sum of objectives. Patel et al (2018) proposed a new row maxima approach to solve MOTP. M. offered a product method to solve MOTP by utilizing a fuzzy membership function. Suggested a straightforward method for determining the optimal linear MOTP compromise solution. To solve MOTP, proposed the Matrix maxima approach with a Pareto optimality criterion. Used fuzzy programming to find the best compromise solution to a multi-objective transportation problem. Used the solving Multi-objective Transportation Problem by row maxima method. Proposed the Geometric Mean Method for Solving Multi-Objective Transportation Problems in Fuzzy Environments [10-18].

Furthermore, how ants can find the shortest paths between food sources and their colony. These ideas are based on ant behavior in the wild. This concept was created using the probabilistic technique known as finding good pathways via graphs. This is known as the Ant Colony Algorithm (ACA), and it was first presented by while traveling in this manner, the ants deposit a chemical compound known as a pheromone, which aids in communication among themselves. When finding the quickest path between food sources and their nest, they look for areas with high pheromone concentrations [19].

Because ants can detect pheromones and choose the most advantageous way. Dorigo, brought the concept of the ant system into the literature. The ant algorithm with elitist ants was proposed by. Following that, many writers researched ACA, including the max-min ant system, the ant algorithm with additional reinforcement, and the best-worst ant system, among others. Many optimization issues, including transportation challenges, have been solved using ant colony techniques [20-23].

In this research, we examine Geometric Mean Combined and numerous adaptations of the ant colony algorithm utilizing Ant Colony Algorithm to Solve Multi-Objective Transportation Problems in Fuzzy Environments to identify the optimal solution.

Preliminaries

In this section, some basic definitions of fuzzy numbers are presented. The Fuzzy set theory was first formulated by [24]. The following definitions of the fuzzy numbers and some operations on it may be useful [25].

Definition

A fuzzy set is a pair (X,μ_A) X is a set and μ_A: $X \to [0,1]$. For all $x \in X$, $\mu_A(X)$ is called the membership fuction of x. If $\mu_A(X)=1$, we say that x is **Fully Included** in (X,μ_A), and if $\mu_A(X)=0$, we say that x is **Not Included** in (X,μ_A). If there exists some $x \in X$ such that $\mu_A(X)=1$, we say that (X,μ_A) is **Normal**. Otherwise, we say that (X,μ_A) is **Subnormal.**

Definition Triangular fuzzy number

A triangular fuzzy number (TFN)$\tilde{A}=(a_1,a_2,a_3)$is FN with membership function

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & for\ x<a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & for\ a_1\le x\le a_2 \\ \dfrac{a_3-x}{a_3-a_1}, & for\ a_2\le x\le a_3 \\ 0, & for\ x\ge a_3 \end{cases}$$

The α-cut of the TFN given by,
$A_\alpha=[a_1+(a_2+a_1)\alpha, a_3+(a_2-a_3)\alpha] \in (0, 1]$.Where a_1,a_2 and a_3 be real numbers with $a_1 \le a_2 \le a_3$.

Definition Geometric mean

The geometric mean is a mean or average that reveals the center tendency or typical value of a set of numbers by multiplying their values together (as opposed to the arithmetic mean which uses their sum) [26, 27]. In general, the geometric mean is defined as the n^{th} root of the product of n numbers, i.e., for a given set of numbers, the geometric mean is the nth root of the product of n numbers $x_1,x_2,\ldots.x_n$, the geometric mean is defined as

$$(\prod_{i=1}^{n} x_i)^{\frac{1}{n}} = \sqrt[n]{x_1 x_2 x_3 \ldots . x_n}$$

Mathematical Model of Modified Ant Colony Algorithm (1991)

Here the new path based on the probability from place i to place j for the k-th ant as Shown in Equation (1)

$$P_{ij} = \frac{\Omega^{\theta}{}_{ij}\mho_{ij}{}^{\varphi}}{\sum_k \Omega^{\theta}{}_{ik}\mho_{ik}{}^{\varphi}} \tag{1}$$

Where, Ω_{ij} and $\mho_{ij}$ are the values of pheromone trail level of the move, and some heuristic information, which correspond to the link (i,j). θ And φ are both parameters used to control the importance of the pheromone trail and heuristic information during component selection. For our scenario we assumed Ω =1 and φ=1/3 in the transition rule,

$$P_{ij}(t) = \begin{cases} \dfrac{(\prod_{j=1}^{3} a_{ij})^{\frac{1}{3}}}{\sum_{i=1}^{3}(\prod_{j=1}^{3} a_{ij})^{\frac{1}{3}}} & ;\ i^{th} \text{ ant visits the } j^{th}\ city \\ 0 & ;\ \text{Otherwise} \end{cases} \tag{2}$$

With

a_{ij}; is cost between node i and node j.

P_{ij} (t); Probability to branch from node i to node j.

Pheromone Update Rule.

After all ants complete their tours, the local update rule of the pheromone trails is applied for each route according to (3),

$$\Omega_{ij}(t+1) = (1-\rho)\Omega_{ij}(t) + \sum_{k=1}^{m} \Delta \frac{\mu}{\text{£}^k} \tag{3}$$

After that, apply the global pheromone update rule in which the amount of pheromone is added to the best route which has the lowest cost.

Here, Y^k is the distance of the best route. µ Is simply a parameter to adjust the amount of pheromone deposited, typically it would be set to 1. We sum $\mu/\text{£}^k$ for every solution which used component (i, j), then that value becomes the amount of pheromone to be deposited on component (i, j). In our case ,

$$\Omega_{ij}(t+1) = (1-\rho)\Omega_{ij}(t)$$

Where Ω_{ij} (t) is the maximum number of Demands or Supplies and $0<\rho\leq 1$.

Mathematical Formulation

The mathematical formulation of the FTP is as follows.

Minimize $\tilde{Z} = \sum_{i=0}^{m}\sum_{j=0}^{n} \hat{c}_{ij}\tilde{X}_{ij}$

Subject to

$\sum_{j=1}^{n} \tilde{X}_{ij} \leq a_i$ For $i = 1,2,3,\ldots,m$

$\sum_{i=1}^{m} \tilde{X}_{ij} \leq b_j$ For $j = 1,2,3,\ldots,n$

$\tilde{X}_{ij} \geq 0$ For $i = 1,2,3,\ldots,m$ and $j = 1,2,3,\ldots,n$

Here, all a_i and b_j are assumed to be positive, and a_i are normally called supplies and b_j are called demands, as shown in below table. The fuzzy cost $\hat{c}_{ij}$ are all non-negative. If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, it is a balanced transportation problem .If this condition is not met, a dummy origin or destination is generally introduced to make the problem balanced.

Proposed Method

In this section, a proposed method, Improved Ant Colony Algorithm, for finding an optimal solution. Following are the steps for solving Fuzzy Transportation Problem.

Algorithm

Step 1: Construct the fuzzy transportation the cost table from the given problem

Step 2: Examine the TP to see if it is balanced, and if not, make it so.

Step 3: Convert fuzzy cost values in the Transportation cost table to crisp cost values by utilizing Geometric Mean.

Step 4: The probability table is then computed using the Modified ACO algorithm..

Step 5: Starting with the $X_{ij}=min(a_i,b_j)$ (unbalanced) or starting with the $X_{ij}=max(a_i,b_j)$ (balanced) probability table to make the first allocation.

Step 6: Assign, Step 5 at the place of the minimum probability cell

Step 7: If the demand in the column (or supply in the row) is satisfied, delete that column (or row), then proceed to the next minimal value in the demand and supply.

Step 8: Repeat this process until all supply and demand are satisfied, then proceed to Step – 10.

Otherwise, proceed to Step 6.

Step 9: Stop and compute the first viable solution.

Illustration Example

Example 1

We take a distribution problem in which a single homogeneous item is to be distributed from three stores (A, B, C) to four different warehouses (P, Q, and R, S). Cost, time and distance for each unit transported is given in the table. Find the minimum time, cost and distance.

Table 1: Cost

Supply Demand	P	Q	R	S	Supply
A	21	16	15	13	11
B	17	18	24	23	13
C	32	27	18	41	19
Demand	6	10	12	15	43

Table 2: Time

Supply Demand	P	Q	R	S	Supply
A	1	2	1	4	11
B	3	3	2	1	13
C	4	2	5	9	19
Demand	6	10	12	15	43

Table 3 :Distance

Supply Demand	P	Q	R	S	Supply
A	11	13	17	14	11
B	16	18	14	10	13
C	21	24	13	10	19
Demand	6	10	12	15	43

Table 4: Step 3

Supply Demand	P	Q	R	S	Supply
A	6.13	7.46	6.34	8.99	11
B	9.34	9.90	8.75	6.13	13
C	13.90	10.90	10.53	15.45	19
Demand	6	10	12	15	43

Table 5: Step 4

Supply Demand	P	Q	R	S	Supply
A	.053	.065	.055	.078	11
B	.082	.086	.076	.053	13
C	.122	.095	.092	.135	19
Demand	6	10	12	15	43

Table 6: Steps 5 & 6and 1st allocation

Supply Demand	P	Q	R	S	Supply
A	.053	.065	.055	.078	11
B	.082	.086	.076	.053	13
C	.122	.095	.092*12	.135	19*7
	6	10	12*0	15	43

Table 7: Steps 6 & 7 and 2nd allocation

Supply Demand	P	Q	R	S	Supply
A	.053	.065	.055	.078	11
B	.082	.086	.076	.053	13
C	.122	.095*7	.092*12	.135	19*7*0
	6	10*3	12*0	15	43

Table 8: Steps 6 & 7and 3rd allocation

Supply Demand	P	Q	R	S	Supply
A	.053	.065*3	.055	.078	11*8
B	.082	.086	.076	.053	13
C	.122	.095*7	.092*12	.135	19*7*0
	6	10*3*0	12*0	15	43

Table 9: Steps 6, 7, 8 and 3rd allocation

Supply Demand	P	Q	R	S	Supply
A	.053*6	.065*3	.055	.078	11*8*2
B	.082	.086	.076	.053	13
C	.122	.095*7	.092*12	.135	19*7*0
	6*0	10*3*0	12*0	15	43

Table 9: Steps 6, 7, 8 and 3rd allocation

Supply Demand	P	Q	R	S	Supply
A	.053*6	.065*3	.055	.078	11*8*2
B	.082	.086	.076	.053	13
C	.122	.095*7	.092*12	.135	19*7*0
	6*0	10*3*0	12*0	15	43

Table 10: Steps 6, 7, 8 and 3rd allocation

Supply Demand	P	Q	R	S	Supply
A	.053*6	.065*3	.055	.078*2	11*8*2*0
B	.082	.086	.076	.053	13
C	.122	.095*7	.092*12	.135	19*7*0
	6*0	10*3*0	12*0	15*13	43

Table 11: Step 9

Supply Demand	P	Q	R	S	Supply
A	.053*6	.065*3	.055	.078*2	11*8*2*0
B	.082	.086	.076	.053*13	13*0
C	.122	.095*7	.092*12	.135	19*7*0
	6*0	10*3*0	12*0	15*13*0	43

The solution is given as: $x_{11}=6$,$x_{12}=3$,$x_{13}=2$,$x_{24}=13$,$x_{32}=7$, $x_{33}=12$
Following are the values of objectives: Minimum Cost = 904 units,
Minimum Time = 107 units,
Minimum distance = 587 units (1 iteration)

Table 12: Comparison between different methods

Method	Minimum cost	Minimum time	Minimum Distance
New row Maxima method [9]	938	117	457
Product Approach [10]	938	132	552
Geometric mean method	904	107	587
GMACOA	904	107	587
LINGO	796	89	527

The findings of the comparisons in Table 12 are depicted using bar graphs, as shown in Fig. 1.

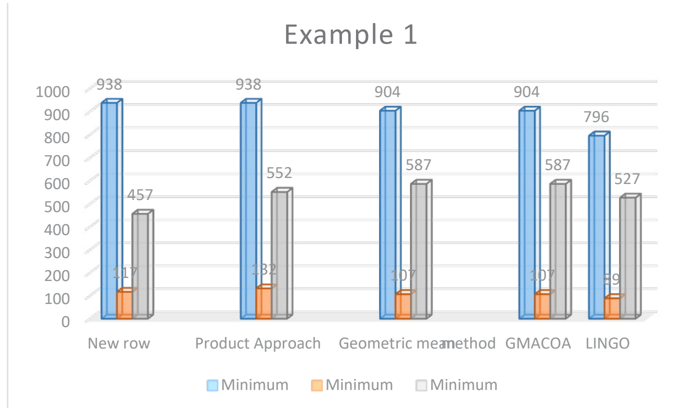

Figure 1: Compares the results of the New Row Maxima technique, the Product Approach, the Geometric Mean method, GMACOA, and the optimal method (LINGO).

According to the simulation findings (Fig. 1 and Table 12), the proposed strategy outperforms the geometric mean method.

Example 2(Khilendra,2020)

Now we take one more example with following characteristics:

Table 13: Cost

Supply Demand	P	Q	R	S	Supply
A	6	4	1	5	14
B	8	9	2	7	16
C	4	3	6	2	5
Demand	6	10	15	4	

Table 14: Time

Supply Demand	P	Q	R	S	Supply
A	13	11	15	20	14
B	17	14	12	13	16
C	18	18	15	12	5
Demand	6	10	15	4	

Table 15: Distance

Supply Demand	P	Q	R	S	Supply
A	6	3	5	4	14
B	5	9	2	7	16
C	5	7	8	6	5
Demand	6	10	15	4	

Table 16: Step 3

Supply Demand	P	Q	R	S	Supply
A	7.76	5.09	4.21	7.36	14
B	8.79	10.42	3.63	8.60	16
C	7.11	7.23	8.96	5.24	5
Demand	6	10	15	4	

Table 17: Step 4

Supply Demand	P	Q	R	S	Supply
A	.091	.060	.049	.087	14
B	.104	.123	.043	.101	16
C	.084	.085	.106	.062	5
Demand	6	10	15	4	

Table 18: Step 5 and 1st allocation

Supply Demand	P	Q	R	S	Supply
A	.091	.060	.049	.087	14
B	.104	.123	.043*15	.101	16*1
C	.084	.085	.106	.062	5
Demand	6	10	15*0	4	

Table 19: Steps 5, 6, 7 and 1st allocation

Supply Demand	P	Q	R	S	Supply
A	.091	.060*10	.049	.087	14*4
B	.104	.123	.043*15	.101	16*1
C	.084	.085	.106	.062	5
Demand	6	10*0	15*0	4	

Table 20: Steps 5-8 and other allocations

Supply Demand	P	Q	R	S	Supply
A	.091*4	.060*10	.049	.087	14*4*0
B	.104*1	.123	.043*15	.101	16*1
C	.084*1	.085	.106	.062*4	5*1
Demand	6*5*1*0	10*0	15*0	4*0	

Table 21: Comparison between different methods

Method	Minimum cost	Minimum time	Minimum Distance
New row Maxima method [9]	122	461	130
Product Approach [10]	114	425	128
Geometric mean method	114	425	118
GMACOA	114	425	118
LINGO	114	424	106

Table 21's comparison data are also depicted using bar graphs and the results Fig. 2.

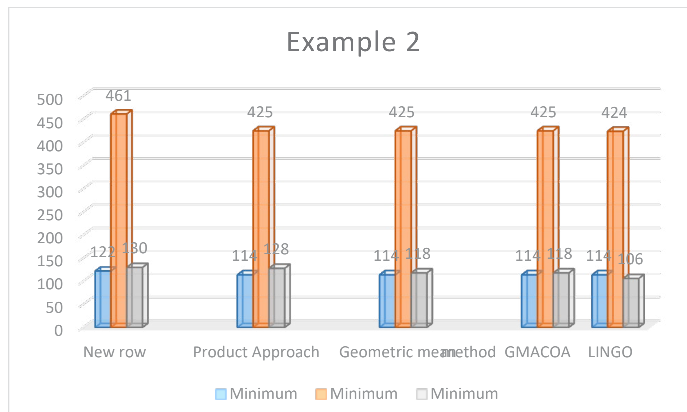

Figure 2: Shows a comparison of the outcomes given by the New Row Maxima approach, the Product Approach, the Geometric Mean method, GMACOA, and the optimal method.

The proposed technique uses the geometric mean method, according to the simulation results (Fig. 2 and Table 21).

Example 3

Table 22: Example

x	D_1	D_2	D_3	Supply
S_1	(1,2,3)	(4,7,10)	(10,14,18)	5
S_2	(2,3,4)	(2,3.4)	(0,1,2)	8
S_3	(1,5,9)	(3,4,5)	(4,7,10)	7
S_4	(0,1,2)	(5,6,7)	(1,2,3)	15
Demand	7	9	18	

Table 22: Step 2

x	D_1	D_2	D_3	Dummy	Supply
S_1	(1,2,3)	(4,7,10)	(10,14,18)	0	5
S_2	(2,3,4)	(2,3.4)	(0,1,2)	0	8
S_3	(1,5,9)	(3,4,5)	(4,7,10)	0	7
S_4	(0,1,2)	(5,6,7)	(1,2,3)	0	15
Demand	7	9	18	1	

Table 23: Step 3

x	D_1	D_2	D_3	Dummy	Supply
S_1	1.81	6.54	13.61	0	5
S_2	2.88	2.88	0	0	8
S_3	3.55	3.91	6.54	0	7
S_4	0	5.94	1.81	0	15
Demand	7	9	18	1	

Table 23: Step 4

x	D_1	D_2	D_3	Dummy	Supply
S_1	.036	.132	.275	0	5
S_2	.058	.058	0	0	8
S_3	.071	.079	.132	0	7
S_4	0	.120	.036	0	15
Demand	7	9	18	1	

Table 24: Step 6 and 1st allocation

x	D_1	D_2	D_3	Dummy	Supply
S_1	.036*4	.132	.275	0*1	5*4*0
S_2	.058	.058	0	0	8
S_3	.071	.079	.132	0	7
S_4	0*3	.120	.036*12	0	15*12*0
Demand	7*3*0	9	18	1	

Table 25: Other steps and other allocations

x	D_1	D_2	D_3	Dummy	Supply
S_1	.036*4	.132	.275	0*1	5*4*0
S_2	.058	.058*2	0*6	0	8*2*0
S_3	.071	.079*7	.132	0	7*0
S_4	0*3	.120	.036*12	0	15*12*0
Demand	7*3*0	9*7*0	18	1	

The solution is given as: $x_{11}=4$, $x_{22}=2$,$x_{23}=6$, $x_{32}=7$,$x_{41}=3$, $x_{43}=12$
Following are the values of objectives: 4(1,2,3)+2(2,3,4)+6(0,1,2)+7(3,4,5)+3(0,1,2)+12(1,2,3)=(41,75,109)=75

Table 24: Initial Solutions Obtained by all Procedures

Method	VAM	SVAM	GVAM	BVAM	RVAM	ASM	ZSM	IZPM	GMA-COA	Optimal
Example 3	82	99	80	84	93	149	79	75	75	75

The comparison data from Table 24 are also represented using bar graphs and the form Fig. 3.

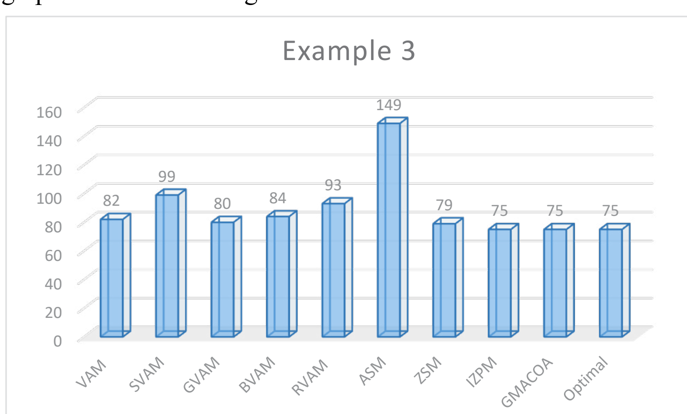

Figure3: A comparison of the results obtained by VAM,SVAM,GVAM,BVAM,RVAM,ASM,ZSM,IZPM,GMACOA,AND Optimal

According to the simulation results (Fig. 3 and Table 24), the suggested strategy employs the IZPM method, and other ways outperform our GMACOA. In addition, the shortest number of iterations resulted in the best solution.

Conclusion

The TP is an essential component of this serious universe. The fundamental purpose of ordinary TPs is to reduce the cost of carrying an item from its origin to its destination. A lot of objectives must be examined and optimized concurrently in some major problems. These are known as multi-objective problems. In this work, instead of using conventional approaches, the geometric mean method combined with the Ant Colony Algorithm is used to solve a multi-objective fuzzy transportation problem in fuzzy environments. When compared to other current approaches, the proposed algorithm provides the best performance. As a result, the higher the IFS, the fewer iterations are required to get the final optimal solution [28, 33].The method is quite straightforward. In this study, however, we provide a novel alternative method, a modified ant colony optimization algorithm that provides an optimal solution to the many different types of TPs.

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