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From Quantum Field Theory to the Contemporary Quantum Mechanics

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Abstract

In this talk we remind how the notion of the so-called clothed particles, put forward in relativistic quantum field theory by Greenberg and Schweber, can be used via the method of unitary clothing transformations (shortly, the UCT method) when finding the eigenstates of the total Hamiltonian H in case of interacting fields with the Yukawa - type couplings. In general, the UCT method is aimed at reduction of the exact eigenvalue problem in the primary Fock space to the model-space problems in the corresponding Hilbert spaces of the contemporary quantum mechanics. In this context we consider an approximate treatment of the physical vacuum, the observable one-particle and two-particle bound and scattering states.

Some recollections

One day in the mid-90s after my lecture delivered for experimentalists from the High Energy Physics Dept. of our Center and devoted to the role of meson exchange currents in electromagnetic interactions with nuclei we were discussing a surprising property of atomic nuclei. Namely, when considering any nucleus as a system of interacting meson and nucleon fields we often forget the spring of nuclear forces and prefer to a handle a set of particles that preserve their property to be physical nucleons with specific interactions (rather phenomenological) between them, i.e., coming to the well known contemporary picture.

At that time I had begun to work on the Laboratory for Theoretical Physics (LTP) JINR and was concerned with a constructive way from quantum field theory (QFT) to the contemporary quantum mechanics. Along this guideline we have found together with Mikhail Shirokov such a way relying upon the idea by Greenberg and Schweber [1] and developing the UCT method [2, 3]. Afterwards, for the twenty past years we have seen how this method may be realized in mesodynamics, quantum electrodynamics (QED) and other field models [3-10]. In particular, starting from the primary Yukawa-type couplings between the "bare" fermions (nucleons and antinucleons) and bosons (π -, η -, ρ -, ω - mesons, etc.) we have built up new families of the interactions ("quasipotentials") between the "clothed" particles (nucleons in the theory of nuclear forces, electrons and positrons in QED, etc.), responsible for physical processes in the corresponding systems (see, e.g., our lecture for the Ettore Majorana School [2], a survey in [6] and our contributions to the Few-Body Conferences in Caen and Guildford [13]. These quasipotentials are hermitean and energy independent that make them attractive in practical calculations. The corresponding four-operator interactions for the 2 - 2 processes (such as $NN \to NN$ and $N\bar{N} \leftrightarrow \pi + \pi$), the five-operator interactions for the 2 - 3 ones (such as $NN \to BNN$) are derived along the chain: bare particles with bare masses → bare particles with physical masses → physical (observable) particles. Although the eigenvalue problems that we have to solve are related to a three-dimensional formalism, our consideration is compatible with the relativistic invariance requirements being fulfilled in the framework of an original procedure proposed to meet the Poincar´e-Lie algebra [8]. In this context, we would like to draw your attention to our relativistic Faddeev calculations (in particular, with the Kharkov potential) for the three-nucleon bound state (triton) and some Nd elastic scattering observables (to be published in Proc. of the FB22 Conference)

Underlying formalism

We will now prove the fundamental theorem: any operator O may be expressed as a sum of products of creation and annihilation operators ...
S. Weinberg
Quantum Theory of Fields, Vol. I, 1995, p. 175.

In accordance with the motto each of ten generators of the Poincar´e group Π may be expressed as a sum of products of the creation and annihilation operators $a^{\dagger}(n)$ and $a(n)$ ($n = 1, 2, ...$) for free particles, e.g., bosons and/or fermions. In the framework of such a corpuscular picture the Hamiltonian of a system of interacting mesons and nucleons can be written as

$$H = \sum_{C=0}^{\infty} \sum_{A=0}^{\infty} H_{CA}, \qquad (1)$$

$$H_{CA} = \sum\!\!\!\!\!\!\int H_{CA}(1', 2', ..., n'_C; 1, 2, ..., n_A) a^\dagger(1') a^\dagger(2') ... a^\dagger(n'_C) a(n_A) ... a(2) a(1), \quad (2)$$

where $C(A)$ denotes the particle-creation (annihilation) number for the operator substructure H_{CA} and

$$H_{CA}(1', 2', ..., C; 1, 2, ..., A) = \delta(\vec{p}_1' + \vec{p}_2' + ... + \vec{p}_C' - \vec{p}_1 - \vec{p}_2 - ... - \vec{p}_A)$$

$$\times h_{CA}(p_1'\mu_1'\xi_1', p_2'\mu_2'\xi_2', ..., p_C'\mu_C'\xi_C'; p_1\mu_1\xi_1, p_2\mu_2\xi_2, ..., p_A\mu_A\xi_A),$$

where c-number coefficients h_{CA} do not contain! delta functions, $a(n) = a(p_n, \mu_n, \xi_n)$ the annihilation operator for a particle of species ξ_n with the momentum $\vec{p}_n$ and polarization μ_n. In its turn, every operator H_{CA} can be represented as

$$H_{CA} = \int H_{CA}(\mathbf{x}) \mathbf{dx} \quad (3)$$

if one uses the formula

$$\delta(\mathbf{p} - \mathbf{p}') = \frac{1}{(2\pi)^3} \int e^{i(\mathbf{p}-\mathbf{p}')\mathbf{x}} \mathbf{dx}$$

Thus, we come to the form $H = \int H(\mathrm{x})\mathrm{dx}$ well known from local field models

$$H = \int H(\mathbf{x}) \mathbf{dx} \quad (4)$$

with the density

$$H(\mathbf{x}) = \sum_{\mathbf{C}=0}^{\infty} \sum_{\mathbf{A}=0}^{\infty} \mathrm{H_{CA}}(\mathbf{x}). \quad (5)$$

For example, in case with $C=A=2$ we have

$$H_{22}(1', 2'; 1, 2) = \delta(\vec{p}_1' + \vec{p}_2' - \vec{p}_1 - \vec{p}_2) h(1'2'; 12),$$

$$H_{22}(\vec{x}) = \frac{1}{(2\pi)^3} \sum\!\!\!\!\!\!\int \exp[-i(\vec{p}_1' + \vec{p}_2' - \vec{p}_1 - \vec{p}_2)\vec{x}] \, h(1'2'; 12) a^\dagger(1') a^\dagger(2') a(2) a(1).$$

Further, we employ the transformation properties with the respect to Π in case of a massive particle with the mass m and spin j:

$$U_F(\Lambda, b) a^\dagger(p, \mu) U_F^{-1}(\Lambda, b) = e^{i\Lambda p b} D^{(j)}_{\mu'\mu}(W(\Lambda, p)) a^\dagger(\Lambda p, \mu'),$$

$\forall \Lambda \in L_+$ and arbitrary spacetime shifts $b = (b^0, \vec{b})$

with the D-function whose argument is the Wigner rotation $W(\Lambda, \rho)$, L_+ the homogeneous (proper) orthochronous Lorentz group. The correspodence $(\Lambda, b) \rightarrow U_F(\Lambda, b)$ realizes the unitary irreducible representation of Π on the space (to be definite) of meson-nucleon states for the operators $a(p, \mu) = a(\vec{p}, \mu)\sqrt{p_0}$ that meet the covariant commutation relations

$$[a(p', \mu'), a^\dagger(p, \mu)]_\pm = p_0 \delta(\vec{p} - \vec{p}') \delta_{\mu'\mu},$$
$$[a(p', \mu'), a(p, \mu)]_\pm = [a^\dagger(p', \mu'), a^\dagger(p, \mu)]_\pm = 0. \quad (6)$$

Here $p_0 = \sqrt{\vec{p}^2 + m^2}$ is the fourth component of the 4-momentum $p = (p_0, \vec{p})$. Often one has to deal with field models where in the Dirac (D) picture

$$U_F(\Lambda, b) H_I(x) U_F^{-1}(\Lambda, b) = H_I(\Lambda x + b), \quad \forall x = (t, \vec{x}). \quad (7)$$

It is not the case, where the pseudoscalar (π and η), vector (ρ and ω) and scalar (δ and σ) meson (boson) fields interact with the 1/2 spin (N and $\bar{N}$ fermion ones via the Yukawa-type couplings

$V = \sum_b V_b = V_s + V_{ps} + V_v$ in

$H_I = V +$ mass and vertex counterterms, (8)

with

$$V_s = g_s \int d\vec{x} \, \bar{\psi}(\vec{x}) \psi(\vec{x}) \varphi_s(\vec{x}), \quad (9)$$

$$V_{ps} = i g_{ps} \int d\vec{x} \, \bar{\psi}(\vec{x}) \gamma_5 \psi(\vec{x}) \varphi_{ps}(\vec{x}) \quad (10)$$

and

$$\mathrm{V_v} = \int d\vec{x} \left\{ g_v \bar{\psi}(\vec{x}) \gamma_\mu \psi(\vec{x}) \varphi_v^\mu(\vec{x}) + \frac{f_v}{4m} \bar{\psi}(\vec{x}) \sigma_{\mu\nu} \psi(\vec{x}) \varphi_v^{\mu\nu}(\vec{x}) \right\}$$
$$+ \int d\vec{x} \left\{ \frac{g_v^2}{2m_v^2} \bar{\psi}(\vec{x}) \gamma_0 \psi(\vec{x}) \bar{\psi}(\vec{x}) \gamma_0 \psi(\vec{x}) + \frac{f_v^2}{4m^2} \bar{\psi}(\vec{x}) \sigma_{0i} \psi(\vec{x}) \bar{\psi}(\vec{x}) \sigma_{0i} \psi(\vec{x}) \right\},$$

where $\varphi_v^{\mu\nu}(\vec{x}) = \partial^\mu \varphi_v^\nu(\vec{x}) - \partial^\nu \varphi_v^\mu(\vec{x})$ the tensor of the vector fields involved (details in [7]). Here we encounter the scalar H_{sc} and nonscalar H_{nonsc} contributions to the interaction densities of $\rho N N$ and $\omega N N$ couplings

$$U_F(\Lambda, a) H_{sc}(x) U_F^{-1}(\Lambda, a) = H_{sc}(\Lambda x + a), \quad (11)$$

$$U_F(\Lambda, a) H_{nonsc}(x) U_F^{-1}(\Lambda, a) \neq H_{nonsc}(\Lambda x + a) \quad (12)$$

It requires a special consideration [8].

Boost generators. Relativistic invariance (RI) as a whole

"To free ourselves from any dependence on pre-existing field theories" (after S.Weinberg), the boost operators $\vec{N} = (N^1, N^2, N^3)$ can be written as

$$\vec{N} = \sum_{C=0}^{\infty} \sum_{A=0}^{\infty} \vec{N}_{CA}, \quad (13)$$

$$\vec{N}_{CA} = \sum\!\!\!\!\!\!\int \vec{N}_{CA}(1', 2', ..., n'_C; 1, 2, ..., n_A) a^\dagger(1') a^\dagger(2') ... a^\dagger(n'_C) a(n_A) ... a(2) a(1) \quad (14)$$

Recently [8] we have developed an algebraic procedure to find links between coefficients H_{CA} and N_{CA}, compatible with commutations

$$[P_i, P_j] = 0,\ [J_i, J_j] = i\varepsilon_{ijk}J_k,\ [J_i, P_j] = i\varepsilon_{ijk}P_k, \tag{15}$$

$$[\vec{P}, H] = 0, [\vec{J}, H] = 0, [J_i, N_j] = i\varepsilon_{ijk}N_k, [P_i, N_j] = i\delta_{ij}H, \tag{16}$$

$$[H, \vec{N}] = i\vec{P}, [N_i, N_j] = -i\varepsilon_{ijk}J_k, \tag{17}$$

$$(i, j, k = 1, 2, 3), \tag{18}$$

where $P = (P^1; P^2; P^3)$ and $J = (J^1; J^2; J^3)$ are the linear and angular momentum operators.

For the instant form of relativistic dynamics after Dirac only the Hamiltonian and boost operators carry interactions, viz., $H = H_F + H_I$, $\vec{N} = \vec{N}_F + \vec{N}_I$ while $\vec{P} = \vec{P}_F$ and $\vec{J} = \vec{J}_F$. After these preliminaries we will show how one can build up operators H_I and $\vec{N}_I$.

Recall that the angular momentum $\vec{J} = \vec{J}_F = \vec{J}\pi + \vec{J}_{ferm}$ with

$$\vec{J}_\pi = \frac{i}{2}\int d\vec{k}\,\vec{k}\times\left(\frac{\partial a^\dagger(\vec{k})}{\partial\vec{k}}a(\vec{k}) - a^\dagger(\vec{k})\frac{\partial a(\vec{k})}{\partial\vec{k}}\right) \tag{19}$$

and $\vec{J}_{ferm} = \vec{L}_{ferm} + \vec{S}_{ferm}$, where

$$\vec{L}_{ferm} = \frac{i}{2}\sum\!\!\!\!\!\!\int d\vec{p}\,\vec{p}\times\left(\frac{\partial b^\dagger(\vec{p}\mu)}{\partial\vec{p}}b(\vec{p}\mu) - b^\dagger(\vec{p}\mu)\frac{\partial b(\vec{p}\mu)}{\partial\vec{p}} + \frac{\partial d^\dagger(\vec{p}\mu)}{\partial\vec{p}}d(\vec{p}\mu) - d^\dagger(\vec{p}\mu)\frac{\partial d(\vec{p}\mu)}{\partial\vec{p}}\right),$$

$$\vec{S}_{ferm} = \frac{1}{2}\sum\!\!\!\!\!\!\int d\vec{p}\chi^\dagger(\mu')\vec{\sigma}\chi(\mu)(b^\dagger(\vec{p}\mu')b(\vec{p}\mu) - d^\dagger(\vec{p}\mu')d(\vec{p}\mu)),$$

the boosts $\vec{N}_F = \vec{N}_\pi + \vec{N}_{ferm}$ with

$$\vec{N}_\pi = \frac{i}{2}\int d\vec{k}\,\omega_{\vec{k}}\left(\frac{\partial a^\dagger(\vec{k})}{\partial\vec{k}}a(\vec{k}) - a^\dagger(\vec{k})\frac{\partial a(\vec{k})}{\partial\vec{k}}\right)$$

and $\vec{N}_{ferm} = \vec{N}^{orb}_{ferm} + \vec{N}^{spin}_{ferm}$, where

$$\vec{N}^{orb}_{ferm} = \frac{i}{2}\sum\!\!\!\!\!\!\int d\vec{p}\,E_{\vec{p}}\left(\frac{\partial b^\dagger(\vec{p}\mu)}{\partial\vec{p}}b(\vec{p}\mu) - b^\dagger(\vec{p}\mu)\frac{\partial b(\vec{p}\mu)}{\partial\vec{p}} + \frac{\partial d^\dagger(\vec{p}\mu)}{\partial\vec{p}}d(\vec{p}\mu) - d^\dagger(\vec{p}\mu)\frac{\partial d(\vec{p}\mu)}{\partial\vec{p}}\right),$$

$$\vec{N}^{spin}_{ferm} = -\frac{1}{2}\sum\!\!\!\!\!\!\int d\vec{p}\,\vec{p}\times\frac{\chi^\dagger(\mu)\vec{\sigma}\chi(\mu)}{E_{\vec{p}}+m}\left(b^\dagger(\vec{p}\mu)b(\vec{p}\mu) + d^\dagger(\vec{p}\mu)d(\vec{p}\mu)\right), \tag{20}$$

where $\omega_{\vec{k}} = \sqrt{\vec{k}^2 + m_\pi^2}$ ($E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$) pion (nucleon) energy and $\chi(\mu)$ is the Pauli spinor.

The UCT method in action

First of all, we will express the total field Hamiltonian through the so-called clothed-particle creation (annihilation) operators α_c, e.g., $a^\dagger_c(a_c)$, $b^\dagger_c(b_c)$ and $d^\dagger_c(d_c)$ via UCTs $W(\alpha_c) = W(\alpha) = \exp R$, $R = -R^\dagger$ in the similarity transformation

$$\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c) \tag{21}$$

that connect the primary set α in the bare-particle representation (BPR) with the new operators in the CPR. A key point of the clothing procedure in question is to remove the so-called bad terms from the Hamiltonian

$$H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha) = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \equiv K(\alpha_c), \tag{22}$$

By definition, such terms prevent physical vacuum $|\Omega)$ (H lowest eigenstate) and one-clothed-particle states $|n\rangle_c = a^\dagger_c(n)|\Omega\rangle$ to be the H eigenvectors for all n included. The bad terms occur every time when any normally ordered product

$$a^\dagger(1')a^\dagger(2')...a^\dagger(n'_C)a(n_A)...a(2)a(1) \tag{23}$$

of class [C.A] embodies, at least, one substructure $\in [k.0]$ (k = 1, 2...) or/and $[k.1]$ ($k =$ 2, 3, ...). In this context all primary Yukawa-type (trilinear) couplings shown above should be eliminated. Respectively, let us write for a boson–fermion system

$$H_I(\alpha) = V(\alpha) + V_{ren}(\alpha)$$

with a primary (trial) interaction $V(\alpha) = V_{bad} + V_{good}$ "good" (e.g., $\in [k.2]$) as antithesis of "bad" while $V_{ren}(\alpha)$ [1.1] + [0.2] + [2.0] consists of "renormalization counterterms". It turns out that the latter are important to ensure RI as a whole, i.e., in Dirac sense.

In order to compare our calculations with those by Bonn collaboration (Machleidt, Holinde, Elster) we have employed $V(\alpha) = V_s + V_{ps} + V_v$. In this talk I do not intend to derive all interactions between the clothed mesons and nucleons in the second order of coupling constants. Instead, for simplicity, we switch off the couplings Vs and Vps and then the clothing itself is prompted by

$$H(\alpha) = K(\alpha_c) \equiv W(\alpha_c)[H_F(\alpha_c) + V_v(\alpha_c) + V_{ren}(\alpha_c)]W^\dagger(\alpha_c)$$

or

$$K(\alpha_c) = H_F(\alpha_c) + V_v^{(1)}(\alpha_c) + [R, H_F] + V_v^{(2)}(\alpha_c) + [R, V_v^{(1)}] + \frac{1}{2}[R,[R,H_F]] + [R, V_v^{(2)}] + \frac{1}{2}[R,[R,V_v^{(1)}] + ...$$

by requiring $[R, H_F] = -V_v^{(1)}$ for the operator R of interest to get

$$H = K(\alpha_c) = K_F + K_I \qquad (23)$$

with a new free part $K_F = H_F(a_c) \sim a^\dagger_c a_c$ and interaction

$$K_I = \frac{1}{2}[R, V_v^{(1)}] + V_v^{(2)} + \frac{1}{3}[R,[R,V_v^{(1)}]] + \ldots \qquad (24)$$

Moreover, after modest effort,

$$\frac{1}{2}\left[R, V_v^{(1)}\right](NN \to NN) = K_v(NN \to NN) + K_{cont}(NN \to NN)$$

Operator $K_{cont}(NN \to NN)$ may be associated with a contact interaction since it does not contain any propagators [7]. It has turned out that this operator cancels completely the non–scalar operator $V^{(2)}$. Such a cancellation is a pleasant feature of the CPR.

In parallel, we have

$$\vec{N}(\alpha) = \vec{B}(\alpha_c) = W(\alpha_c)\{\vec{N}_F(\alpha) + \vec{N}_I(\alpha) + \vec{N}_{ren}(\alpha)\}W^\dagger(\alpha_c)$$

with

$$\vec{N}_I = -\int \vec{x}V_v(\vec{x})d\vec{x} = -\int \vec{x}\{V_v^{(1)}(\vec{x}) + V_v^{(2)}(\vec{x})\}d\vec{x} = \vec{N}_I^{(1)} + \vec{N}_I^{(2)}$$

As before [2,3]. we find that the boost generator in CPR gets a structure similar to $K(\alpha_c)$

$$\vec{B}(\alpha_c) = \vec{B}_F + \vec{B}_I.$$

Here $\vec{B}_F = \vec{N}_F(\alpha_c)$ is the boost operator for noninteracting clothed particles (in our case the clothed fermions and vector mesons) and $\vec{B}_I$ incorporates the contributions induced by interactions between them

$$\vec{B}_I = +\frac{1}{2}[R, \vec{N}_I^{(1)}] + \frac{1}{3}[R,[R,\vec{N}_I^{(1)}]] + \ldots$$

Relativistic interactions in meson-nucleon systems

The CPR interaction operators are contained in

$$K_I \sim a_c^\dagger b_c^\dagger a_c b_c(\pi N \to \pi N) + b_c^\dagger b_c^\dagger b_c b_c(NN \to NN) + d_c^\dagger d_c^\dagger d_c d_c(\bar{N}\bar{N} \to \bar{N}\bar{N})$$

$$+ b_c^\dagger b_c^\dagger b_c^\dagger b_c b_c b_c(NNN \to NNN) + \ldots + [a_c^\dagger a_c^\dagger b_c d_c + H.c.](N\bar{N} \leftrightarrow 2\pi) + \ldots$$

$$+ [a_c^\dagger b_c^\dagger b_c^\dagger b_c b_c + H.c.](NN \leftrightarrow \pi NN) + \ldots$$

In particular, we get an alternative for deriving the usual separation

$$H = K + V$$

with the one-body operator of kinetic energy K and the interactions between the clothed (physical) nucleons

$$V = \sum_{i<j}^{N} V(i,j) + \sum_{i<j<k}^{N} V(i,j,k) + \cdots,$$

where we find the two-body $V(i,j)$ and three-body $V(i,j,k)$ forces, etc., reminiscent of famous pages in the text books on nuclear physics.

In addition, the pion-nucleon interaction operator looks as

$$K(\pi N \to \pi N) = \int d\vec{p}_1 d\vec{p}_2 d\vec{k}_1 d\vec{k}_2\, V_{\pi N}(\vec{k}_2, \vec{p}_2; \vec{k}_1, \vec{p}_1)\, a_c^\dagger(\vec{k}_2) b_c^\dagger(\vec{p}_2) a_c(\vec{k}_1) b_c(\vec{p}_1),$$

$$V_{\pi N}(\vec{k}_2, \vec{p}_2; \vec{k}_1, \vec{p}_1) = \frac{g^2}{2(2\pi)^3}\frac{m}{\sqrt{\omega_{\vec{k}_1}\omega_{\vec{k}_2}E_{\vec{p}_1}E_{\vec{p}_2}}}\delta(\vec{p}_1 + \vec{k}_1 - \vec{p}_2 - \vec{k}_2)$$

$$\bar{u}(\vec{p}_2)\left\{\frac{1}{2}\left[\frac{1}{\hat{p}_1 + \hat{k}_1 + m} + \frac{1}{\hat{p}_2 + \hat{k}_2 + m}\right] + \frac{1}{2}\left[\frac{1}{\hat{p}_1 - \hat{k}_2 + m} + \frac{1}{\hat{p}_2 - \hat{k}_1 + m}\right]\right\}u(\vec{p}_1)$$

with the corresponding πN quasipotential in momentum space,

$$\tilde{V}_{\pi N}(\vec{k}_2, \vec{p}_2; \vec{k}_1, \vec{p}_1) = \left\langle a_c^\dagger(\vec{k}_2) b_c^\dagger(\vec{p}_2)\Omega | K(\pi N \to \pi N) | a_c^\dagger(\vec{k}_1) b_c^\dagger(\vec{p}_1)\Omega \right\rangle$$

After normal ordering of the fermion creation and destruction operators we arrive to the $NN \to NN$ interaction operator:

$$K_{NN} = \int d\vec{p}_1 d\vec{p}_2 d\vec{p}_1' d\vec{p}_2' V_{NN}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) b_c^\dagger(\vec{p}_1') b_c^\dagger(\vec{p}_2') b_c(\vec{p}_1) b_c(\vec{p}_2),$$

$$V_{NN}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) = -\frac{1}{2}\frac{g^2}{(2\pi)^3}\frac{m^2}{\sqrt{E_{\vec{p}_1}E_{\vec{p}_2}E_{\vec{p}_1'}E_{\vec{p}_2'}}}\delta(\vec{p}_1' + \vec{p}_2' - \vec{p}_1 - \vec{p}_2)$$

$$\times \bar{u}(\vec{p}_1')\gamma_5 u(\vec{p}_1)\frac{1}{(p_1 - p_1')^2 - \mu^2}\bar{u}(\vec{p}_2')\gamma_5 u(\vec{p}_2),$$

The corresponding relativistic and properly symmetrized NN quasipotential is

$$\tilde{V}_{NN}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) = \left\langle b_c^\dagger(\vec{p}_1') b_c^\dagger(\vec{p}_2')\Omega \mid K_{NN} \mid b_c^\dagger(\vec{p}_1) b_c^\dagger(\vec{p}_2)\Omega \right\rangle$$

or in the terms of the Feynman-like 'propagators' :

$$\tilde{V}_{NN}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) = -\frac{1}{2}\frac{g^2}{(2\pi)^3}\frac{m^2}{2\sqrt{E_{\vec{p}_1}E_{\vec{p}_2}E_{\vec{p}_1'}E_{\vec{p}_2'}}}\delta(\vec{p}_1' + \vec{p}_2' - \vec{p}_1 - \vec{p}_2)$$

$$\times \bar{u}(\vec{p}_1')\gamma_5 u(\vec{p}_1)\frac{1}{2}\left\{\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_2 - p_2')^2 - \mu^2}\right\}$$

$$\bar{u}(\vec{p}_2')\gamma_5 u(\vec{p}_2) - (1 \leftrightarrow 2). \qquad (25)$$

Distinctive feature of the potential (25) is the presence of covariant (Feynman-like) "propagator",

$$\frac{1}{2}\left\{\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_2 - p_2')^2 - \mu^2}\right\}.$$

On the energy shell for the NN scattering, that is

$$E_i \equiv E_{\vec{p}_1} + E_{\vec{p}_2} = E_{\vec{p}_1'} + E_{\vec{p}_2'} \equiv E_f,$$

this expression is converted into the genuine Feynman propagator. It is typical of other interactions in question.

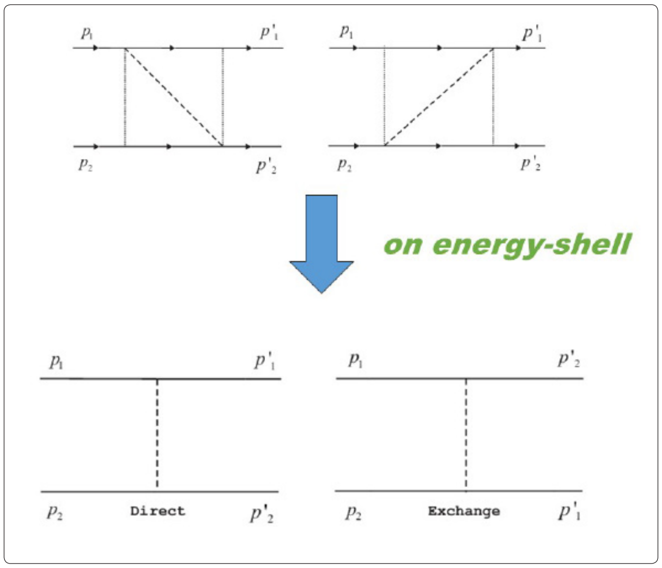

Figure 1: The one-meson-exchange off-energy-shell graphs (upper) and Feynman diagrams (lower) for NN scattering

Potential B by Bonn group can be obtained from UCT quasipotentials with help of replacements

- for boson propagators

$$[(p'-p)^2 - m_b^2]^{-1} \longrightarrow -[\vec{p}' - \vec{p})^2 + m_b^2]^{-1}$$

- for cutoff functions

$$\left[\frac{\Lambda_b^2 - m_b^2}{\Lambda_b^2 - (p'-p)^2}\right]^{n_b} \longrightarrow \left[\frac{\Lambda_b^2 - m_b^2}{\Lambda_b^2 + (\vec{p}' - \vec{p})^2}\right]^{n_b}$$

- omitting off-energy-shell correction in tensor-tensor term

$$\frac{f_v{}^2}{4m^2}(E_{p'} - E_p)^2 \bar{u}(\vec{p}')[\gamma_0\gamma_\nu - g_{0\nu}]u(\vec{p})\bar{u}(-\vec{p}')[\gamma^0\gamma^\nu - g^{0\nu}]u(-\vec{p}) \longrightarrow 0$$

Theory and experiment

We will show our calculations in the CPR versus those with the Bonn potential. In this context we have tried to set links between the Kharkov potential built up by using the UCT method and the high-precision covariant one-boson-exchange potentials for the neutron-proton scattering below 350 MeV [7,11,12, 14, 15].

Calculations of the *np* phase shifts

It has turned out that the best-fit values of the adjustable parameters (the coupling constants and boson form factor masses Λ_b), which provide a fair treatment of the available neutron-proton scattering data below the pion production threshold can be considerably different for the potential models used in our analysis (Table 1).

The energy dependences displayed with the UCT curves in Figures. 2, 3 and 4. have been obtained by solving the partial Lippmann-Schwinger equations (coupled and uncoupled) for the *R*-matrix of the nucleon-nucleon scattering. The rhombs are original OBEP values. Other details are in [7].

Table 1: The best-fit meson parameters for the two models. Column *Bonn* is taken from Table A.1 in [14], while columns *UCT3* and *UCT4* fit (via a least squares procedure from [12]), respectively, the OBEP values in Table 1 of survey [14] and the phase shifts for the model *WJC* -1 (see Tables VII - VIII in [15]). All masses are in MeV.

Meson		Bonn B	UCT3	UCT4
π	$g_\pi^2/4\pi$	14.4	14.67	14.31
	Λ_π	1700	2497	2364.25
	m_π	138.03	138.03	138.03
η	$g_\eta^2/4\pi$	3	6.11	4.67
	Λ_η	1500	955.0	1188.87
	m_η	548.8	548.8	
p	$g_p^2/4\pi$	0.9	1.54	1.38
	Λ_p	1850	1483	1469.78
	f_p/g_p	6.1	5.2	5.75
	m_p	769	769	769
ω	$g_\pi^2/4\pi$	24.5	28.13	28.25
	Λ_ω	1850	2061	2017.27
	m_ω	782.6	782.6	782.6
δ	$g_\omega^2/4\pi$	2.488	2.04	1.85
	Λ_δ	2000	2349.97	2004.05
	m_δ	983	983	983
σ, T = 0; T = 1	$g_\sigma^2/4\pi$	18.3773, 8.9437	18.576, 11.11	19.20,10.93
	Λ_σ	2000, 1900	1611.54, 1986	1727.02, 2241.14
	m_σ	720, 550	713.04, 565.4	721.58, 567.03

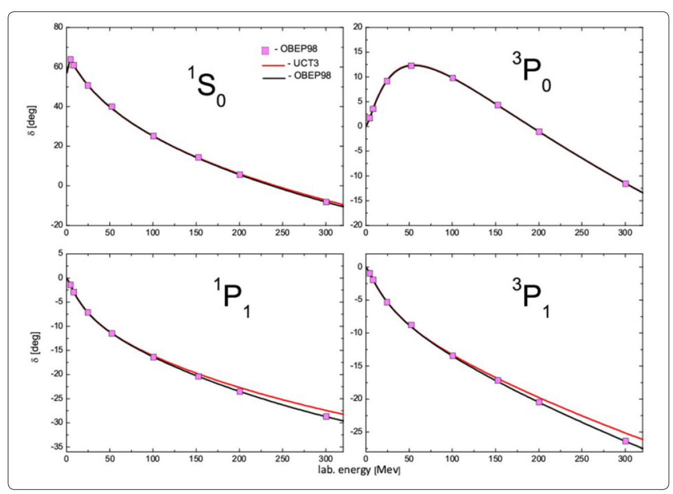

Figure 2: Neutron-proton phase-shifts for the uncoupled partial waves vs the nucleon kinetic energy in the lab. frame

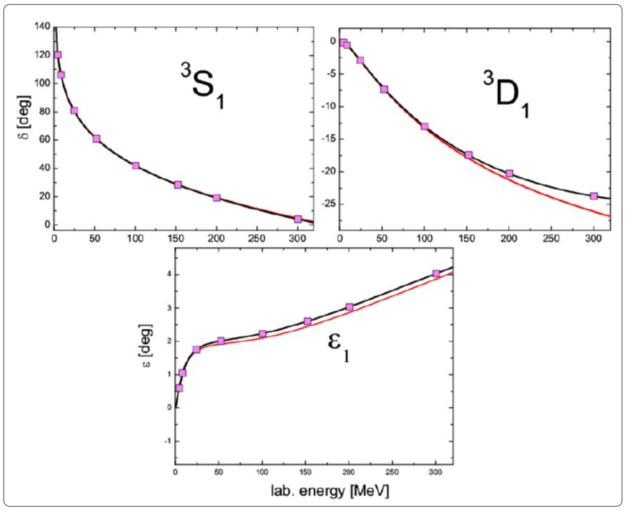

Figure 3: The same as in Figure 2 for the coupled waves and the mixing parameter ε_1 that regulates $^3S_1$ - $^3D_1$ transitions

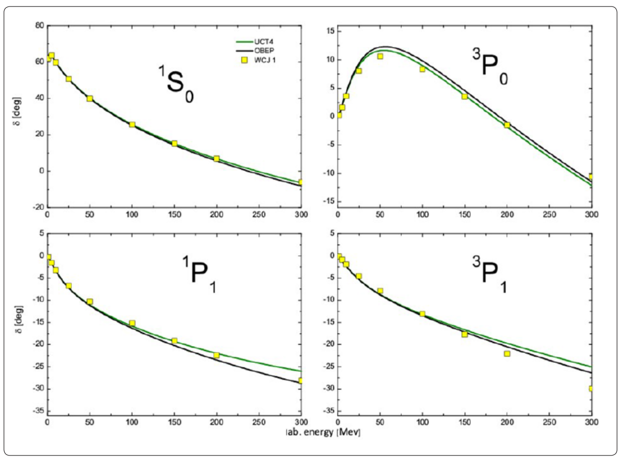

Figure 4: The same as in Figure 2, but with the respect to a high-precision phase shifts [15]

Here we show off-energy-shell *R*-matrices $R(p', p_0)$ for the first partial waves. Recall that on-shell *R*-matrix elements $R(p_0; p_0)$ are proportional to the tangent of the phase shift $\tan\delta(p_0)$.

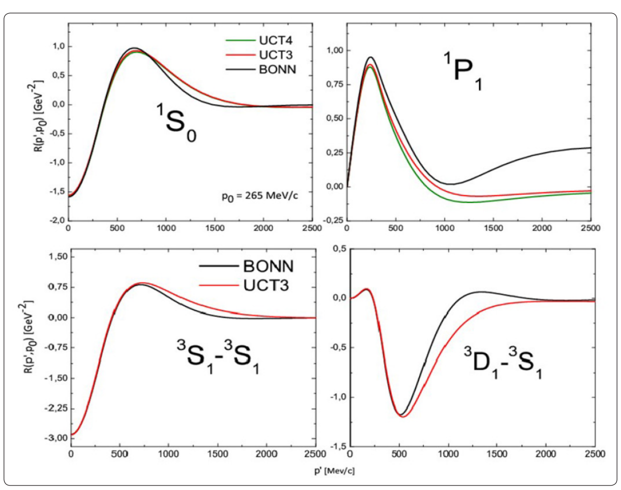

Figure 5: Half-off-shell R-matrices for uncoupled waves at lab. energy equal to 150 MeV (p_0 = 265 MeV)

Figure 5 demonstrates that a good on-energy-shell t-matrix does not mean the same for off-shell *R* - matrix elements.

Deuteron and triton properties in momentum space

In this connection some results of our calculations are collected in Tables 2 and 3.

Table 2: Deuteron and low-energy parameters. The experimental values are from Table 4.2 [14]

Parameter	Bonn B	UCT	Experiment
a_s (fm)	-23.71	-23.57	-23.748±0.010
r_s (fm)	2.71	2.65	2.75±0.05
a_s (fm)	5.426	5.44	5.419±0.007
r_t (fm)	1.761	1.79	1.754±0.008
ε_d (Mev)	2.223	2.224	2.224575
P_D (%)	4.99	4.89	

Table 3: Triton binding energies of Kharkov potential versus other popular solutions (in MeV)

Potentials	Relativistic (Nonrelativistic)	Difference
Kharkov (UCT3)	-7.72 (-7.838)	0.066
Bonn	-8.14	0.099
CD-Bonn	-8.150(-8.248)	

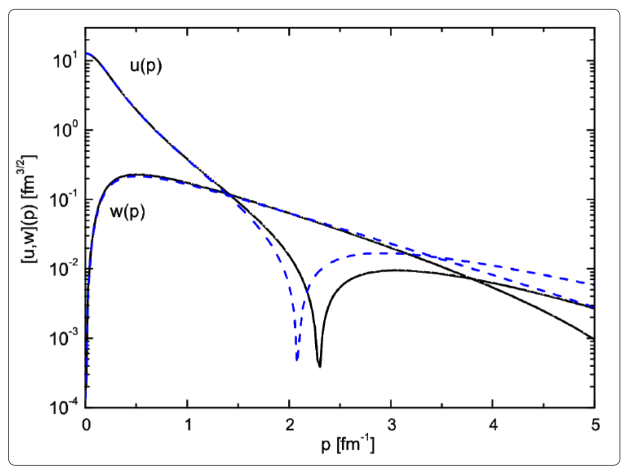

Figure 6: Deuteron wave functions $\psi^d_0(p) = u(p)$ and $\psi^d_2(p) = w(p)$. Solid(dotted) curves for Bonn B (Kharkov) potential

Deuteron states are normalized as $\int_0^\infty p^2 dp \left[\psi_0^2(p) + \psi_2^2(p)\right] = 1$.

At last, we see in Figure 7 the model dependence of the nucleon momentum distributions in the triton and deuteron.

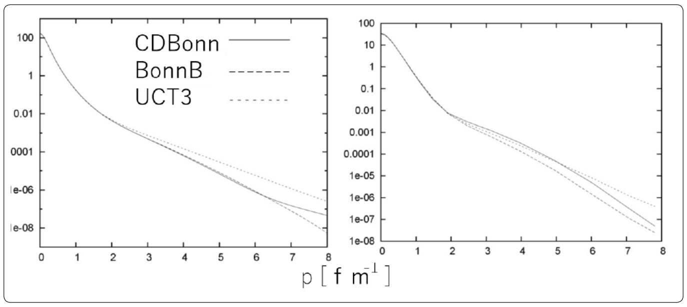

Figure 7: Deuteron (left) and triton (right) nucleon momentum distributions

Summary

- Starting from a total Hamiltonian for the interacting meson and nucleon fields, we come to the Hamiltonian and boost generator in CPR, whose interaction parts consist of new relativistic interactions responsible for physical (not virtual) processes, particularly, in the system of bosons (π–, η–, *ρ*–, ω–, δ– and σ–mesons) and fermions (nucleons and antinucleons).

- The corresponding quasipotentials (these essentially nonlocal objects) for binary processes $NN \to NN$, $N\bar{N} \to N\bar{N}$, etc. are hermitien and energy independent. It makes them attractive for various applications in nuclear physics. They embody the off-shell and recoil effects (the latter in all orders of the $1/c^2$ - expansion) without addressing to any off-shell extrapolations of the *S*-matrix for the NN scattering.

- We have seen the successful applications of the Kharkov potential in describing different 3N observables.

- As a whole, persistent clouds of virtual particles are no longer explicitly contained in the CPR, and their influence is included in properties of clothed particles (these quasiparticles of UCT method). In addition, we would like to stress that problem of the mass and vertex renormalizations is intimately interwoven with constructing the interactions between clothed nucleons. Renormalized quantities are calculated step by step in the course of the clothing procedure unlike some approaches, where they are introduced by "hands".

Acknowledgements

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