## Short Communication

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# Finite Symmetries, Orientations of Bases, Extended CPT for Gravity and the Standard Model 

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The author suggested three 8-dimensional vector spaces with different multiplications and number of generators for SI with the $\mathrm{SU}(3) 8$ gluon GellMann matrices $\lambda \mathrm{k}$, the signed quaternion $\mathrm{SU}(2)$ generators (Pauli matrices $\sigma \mathrm{j}, \mathrm{j}=1,2,3$ and id ) of the weak interaction WI and newly added is in the MINT-Wigris project the octonians which have seven Gleason measuring frames GF, not only the weak Pauli spin GF. Developing a unified theory for the four basi interactions the doubling of quaternion numbers to octonians is necessary. The use of the GF real cross product was necessary to double up the complex numbers to quaternions. As projections the $\lambda \mathrm{k}$ matrices are 3-dimensional blown up oj matrices with a row and column of coordinates 0 added. The blown up $\sigma 3$ matrices are linearly independent and give only 2 not 3 gluons as geometrical invariants. The 3 WI invariants are the weak bosons $\mathrm{W}+$, W - and $\mathrm{Z}^{\circ}$ and one more invariant photon for EMI for the electromagnetic interaction with the $U(1)$ symmetry, a circle.

For the octonians there are many symmetries, not just one as for EMI, WI, SI. Gravity on this base is a multivalued task and it is in superposition with other interactions or forces.

A first consideration that quasiparticles like spin, rgb-gravitons, electromagnetic induction (magnons), phonons, rotons etc. need for their energies geometries a conic whirl character. Whirls are computed mathematically different from waves. In motion a vector can have its initial point as tip fixed and rotates on a cones surface with its upper endpoint, tracing out a circle. For the wave-particlewhirl characters are suggested three 2-dimensional bases where the octonian coordinates are now listed by their indices $0,1,2, \ldots, 7$. The octonian GF 167 is used for 17 as a G-compass, The G-matrix of order 6 is the scaled Schwarzschild radius Rs matrix with its rows for the extended base (1 100 ), ( 1000 ). When (ijk) from the quaternions are taken as first row and the usual matrix determinant is computed the vector is $0 \cdot i+0 \cdot j+1 \cdot \mathrm{k}$. The triple $\mathrm{i}, \mathrm{j}, \mathrm{k}$ can be taken as a left-hand, lh screw.

This is for matter waves with the particle character. For the wave character the scaled Minkowski scaling factor M matrix of order 2 gives as base $\left(\begin{array}{lll}-1 & 1 & 0\end{array}\right),\left(\begin{array}{lll}0 & 1 & 0\end{array}\right)$ and applying the former calculation -k is obtained for a right hand rh screw orientation and the wave character. For such waves the exponential function exp on 7 has a polar angles period $2 \pi$. For whirls a base ( $\left.\begin{array}{lll}1 & 1 & 0\end{array}\right),\left(\begin{array}{lll}-1 & -1 & 0\end{array}\right)$ is suitable and $0 \cdot \mathrm{k}$ is otained for a plane having for the unit circle mpo
counter clockwise and cw clockwise orientation for rotations. As an example the octonina 145 GF has on the circle 1 an electrical charge e0 for electrons cw and mpo for positrons rotating, the magnetic flow quantum $\Phi_{0}=h /\left(2 e_{0}\right)$, $h$ the Planck constant, is presented as a rotating vector on a cone with this bounding circle and tip the initial point of $\Phi 0$ with an up or down directed whirl as for spin. This is called a whirl inversion. The G inversion is for radius of mass systems at the Schwarzschild radius $r^{6} r=R^{2}$ and for speeds at the speed of light $v^{\star} v=c^{2}$, $c$ speed of light.

For a $r g b$-graviton whirl as superposition of three SI red, green, blue color charge whirls the 126 SI whirl are $\lambda \mathrm{j}, \mathrm{j}=1,2,3$. The rotation for $r$ is always $c w$, for green mpo and the rh triple rgb has its endpoints on $+x,+y,-z$ for the location of a nucleon triangle in space with three quarks at the vectors endpoints. The triangle symmetry has three reference points $0,1, \infty$ located at the sides connecting the vertices $+x+y,+x-z,+y-z$. They are degenerate numerical orbit of the triangles symmetry which generates with complex cross ratios six energies E(pot) with the Moebius transformation MT z for scalings, the MT $1 / z$ for potentials and frequency (kinetic energy) and ( $\mathrm{z}-1$ )/z for rotational energy. In the nucleon GF symmetry with a variable complex number z as tip the cross products ( $\mathrm{z}, \mathrm{a} ; \mathrm{b}, \mathrm{c}$ ) with the permuted numbers $0,1, \infty$ have then for particles and barycenters cross ratios the permutation $(0 ; 1, \infty)$ to $(\infty ; 1,0)$ for matter waves, for waves $(1 ; 0, \infty)$ to $(\infty ; 0,1)$ and for whirls $(0 ; \infty, 1)$ to $(1 ; \infty, 0)$. In these couplings the first pairs for the octonians 1,5 as position x and momentum p uncertainty, in a formula known as $\lambda \mathrm{p}=\mathrm{h}, \lambda$ wave length, 6,4 energy/frequency and time $\mathrm{E}=\mathrm{hf}$, f as inverse time interval, uncertainty and angle angular momentum uncertainty $\varphi \mathrm{J}$ $=\mathrm{h}$ for 2,3 .

For the Schwarzschild scaling the octonian 145 GF is taken with radius and its metrical differential dr on 1 with scaling factor $1 / \mathrm{cos}$ $\beta$ of Schwarzschild metric, also for the GF 145 mass (matter wave) 1 norming, normed second cosmic speed $\mathrm{v} 12 / \mathrm{c}^{2}$ as $\sin ^{2} \beta=\mathrm{Rs} / 2 \mathrm{r}$ for the 145 GF taken on 5 and $\mathrm{v} 2=\mathrm{v} 1 \sqrt{ } 2$ on 4 as time $t$ coordinate. In Planck numbers $\mathrm{ct}=\mathrm{Gm} / \mathrm{c}^{2}$ for equal time 4 and mass 5. Using the Minkowski cone line $\mathrm{r}=\mathrm{ct}$, the Schwarzschild radius Rs/2 $=\mathrm{r}$ $=\mathrm{Gm} / \mathrm{c}^{2}$ (scaled) occurs. 145 is also for electromagnetism which is not repeated here. For Minkowski metric the G compass matrix with Rs as scalar is replaced by the M matrix and $\sin \theta=\mathrm{v} / \mathrm{c}$, v relative speed, as scalar and the SI GF 126 of $r g b$-gravitons is chosen. On 1 is taken the length $x \cdot \cos \theta$ scaling, on 6 the kinetic energy $\mathrm{mv}^{2} / 2$
scaling as $\mathrm{p}=\mathrm{mv}$ with $(\mathrm{c} / 2) \cdot \sin \theta$ taken as momentums v and for heat the scaling with $(\sin \theta) / k$ in Planck numbers for heat. Energy is here taken as (waves) kinetic momentum. The Minkowski watch is available for rescaling energy units, not only length or time. In both cases, Minkowksi and Schwarzschild metrics, areas of length and time or of their differentials are preserved.

For whirlsangular momentum $\mathrm{J}=\mathrm{mr}^{2}$ as inertial mass replaces mass on 5 and on speed is angular speed $\omega=2 \pi$. The GF 356 can be chosen. An area not length scaling (for mass on a disk) is involved. If the rotational energy scaling is similar as for waves, $m$ is replaced by $J$ on 5. Then the former relativistic sinus speed is replaced by $\omega$ and p by angular momentum $\mathrm{L}=\mathrm{J} \omega$ on 3, rotational energy on 6 is then $\mathrm{L} \omega / 2.356$ is also used for the SI rotor where the three force vectors rotational on 3, kinetic on 6 mass potential on 5 are attached to the three quarks of a nucleon. In this combinationforce integrations are performed, reversing the Heisenberg differentiations with $\mathrm{d} / \mathrm{dx}(\mathrm{d} / \mathrm{dr})$ for momentum, $\mathrm{d} / \mathrm{dt}$ for linear kineticand angular rotation energies. Integrations are also radial about areas and volumes.

There are three rotors for POT with $\mathrm{a} / \mathrm{r}^{2}$ integrated for the electrial dr on 1 and gravity on 5 potentials. The 3,6 for rotational and kinetic time dt integrations and for magnetic field strength on 4 an area integration/differentiation is for the flow through the area and induction. For heat on 2 and pressure a volume dV integration/ differentiation is used in pV as scaled temperature. Density of matter/ mass $m$ or energy inside a scaled volume $r^{3}$ is then in Planck numbers obtained from $(\mathrm{hc} / \mathrm{G}) /\left(\mathrm{hG} / \mathrm{c}^{3}\right)^{3}=\left(1 / \mathrm{h}^{2} \mathrm{G} 4\right) \mathrm{c}^{10}$ is square root. The SI and WI rotors are for 2,3 and 4,6 , SI combining anangle (entropy in a volume) and angular momentum (pressure on the V surface) and WI combining time (magnetic field transversal crossing an electrical currents loops area) and momentum/frequency (induction as cross product and rotational momentum for the loop).


Figure 1: hedgehog with input/output vectors for energy exchange of a nucleon or atomic kernel AK, 6-roll mill for a quarkgluon plasma as flow about the three rotors coupled energy rolls 15 red turquoise, 23 green magenta, 46 yellow blue

For differentiations the hedgehog figure applies, for integrations the SI rotor dynamics which is a representations of the triangle symmetry D3 group. The rgb-graviton tetrahedron of a nucleon is described in the literature, having the S4 symmetry and the ehdegehog for six energies exchange of the atomic kernels composed of nucleons is described as the hedgehog. The quotient of the $\mathrm{S}_{4}$ group by its normal Klein subgroup Z 2 xZ 2 is $\mathrm{D}_{3}$. Th quotients factor represents the CPT symmetry of Physics ( C conjugation, P space parity, T time reversal).

Parity sets the projective norming of the Riemannian sphere $S^{2}$ to the projective plane on which the hedgehog input/output vectors rotate
between up/down positions on a Moebius strip inside the polar cap $\mathrm{P}^{2}$. The AK is like a 4-dimensional bubble observable in space and has 3 bounding spheres, weak Heegard splitting into the six caps as hemispheres. Conjugation is for the reversal of all quantum numbers of a nano system like a nucleon or fermions. T is for instance for sound expansions in a Mach conic whirl towards the past of a system in motion like an airplane or the sound of a big bang.

Earlier a degenerate orbit of symmetry groups in form of $0,1, \infty$ is mentioned. There are more such numerical degenerate orbits in use. The $\mathrm{D}_{3}$ basic spin orbit $1 / 2,|-1|, 2$ is for fermions, bosond $r g b$-graviton spins. Another is for cubic roots as vertices of a nucleon triangle. The G-orbits of the G-compass with the polynomial $z^{6}-1$ are for $\pi / 3$ orbits in form of the six electrical charges, the color charge orbits where a charge spreads out in the needles motion between two sixth ${ }^{\text {‘ }}$ roots on the enclosed area of the compass, also mass GFs with 3 real or 6 complex mass values are produced. For the electromagentic or neutral masses applies 3 for a real GF and for quarks a complex scaled GF which allows then also complex rescalings of coordinate triples. Six energies are set on the color charges. The six octonian coordinates for space xyz are 1 x or radius on red, 2 y or $\varphi$ as polar angle on green, 3 z or $\theta$ as spherical angle, 4 t time on yellow.

For 5,6 a projective Einstein plane with coordinates [m,f,w] and the mass frequency line $\mathrm{mc}^{2}=\mathrm{hf}$. The projective w -line at infinity allows with $\mathrm{w}=0$ the quotient $[\mathrm{m} / \mathrm{f}, 1]$ closed by $[1,0]$ for setting a masses barycenter with Higgs bosons or a Higgs field. The variable $u=m / f$, not the Einstein lines projective point $\mathrm{h} / \mathrm{c}^{2}$ at infinity on the w -line, can be interpreted as $\mathrm{d} \omega$ differentiatin grotational energies $\omega^{2} / 2$ to $\omega$ of angular momentum orreversely an integration, as in the case of kinetic energy $\mathrm{d}^{2} \mathrm{x} / \mathrm{dt}^{2}$, a dt integration of $\mathrm{d}^{2} \varphi / \mathrm{dt}^{2}$ to $\mathrm{d} \varphi / \mathrm{dt}$. Speed $v=d x / d t$ or angular speed $\omega$ are substituted for $u$.

From the GF xyz-triple vectors with a bounding $\mathrm{S}^{2}$ sphere carrying the 3 vectors weights for a Gleason mesuring operator it is observedd that as well real positive as non-zero (complex or quaternionic measured) numbers can be used for the weights. Also the orientations of the vectors are important. The cross product is only one orientation, but there can be the up/down orientations +++ or- - - for generated spin like vector $\mathrm{s}=(\mathrm{sx}, \mathrm{sy}, \mathrm{sz})$ triples ( 3 weights are equal). -++ is for rh-srews, +- - for lh screws in space, +-+ for mpo rotations, -+- for cw rotations in a plane (two weights can be equal), ++ - for a uud-quark proton and --+ for a ddu neutron (two rgb-graviton tetrahedrons in deuteron exchangeing isospin).

As projective Gleason measures they generate many measuring quadrics and shapes for energy location and transfer. The many shapes of projective normal forms in 2,3 or 4 dimensions are observed. Fractal shapes like cusps for wrinklon quasiparticles, for instance studied in catastrophe theory (the 6-roll mill belongs to this) are observed breaking ocean waves or ships motion in water moving about it. For the known list of quasiparticles fratcal shapes arising from catastrophe theory can be studied as research project. Projective correlations changing dimensions are observed for fiber bundles. The Hopf geometry for electromagnetism has the $S^{3}$ unit sphere in spacetime and the three Pauli matrices map it down to the Riemannian sphere with the Hopf fiber bundle circle as fiber. Points of $\mathrm{S}^{2}$ are blown up to circles in $\mathrm{S}^{3}$. Two MINT-Wigris fiber bundles are for SI and atomic kernels. The SI geometry is a trivial fiber bundle $\mathrm{S}^{3} \mathrm{xS} 5$ and the 3-dimensional sphere in the complex octonian

3-dimensional space 123456 maps with a normed $\mathrm{S}^{1}$ fiber down to the atomic kernel or nucleon bubble $\mathrm{CP}^{2}$. It has a 4-dimensional inner spacetime and dynamics and a bounding Riemannian sphere. Three such bounding spheres are allowed for the hedgehog.

The three 8-dimensional octonian, $\mathrm{SU}(3)$, signed quaternionen spaces are sufficient to describe with their subspaces, new metrics, orbits in them for shapes or numbers, new symmetries and operators the known facts for gravity along the scheme of the (revised) standard model. As 4-dimensional coordinate systems are used (possibly togethet with orthogonal time, or on a space coordinate located) orthogonal Euclidean xyz-space coordinates, spacetime with Minkowski or Schwarzschild metric, spherical coordinates, cylindrical coordinates for EMI and barycentrical coordinates for mass. Not all is repeated in this report, but can be read in the publications of the author. Her articles needed also revisions later on, not only the standard model [1-12].

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