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Review Article

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# Estimating Volatility of Daily Price Returns of Nigerian Stock Market

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**Abstract**
*The financial markets are opened to market risk. Modelling the volatility in daily stock prices entails studying the particular error distribution that is most appropriate for the model. Considering a particular Nigeria stock market from April 1, 2016 to December 16, this study estimates both the symmetric and asymmetric volatilitymodels.The ARMA-GARCH, ARMA-EGARCH models were employed with the error distributions such as normal distribution, student t-distribution and skewed student t- distribution. The ARMA (2,1)-EGARCH (1,1) with student t-distribution was seen to be the most appropriate model. A volatility forecasting accuracy was determined by using the mean absolute scaled error (MASE) to predict the values of the stock market prices for the next 20 years and the result showed that the model was appropriate for predicting volatility.Hence volatility prediction would help inachieving a sound policy decision.*

**Keywords:** GARCH, EGARCH, Stochastic Volatility Model, Error Distribution.

## Introduction

The financial market is affected by volatility that is the rate at which the prices of assets rise and fall given a particular set of returns.

In a study of testing volatility in Nigeria Stock market using GARCH models, Ngozi Atoi (2014) explains that through the mobilization of long-term money for future investment, the capital market section of the financial market contributes significantly to the process of economic expansion around the world. For instance, the stock market in Nigeria aids in the long-term financing of government development projects, acts as a funding source for long-term investments in the private sector, and acted as a catalyst for the consolidation of the banking system in 2004 and 2005 [1].

A stock market is a recognized legal environment where shares of numerous businesses or organizations can be traded. A rising stock market is regarded as a sign of a developing economy. It is frequently seen as the main metric of a nation's economic health and development. Simeyo Otienoetal [2]. Similarly, in another study of modelling volatility in selected Nigerian stock Market, Ekum Matthew Iwada and Owolabi Toyin Omoyeni suggest that Investment analysts, economists, and policymakers place a lot of attention on the stock market since it can be used to gauge changes in overall economic activity using the stock prices of listed companies on the Nigerian Stock Exchange (NSE) [3].

The stock market is exposed to risk and this risk which is the risk of losses in position brought about the movements on market variables like prices and volatility. In addition, volatility is the rate at which the price of assets rises and falls given a particular set of returns. Also, it is a measure which investors look at before making any trading decisions. U. Usman etal explains that in these situations, the assumption of constant variance (homoscedasticity) is unsuitable since the stock market demonstrates fluctuations in variance over time. The variance in the financial statistics can be as a result of volatility in the financial market [4]. On the other hand, in a study, Pushpaetal examines volatility prediction as a crucial instrument in financial economics for risk management and asset allocation since knowledge of upcoming volatility can help investors reduce losses [5].

Volatility has been a major concern to stock market; due to this fact many studies have modeled volatility with several volatility models. Just like in the study of modelling and forecasting of all India monthly average wholesale price volatility of onion. The GARCH and EGARCH techniques were applied. While another study on fitting the Nigeria stock market return series using GARCH models made use of several. However, this current study will estimates the first order symmetric and asymmetric volatility models Nigeria stock market from April 1, 2016 to December 16, each model in Normal, Student's-t and skewed student's –t error distributions then selecting the best forecasting volatility model with the most appropriate error distribution. And comparing the best model to the stochastic volatility model.

## Methodology

### ARCH (q) Model

ARCH models (Autoregressive Conditional Heteroscedasticare formed on the basis of the variance of the error term at time t which is dependent on the realized values of the squared error terms in past time periods. That is, it is able to capture the behaviour of volatility as a function of time. More so, it takes into account the number of stylized facts that characterize the majority of financial series such as persistence, volatility clusters and leptokurtic behaviour of data. The model is specified as

Let $Y_t$ the return be explained by some variables $X_t$ which can be an ARMA (p, q) model.

$Y_t$- $X_t\beta + \varepsilon_t$, where $\varepsilon_t / I_{t-1} \sim N(0, \delta_t)$. $I_{t-1}$ is all the information available in the returns data up to the time t-1 The ARCH model allows you to set the conditional variance $\delta_t^2$.

$$\text{that is,}\delta_t^2 = \alpha_0 + \alpha_1\, \varepsilon_{t-1}^2 + \alpha_2\, \varepsilon_{t-2}^2 + \ldots + \alpha_q\, \varepsilon_{t-q}^2$$

$$\varepsilon_t = \delta_t\, z_t z_t \sim N(0,1)$$

Where

$\delta_t$ is the conditional variance

$z_t$ is the standard residue

$\varepsilon_t$ is the conditional errors of the asset at the time t

$\alpha_0 \ldots, \alpha_i$ are real parameters

q is order of ARCH process

The value of the conditional variance $\delta_t^2$ must be positive and negative variance at any point in time would be meaningless.

## Generalized-ARCH Model (GARCH)

*let* $Y_t = X_t\beta + \varepsilon_t$

The return be described by some variables$X_t$ .Where $\varepsilon_t / I_{t-1} \sim N$ $(0, \delta_t)$

The GARCH process allows you to set the conditional variance $\delta_t^2$.

$$\text{That is } \delta_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, \epsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j\, h_{t}\text{- j}$$

$\alpha_0$ the parameter Representing the long=term average variance

$\alpha_i$ the parameter that measures the sensitivity to conditional volatility

$\beta_j$ the parameter that measures the persistence of conditional volatility

To get a positive condition variance, one must adhere to the conditions

$\alpha_0 > 0$ and$\alpha_{i \geq}$ 0 for all I =1,2,3,….,q

$\beta_j$ ≥0 for allj =1, 2, 3,…, pq >0 and p≥ 0

## Exponential GARCH (EGARCH) Model

The EGARCH is an asymmetric GARCH model that identifies not only the conditional variance but the logarithm of the conditional volatility.

It is generally recognized that EGARCH model gives a much more in sample in-sample fit than other types of GARCH models.

The EGARH model was put forward by Nelson and the model includes the leverage effects in its equation.

$$\ln(\delta_t^2) = \omega + \beta \ln(\delta_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} + \alpha \left[ \frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

Variance is always positive because

$$\delta_t^2 = \log(\delta_t^2)$$

Even if the parameters are negative, the variance will still be positive. Leverage effect indicates that if the relationship between volatility and returns is negative then

$$\gamma < 0$$

The α shows the symmetric effect, the βmeasures the persistence in conditional volatility shock and reflects the asymmetric performance.

## Error Distribution

This study introduces both the normal and the non- normal innovations. That is we are going to make use of three distributions, the normal distribution, the student- t distribution and the skewed student-t distribution.

## Normal Distribution

The probability density function of $X_t$ is shown as

$$f(X_t) = \frac{1}{\sqrt{2\pi\delta^2}} \exp\left\{-\frac{1}{2}(X_t - \mu)^2\right\}$$

*where μ is mean and δ is the standard deviation*

## Student t-Distribution

The probability density function$X_t$ is given as

$$f(X_t) = \frac{\gamma\left(\frac{v+1}{2}\right)}{\gamma\left(\frac{v}{2}\right)\sqrt{(v-2)\pi}} \left(1 + \frac{x_t^2}{v-2}\right)^{\frac{1}{2}(v+1)}$$

Where v is the number of degrees of freedom, $2 < \nu \leq \infty$, and is gamma function. When v →∞, the student t-distribution is almost equals to the normal distribution. The smaller the v the heavier the tails.

## Skewed Student-t Distribution

The Student t distribution is more effective in generating values that lie far from its mean and capturing heavy tail data sets. On the other hand, it is a symmetric distribution that cannot capture asymmetry. To make room for asymmetry and long tailed data. Hansen (1994) put forward the skewed t distribution whereas retaining the property of a zero mean and variance equal to the Skewed t distribution with degree of freedom = ν degrees of freedom has the following density, where f(x) is the density of the t distribution, with = ν degrees of freedom [6]:

$$f(x) = \frac{2}{\gamma + \frac{1}{\gamma}} f(\gamma x) \; for \; x < 0$$

$$\text{And} f(x) = \frac{2}{\gamma + \frac{1}{\gamma}} f\left(\frac{x}{\gamma}\right) \; for \; x \geq 0$$

## Selection of Model

Selection of model when making comparison among several specification of ARMA-GARCH models.the appropriate model is selected by making use of the Akaike Information Criteria (AIC) and the Bayesian Information Criteria(BIC). The AIC and BIC can be calculated as

AIC = -2ln(residual sum of squares) +2k

BIC = -2ln(residual sum of squares) +ln(N)k

Where N is the number of observations and k is the parameters estimated or estimated coefficients. The least value of AIC and BIC is selected as the more appropriate model when making comparison among models.

## Evaluation of Model

In evaluating the volatility forecasting performance of models, some statistical measures are considered here.

## Data

A daily time series daily price data of Fidelity Bank Nigeria from April-2016 to December- 2022 (total number of observation 1664). Which the GARCH models are applied for prediction. In other to treat volatility, we compute the daily yield as follows:

$$r_t = \left(\frac{p_t}{p_{t-1}}\right) x \; 100$$

Where $p_t$ is the Fidelity bank stock price from April 1 2016 to December 16 2022

**Table 1: descriptive statistics of daily price returns of Fidelity Bank Nigeria**

| N | Mean | Min | Max | SD | Skew | Kurt |
|---|---|---|---|---|---|---|
| 1664 | 0.03 | -9.48 | 116.93 | 3.84 | 0.20 | 2.59 |

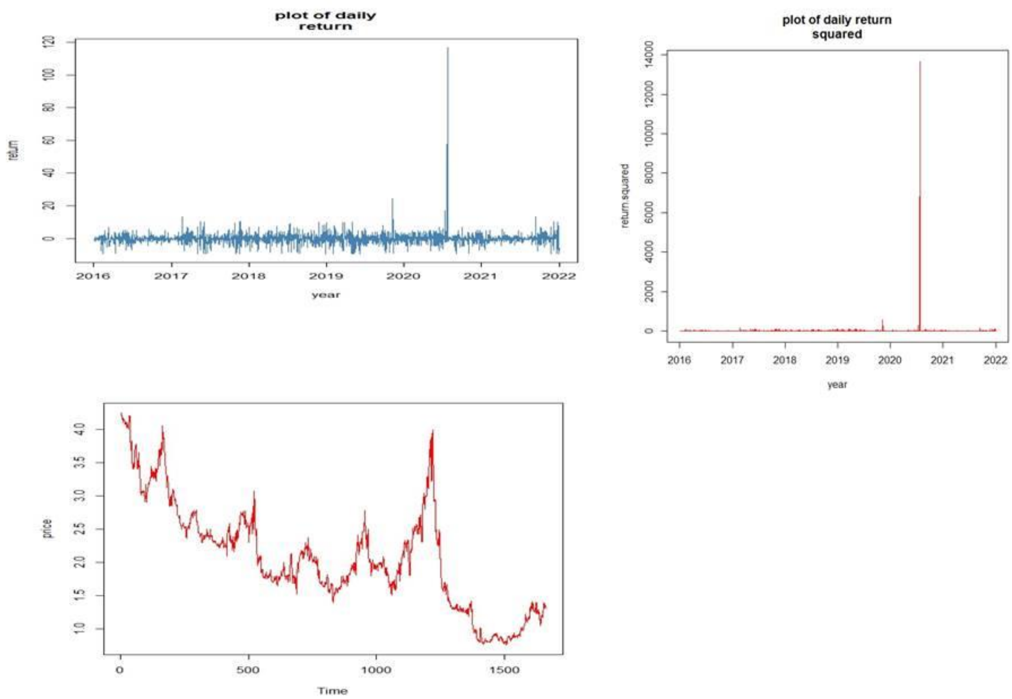


**Figure 1:** Plots of Daily returns

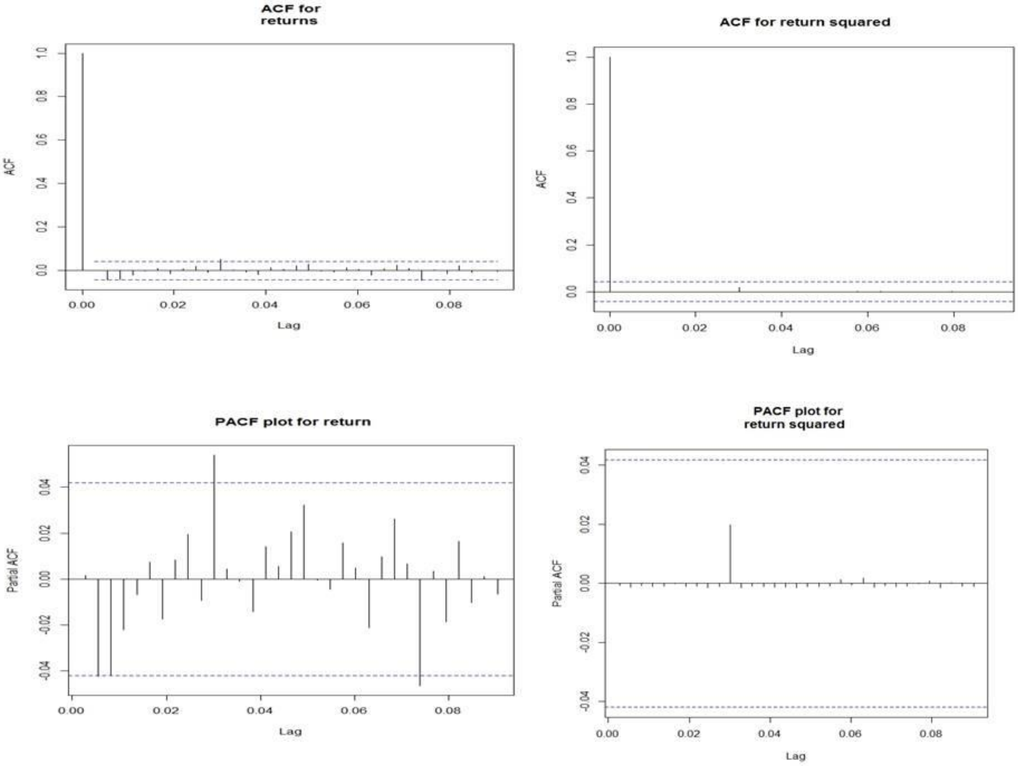


**Figure 2:** The autocorrelation of the series sample

Table 1 shows the skewness is 0.20 that means is positively skewed, it is not zero, we can say that the return is asymmetric. While the kurtosis is seen to be 2.59 which is less than three and this means that the distribution of return is said platykurtic that is the distribution tend s to produce fewer and less outliers than the normal distribution.

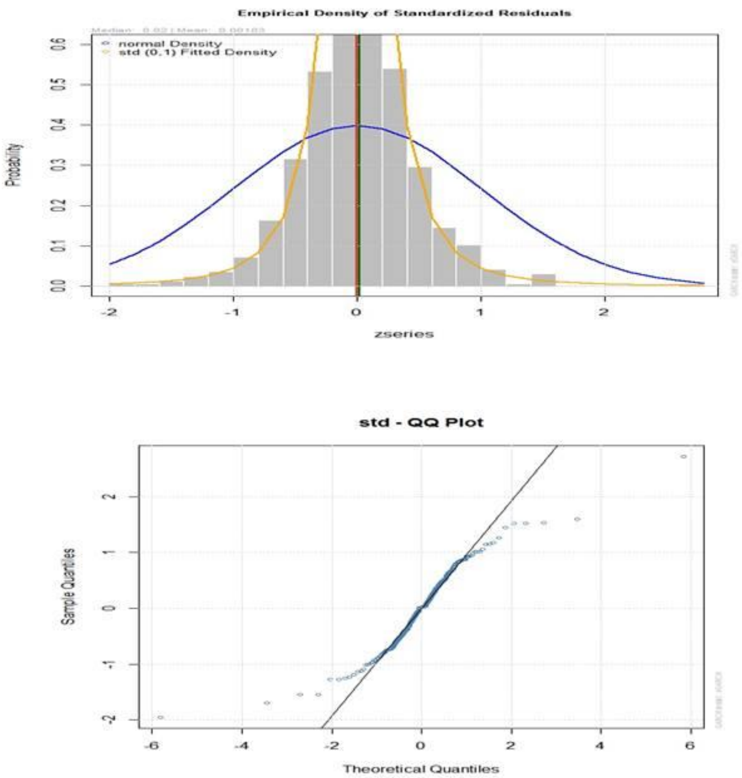


**Figure 3:** the Density and the QQ plot of Standardized Residuals of ARMA (2.1)-EGARCH (1, 1) with Student t-Distribution

## Results

In this result, ARMA-GARCH and ARMA-EGARCH models are used in estimating are forecasting the daily returns of the aforementioned stock market in Nigeria under different error distributions. That the normal distribution, student t-distribution and the skewed student t-distribution. Comparison is made using forecasting measures such as MASE, then choosing the appropriate volatility prediction model.

## Model Selection for Mean Equation and Variance Equation

In selecting the appropriate mean equation ARMA (p, q) and Variance equation GARCH (p, q), EGARCH (p, q) model for the daily price returns. The autocorrelation and the partial autocorrelation of the daily returns are observed. Thus the other of p and q can be obtained.

The Autoregressive integrated moving average (ARIMA 2,0,1) is selected within the candidate models by thoroughly examining the selection of parameters. Comparing ARMA (2,1)-GARCH(1,1) and ARMA(2,1)-EGARCH(1,1) models under three error distributions by using the AIC and BIC values for different models, the EGARCH(1,1) with student t-distribution is seen to be the best possible candidate model. The estimated parameters of ARMA (2,1)-EGARCH(1,1) model with student t-distribution is represented in Table 4.

The variance equation of the estimated EGARCH (1,1) model is shown below

$$\ln(\delta_t^2) = 0.121 + 0.971\ln(\delta_{t-1}^2) + 0.521\frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} + 0.148\left[\frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right]$$

**Table 3: Selection criteria of model**

| Model | Distribution | AIC | BIC |
|---|---|---|---|
| ARMA (2,1)-GARCH (1,1) | Normal distribution | 5.337 | 5.339 |
| ARMA (2,1)-GARCH (1,1) | Student t-distribution | 4.831 | 4.856 |
| ARMA (2,1)-GARCH (1,1) | Skewed student t-distribution | 4.832 | 4.860 |
| ARMA (2,1)-EGARCH (1,1) | Normal Distribution | 4.596 | 4.642 |
| ARMA (2,1)-EGARCH (1,1) | Student t-distribution | 4.443 | 4.494 |
| ARMA (2,1)-EGARCH (1,1) | Skewed student t-distribution | 4.444 | 4.501 |

**Table 4: Coefficient Estimates of ARMA (2,1) –EGARCH (1.1) M odel with Student t=distribution**

| | Parameter | Standard Error | t-Statistics | P-Value |
|---|---|---|---|---|
| Mean Equation | | | | |
| $\mu_0$ | -0.109 | 0.040 | -2.711 | 0.007 |
| AR (1) | -0.354 | 0.041 | -8.631 | <0.001 |
| AR (2) | -0.052 | 0.018 | -2.930 | 0.003 |
| MA (1) | 0.363 | 0.041 | 8.831 | <0.001 |
| Variance Equation | | | | |
| Ω | 0.121 | 0.041 | 2.983 | 0.003 |
| $\alpha_1$ | 0.148 | 0.123 | 1.199 | 0.045 |
| $\beta_1$ | 0.971 | 0.004 | 253.730 | <0.001 |
| $\gamma$ | 0.521 | 0.356 | 1.465 | 0.050 |
| $v$ | 2.120 | 0.135 | 15.757 | <0.001 |
| $\alpha_1+\beta_1$ | 1.119 | | | |

The table 4 shows that all the ARMA(2,1)-EGARCH(1,1) model parameters are statistically significant and the shock persistence parameter ($\beta_1$=0.971) is near to the unity which means that the conditional variance has a long memory and the volatility shock is reasonably persistent. More so, the leverage effect shown by the EGARCH (1,1) model, indicates that the leverage effect coefficient ($\gamma$=0.521) is positive and statistically significant, implying that the positive shocks have small influence on the future volatility unlike the negative shocks which have a greater influence or impact on the future volatility. That is to say, the more leverage you have the more risk you face. So if there is a drop in the value of stock (negative shock/return) there is an increase in financial leverage which makes the stock risker thus increasing volatility.

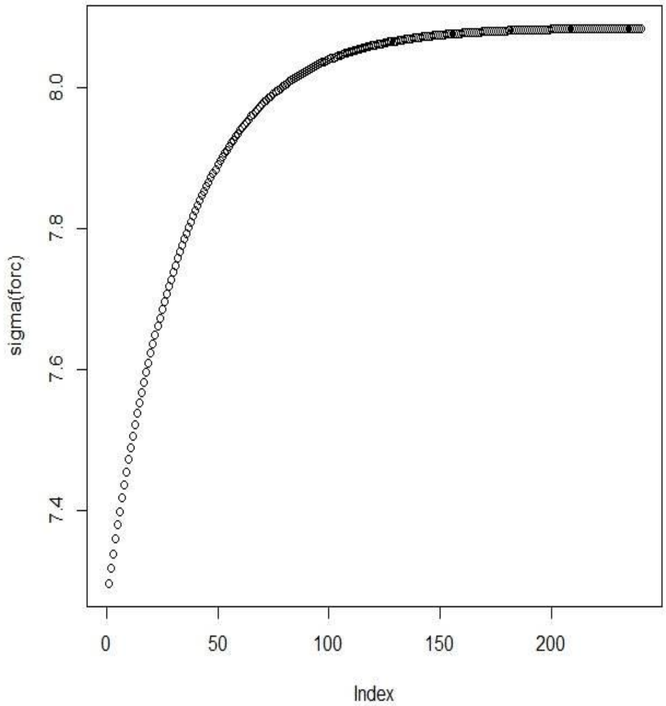


**Figure 4:** Plot showing the Forecast of Volatility for the next 20 years

The Fig 3 show the volatility forecast for the next 20 years (240 months) of the Fidelity bank stock return for the next 20 years. Here we can observe that based on this model, we expect the volatility to potentially increase in the next 120 months (10 years) and remains at the same level for the remaining years as seen in the plot.

### Diagnostic Check for the Selected Model

In determinig whether the model is appropriate, the Ljung-Box test and the ARCH-LM are used in testing serial correlation (autocorrelation). Basically if the p-value is lower than the 0.05, then the null hypothesis is rejected, as we can see from Table 5 and Table 6, the p-value of the two tests are seen to be greater than 0.05. that is, no enough evidence to reject the null hypothesis for all the lags, that is no serial correlation and no ARCH effect in ARMA (2,1)-EGARCH(1,1) model.

**Table 5: Ljung- Box squared residual test for ARMA(2,1) –EGARCH(1,1) model with Student t- distribution**

| ARMA(2,1) –EGARCH(1,1) model with Student t- distribution | | |
|---|---|---|
| Ljung-Box squared residual | | |
| | Statistics | p-value |
| Lag [3] | 4.60 | 0.10 |
| Lag [5] | 4.86 | 0.16 |
| Lag [7] | 6.53 | 0.24 |

**Table 6: ARCH Effect test for ARMA(2,1) –EGARCH(1,1) model with Student t- distribution**

| ARMA(2,1) –EGARCH(1,1) model with Student t- distribution | | |
|---|---|---|
| ARCH test | | |
| | Statistics | p-value |
| Lag [3] | 0.38 | 0.57 |
| Lag [5] | 0.77 | 0.80 |
| Lag [7] | 2.43 | 0.63 |

### Forecast Accuracy

In other to determine the volatility forecasting performance of models, the Mean absolute scaled error (MASE) is used. The MASE measures the forecast error which is compared to the error of naïve forecast. In this study, we predicted the values of fidelity prices for the next 20 years and the MASE calculated with observed values is given as one (1) approximately, which indicates that the ARMA (2,1)-EGARCH (1,1) with model is appropriate in predicting volatility.

### Conclusion

The study evaluated the daily stock prices of Fidelity Bank from April 2016 to December 2022. Both the symmetric and the Asymmetric GARCH models were considered with three different error distributions.The Autoregressive integrated moving average (ARIMA 2,0,1) was selected within the candidate models by thoroughly examining the selection of parameters. Then the ARMA-GARCH, ARMA-EGARCH models were employed with the error distributions such as normal distribution, student t-distribution and skewed student t- distribution. The ARMA (2,1)-EGARCH (1,1) with student t-distribution was seen to the most appropriate model. A volatility forecasting accuracy was determined by using the mean absolute scaled error to predict the values of the stock market prices for the next 20 years and the result showed that the model was appropriate for predicting volatility. Furthermore, serial correlation was checked by using the Ljung-Box test and the ARCH effect test and there was no autocorrelation or serial correlation. Finally the descriptive statistics were estimated and the results were shown in Table 1 also the Density and QQplots of the standardizes residuals for ARMA-EGARCH model are seen in Fig 3.

## Conflicting Interests

Authors have declared that there is no conflicting of interest

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