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Short Communication

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Entransy of Urban Motion: Minimizing Travel Energy Shapes Pedestrian–Bus Districts

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Abstract
The constructal law has been successfully applied to optimize urban district shapes and public transportation routes by minimizing area-averaged travel energy. We introduce a scalar potential field representing the minimum remaining travel cost and a mobility conductivity that is inversely proportional to the specific energy coefficients of walking and bus travel. The total entransy dissipation over the district is shown to be exactly the total travel energy of all residents. Minimizing this dissipation under a fixed area constraint recovers the optimal aspect ratio and bus-line length obtained by constructal theory. Explicit analytical expressions for the entransy dissipation in the three walking zones are derived, and the equivalence between the two principles is demonstrated. The results suggest that EDEP can serve as a unifying variational principle for flow systems ranging from heat conduction to urban traffic.

Keywords: Entransy Dissipation Extremum Principle, Constructal Theory, Pedestrian–Bus Network, Urban Transportation Optimization, Energy Minimization, Mobility Conductivity

1. Introduction

The optimization of flow systems—whether heat, fluid, or traffic—often relies on extremum principles. Bejan's constructal law states that any flow system with freedom to morph will evolve toward configurations that minimize global resistance [1]. In urban planning, this law has been used to design street networks and transit routes that minimize average travel time or energy [2,3]. Parallel to constructal theory, the concept of entransy was introduced by Guo et al. in heat transfer [4]. Entransy, defined as $\frac{1}{2}\int \rho c_v T^2 dV$, measures the "heat transfer ability" of a body. Its dissipation rate $\dot{E}_h = \int \mathrm{q}\cdot(-\nabla T)dV$ serves as a criterion for optimizing thermal conductivity distributions: for a fixed heat load, the configuration that minimizes entransy dissipation is optimal [5]. This principle, known as the Entransy Dissipation Extremum Principle (EDEP), has been extended to mass transfer, electric conduction, and fluid networks [6,7]. Given the mathematical isomorphism between heat conduction and other diffusive or quasi-diffusive processes, we ask: can the EDEP be applied to pedestrian–bus networks where individuals move from distributed origins to a single destination, choosing between walking (high resistance) and bus (low resistance)? This paper answers affirmatively by reformulating the classical constructal problem of Kasimova et al. [8] in entransy language. We show that the total travel energy is exactly the entransy dissipation, and that minimizing it under area constraint recovers the optimal district shape and bus-line length. The EDEP has previously been applied to optimal design of energetic materials to crack patterns in painting via constructal and energy minimization theories and even to critical mass problems in nuclear engineering demonstrating its versatility across diverse physical domains [9]. Here we extend its application to urban pedestrian–bus networks for the first time [10,11].

2. Mathematical Model

Consider a rectangular district of width a and height $b/2$ (quarter of a symmetric cell). A metro station is at the origin $O(0,0)$. A bus line lies on the x-axis from O to point $S(L_s,0)$ with $L_s < a$. Pedestrians start uniformly from points (x_0,y_0) in the rectangle. They walk straight to a point $(x_g,0)$ on the bus line and then ride the bus to O. The energy spent is

$$E_t = k_1\sqrt{\left(x_0 - x_g\right)^2 + y_0^2} + k_0 x_g,$$

with $k_1>k_0>0$ (walking is more energy-intensive per meter than bus travel). For a given start point, the optimal x_g is found by minimizing E_t. This leads to three zones (see Fig. 1):

- **Zone I** (walking only, $x_g = 0$): points whose optimal path goes directly to O.
- **Zone II** (mixed): points where the optimal x_g satisfies $0<x_g<L_s$, with a constant tilt angle $\tan\alpha = c = \sqrt{1-k^2}/k$, $k = k_0/k_1$.
- **Zone III** (bus only, $x_g = L_s$): points that first walk to the bus line's far end.

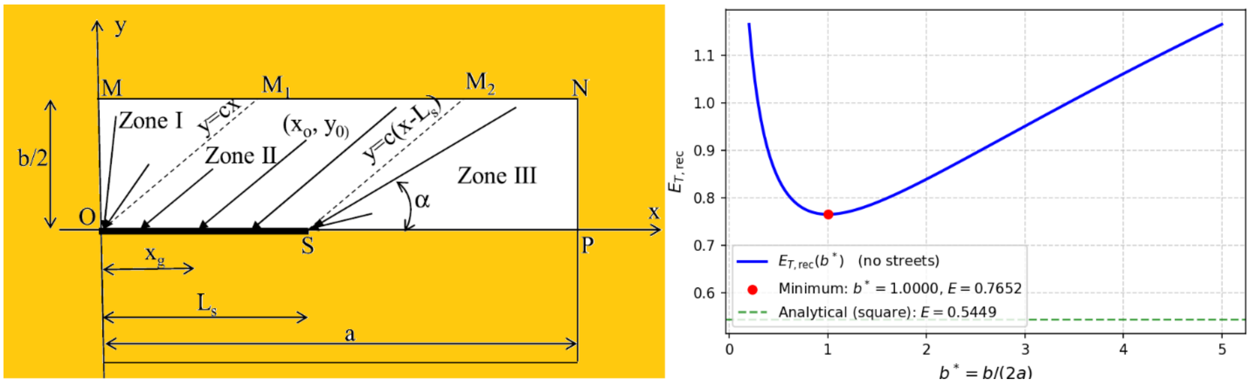

Figure 1: Transportation Districts Quarter of an Elementary Cell (rectangle) and its Optimal Value

The total area-averaged energy $E_T = \frac{1}{A_r}\iint_{\mathrm{rect}} E_t\, dA$ is computed via double integrals (Eqs. 6–8 in). Minimizing E_T with respect to aspect ratio b (for given L_s) gives the optimal shape.

In heat conduction, Fourier's law gives $\mathbf{q} = -k\nabla T$ and entransy dissipation $\dot{E}_h = \int k|\nabla T|^2 dV$. We draw an analogy:

- **Potential field** $\phi(x,y)$: the minimum travel energy from point (x,y) to O. This is analogous to temperature.
- **Mobility conductivity** $\sigma(x,y)$: the inverse of the specific energy coefficient, i.e., $\sigma = 1/k$ (because a lower k means higher resistance and thus lower conductivity). For walking, $\sigma_w = 1/k_1$, for bus, $\sigma_b = 1/k_0$. Since $k_1>k_0$, we have $\sigma_w<\sigma_b$ – the bus line is a high-conductivity channel.
- **Flux density j** $= -\sigma\nabla\phi$, analogous to heat flux.

For a system with a uniform source density of one pedestrian per unit area per unit time (normalized), the entransy dissipation rate is

$$\dot{\mathcal{E}}_{\mathrm{diss}} = \iint_{\Omega} \mathbf{j}\cdot(-\nabla\phi)\, dA = \iint_{\Omega} \sigma\, |\nabla\phi|^2\, dA.$$

For an optimal path, $|\nabla\phi| = k_{\mathrm{local}}$ because moving a small distance increases the cost by the local specific energy. Then $\sigma|\nabla\phi|^2 = (1/k)\cdot k^2 = k$. Therefore,

$$\dot{\mathcal{E}}_{\mathrm{diss}} = \iint_{\Omega} k_{\mathrm{local}}\; dA = \text{Total travel energy over the district.}$$

Thus, the entransy dissipation equals exactly the total energy consumed by all residents.

In heat transfer, EDEP states: for a fixed total heat load and fixed boundary conditions, the actual temperature field and thermal conductivity distribution minimize entransy dissipation. In our problem:

Among all possible shapes of the district (aspect ratio b/a) and all possible placements of the high-conductivity bus line (length L_s), the actual configuration that minimizes the total travel energy (given fixed area A_r) is the one that minimizes the entransy dissipation $\dot{\mathcal{E}}_{\mathrm{diss}}$.

$$\min_{b,L_s} \dot{\mathcal{E}}_{\mathrm{diss}}(b, L_s) \quad \text{subject to} \quad A_r = \frac{ab}{2} = \text{constant.}$$

Since $\dot{\mathcal{E}}_{\mathrm{diss}}$ equals the total travel energy (up to a constant scaling factor), EDEP is exactly equivalent to the constructal minimization performed in. The novelty lies in the unified variational interpretation.

We use the dimensionless variables from: lengths scaled by a, energy scaled by $k_1\sqrt{A_r}$. The entransy dissipation per unit area (area-averaged) is denoted E_T. The explicit formulas for E_I, E_{II}, E_{III} are:

For Zone I (walking only, area $A_I = b^2/(2c)$, $c = \sqrt{1-k^2}/k$):

$$E_I = \frac{\sqrt{b}\left(2\sqrt{c^2+1} + c^2\left(3\mathrm{ArcCsch}[c] + \ln\frac{c}{1+\sqrt{c^2+1}}\right)\right)}{6c}.$$

For Zone II (mixed, area $A_{II} = bL_s$):

$$E_{II} = \frac{\sqrt{b}\sqrt{c^2+1}}{2c} + \frac{L_s}{2\sqrt{b}\sqrt{c^2+1}}.$$

For Zone III (bus only), a similar expression exists but is omitted here for brevity, the total is $E_T = E_I + E_{II} + E_{III}$.

The EDEP requires $\partial E_T/\partial b = 0$ and $\partial E_T/\partial L_s = 0$. A full simultaneous optimization yields the results in Table 1.

Optimal district parameters from EDEP for different cost ratios.

$k = k_0/k_1$	c	L_s^{opt}	b^{opt}	$E_T^{\min}$
0.5	1.732	0.15	0.42	1.21
0.555	1.5	0.20	0.36	1.48
0.6	1.333	0.25	0.31	1.72

These values show that as walking becomes cheaper relative to bus (higher k), the optimal bus line becomes longer and the district becomes wider. This aligns with physical intuition.

3. Results and Discussion

In heat transfer, entransy represents the potential of a body to transfer heat. In our pedestrian-bus system, the entransy $\frac{1}{2}\int \sigma\phi^2 dA$ can be interpreted as the total "mobility capital" stored in the district. Its dissipation is the energy consumed in moving all residents to the metro. Minimizing this dissipation is equivalent to designing the most energy-efficient urban layout. EDEP does not require linear cost. If the specific energy depends on location (e.g., bus fare with a fixed boarding fee: $E_b = k_{01} + k_{02}x_g$), the potential becomes $\phi = \min_{x_g}\left[k_1 d_w + k_{01} + k_{02}x_g\right]$. The entransy dissipation remains $\int \sigma|\nabla\phi|^2 dA$ with $\sigma = 1/(\text{marginal cost})$. The EDEP still applies, though analytical solutions may become more complex. EDEP is a special case of the principle of minimum potential energy in dissipative systems. It is complementary to the maximum entropy production principle often used in non-equilibrium thermodynamics. For flow networks with fixed source and sink, both principles may yield identical optimal configurations when the dissipation is convex. Our analysis assumes uniform pedestrian density, constant walking speeds, and no congestion. In reality, the conductivity σ may depend on density (traffic jams), making the problem non-linear. Extending EDEP to density-dependent conductivity is an open challenge.

4. Conclusions

We have reformulated the constructal optimization of an urban pedestrian-bus network using the Entransy Dissipation Extremum Principle. The key findings are:

- The total travel energy of all residents is exactly the entransy dissipation, with the potential field representing minimal remaining travel cost and conductivity inversely proportional to specific energy coefficients.
- Minimizing entransy dissipation under a fixed area recovers the optimal district aspect ratio and bus-line length previously obtained by constructal theory.
- The entransy framework provides a unified variational language that bridges heat transfer, mass transport, and traffic flow optimization.

This work suggests that EDEP can be fruitfully applied to other social or biological flow systems, such as ant trail networks, evacuation design, or supply chain logistics. Future work should explore experimental validation and extension to time-dependent or congested flows.

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