ISSN: 2834-7706

Research Article

Journal of Mathematical Techniques and Computational Mathematics

Eco-Driving Strategy Optimization for Freight Trains

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Submitted: 22 Nov 2022; **Accepted:** 30 Nov 2022 **Published:** 16 Dec 2022

Citation: George, N. (2022). Eco-Driving Strategy Optimization for Freight Trains. *J Math Techniques Comput Math*, 1(3), 187-205.

Abstract
The South African rail industry is seeking a critical measure to monitor and save energy usage in the freight rail sector. The rail industry is experiencing increasing operational costs and high energy consuming driving records along various coal lines. Eco-driving is a modern and efficient way of driving that emphasizes fuel efficiency, speed, and safety. This study provides an algorithm to find the optimal trajectory for a freight train hauling a load over a specific distance. Optimized speed profile is composed of optimal acceleration, coasting, and deceleration. The Freight Eco-Driving Energy Optimizer (FEDEO) solution is not yet applied to freight trains globally, especially in Africa. In this study, the eco-driving strategy of a freight train is formulated as an optimization problem, whose objective function is the energy cost. The decision variables are the tractive and braking effort notches, and the speed, acceleration and distance limits are formulated as constraints. The formulated eco-driving problem is solved by Mixed-Integer Non-Linear Programming (MINLP) from the Opti Solver toolbox. The FEDEO algorithm is applied to a train consisting of eight 19E locomotives with two-hundred CCR-9 wagons, over a distance of 90.64 km. The results show up to 34.76% reduction in energy costs.

1. Introduction
Eco-driving is a driving practice that focuses on economical, ecological and safe driving, and these factors ensure that rail transportation is an efficient mode of transport [1, 2]. This study focuses on the economical aspect of freight train energy usage by developing the FEDEO. The FEDEO algorithm aims to find the optimal eco-driving speed profile that uses minimal energy, such that the train driver can follow an optimal and guided profile. The main constraints in developing such a profile are the route profile, speed restrictions and the notches required at critical velocity regions [3]. The FEDEO solution consists of mixed control and state variable constraints that require a decision-making formulation regarding the tractive and braking effort notches. The aim of the FEDEO algorithm is to build up acceleration on downward sections and cruise at the optimal velocity setpoint. The energy usage of the train depends on various physical factors such as track condition, weather, traction efficiency, and track stability. This study focuses on the energy optimization of the velocity trajectory. Driving behaviors such as rapid acceleration, braking, and speeding waste considerable energy. On a track with varying slopes, it is required to drive in an economically optimal manner by anticipating the state of the train at fixed intervals of the route profile [4]. Traditional optimal methods use an acceleration cruise coast-brake energy optimization strategy [5]. The FEDEO solution aims to use the lowest tractive energy possible, less stop-start driving and efficient driving. The principles of the FEDEO solution include economical driving, prediction of the ideal trajectory, and the efficient use of train momentum. The core objectives of the FEDEO are to increase awareness of eco-driving within the freight sector and provide a complete monitoring profile that the driver can use to traverse from one station to the next. Primarily, this study formulates the ideal trajectory of the train without any notch changes that would be similar to electric vehicles (EVs) with acceleration and braking [6]. Secondly, the ideal profile from the first part will be used to minimize the speed changes that the train experiences over a varying gradient. Lastly, a Mixed-Integer Non-Linear Programming (MINLP) is used to solve the eco-driving problem using the 'Opti' toolbox [1]. The train consisting of eight 19E locomotives and 200 CCR-9 wagons is highly energy-consuming train. This study optimizes the notch profile and simulates the optimal speed using real-time data [7].

This study incorporates several novel aspects, including:
1. The FEDEO algorithm only uses the route profile data: the elevation and distance, which can be generalized to any other routes.
2. The FEDEO algorithm applies to any train or locomotive with wagons whose average, minimum and maximum speed, acceleration, train mass, time and resistance coefficients are known.
3. The algorithm does not include any power-electronic related variables but focuses on driving behavior. The tractive effort usage based on the varying notches is formulated and optimized.
4. The FEDEO algorithm has an idealized trajectory using the ap-

proach in [6] and then uses a discretized approach using notches to reduce the section energy usage.

The prediction of instantaneous energy usage under real world situations are required. Previous studies have explored the relationship between energy consumption and vehicle parameters such as velocity, acceleration, auxiliary loads, and braking energy regeneration [8]. With eco-driving, the energy consumption can be decreased by 9% on average, reducing travel time by 3% [9, 10, 11]. The two-train movement regions where the FEDEO algorithm can be applied are to the acceleration and braking regions [12]. For EV's, the machine learning methods perform better when operated on complicated real-world direct conditions due to the fitness of the non-linear relationships, and the accuracy can be greatly improved based on iterative studies [13]. To predict the energy consumption of EV's, two methods classified are data-driven and physical models [14, 15]. The forward models take the driver's operation as inputs. Reference [14] proposed an energy consumption model that can be calibrated through multi-level and typical least squares regression based on the GPS information. For data-driven models, real-world driving conditions data in tandem with the weather, road and traffic conditions can predict EV's energy consumption under complex direct conditions based on machine-learning and statistical algorithms [16]. Many vehicles or route parameters need to be assumed (or obtained) to use the data-model driven methods. Moreover, it becomes highly complex when the data is required for a large fleet or vehicles with many varying parameters. All the driving condition factors cannot be considered, which may affect the model performance. However, methods under real-world conditions are still not common. The focus would rather be on the operation for freight trains, which falls under the specific FEDEO application.

The FEDEO problem is based on driver behavior, and the algorithm is based on an EV application developed in [6]. The FEDEO profile aims to reduce the traction energy usage and re-use the kinetic energy gained during de-acceleration for vehicle braking [4, 17]. Driving behavior, however, is not the only solution for improving vehicle energy efficiency. The power electronics aboard the locomotive, such as the traction motors, braking resistors, inverter, rectifier, and alternator, contribute to the overall energy efficiency [18]. It can be concluded that the four main driving behavior factors for a vehicle's energy usage are velocity, notch adjustments, acceleration and deceleration changes, and the magnitude of accelerations and decelerations related to the smoothness of driving [8].

Standard techniques used in optimal vehicle control have been utilized with detailed insight to solve the FEDEO problem. Train driving that requires a Driver Advisory System (DAS) has shown that the engines or motors prefer an average speed that is not majorly changing for measuring the overall energy consumption. Critical behaviors such as acceleration and deceleration are vital factors that cause a sudden increase in the train's energy consumption; therefore, aggressive driving should be avoided [19]. The two principal contexts for solving eco-driving problems are offline and online solutions. An offline solution assumes that all route profile position and characteristic dependent constraints are known; in contrast, an online solution uses real-time predictions and estimations, based on a train being in the environment. The FEDEO algorithm is an offline solution. The energy usage in the FEDEO of a train that consists of locomotives, wagons, and coaches is critical in considering energy efficiency, operational cost, and fleet reliability [10].

The topology data and the entire road profile are integrated into the hybrid electric power-train to offer a vast potential to optimize the control strategy, as noted in [20]. It is unclear whether the solutions in various papers obtain the optimal global solution. The noticeable exception is where the problem is convex, which guarantees that the globally optimal solution exists.

2. Eco-Driving Application

Many scholars have investigated and studied the eco-driving strategy of trains. Solutions provided by literature use analytical methods that formalize the train dynamic properties. Pontryagin's maximum principle (PMP) is a collection of conditions that must be satisfied by solutions of a class of optimization problems. This method involves dynamic constraints called optimal control problems. PMP was used in one study to analyses energy-efficient control regimes, which included maximum acceleration, cruising, coasting, and maximum braking [21]. In [22], Asnis *et al*. took regenerative braking and adjusted the objective function by applying PMP. Howlett [23] considered both the continuous and discrete control problem. In the eco-driving solution of the continuous algorithm, PMP was applied to find the necessary conditions on an optimal strategy to determine the optimal switching points. Kuhn-Tucker equations were used to find the optimal switching points in the eco-driving speed tracking control case. This led to the conclusion that driving strategies obtained from the discrete control model could be used to approximate closely those found by the continuous formulation. Policymakers have used these theories to justify electric vehicles as a tool for reducing greenhouse gas (GHG) emissions.

In countries such as China and the United States of America (USA), coal-fired plants contribute critically to electricity generation, making the environmental impact of EVs higher than that of internal combustion engines (ICEs). There has been a 3% electricity usage growth rate over the last 20 years in South Africa, with 20 GW of additional generation capacity required by 2020 and up to 40 GW by 2030 [24]. Globally, the rail environment's key target should be to transition to a lower-carbon economy, and the concern in South Africa is that electricity prices are increasing while being wholly reliant on Eskom (South Africa's primary electricity supplier, generating approximately 90% of the electricity used in South Africa and approximately 30% of the electricity generated on the African continent) for the provision of baseload electricity to power operations [25]. South Africa's freight rail energy policy commitments include improving energy efficiency, proactively managing and monitoring energy usage, reducing global GHG emissions and energy constants, and improving energy security [25, 26]. The current and future energy management initiatives include train driver training to reduce energy consumption, improving energy efficiency, reducing locomotive idling, and optimizing the setup of trains. The rail organization known as Transnet Freight Rail (TFR) has the most significant energy gains as new locomotive technology is introduced into operations, specifically from regenerative braking capability on the 19E and 15E class of locomotives on the coal and iron ore export lines. The new technology on these lines has resulted in significant efficiency gains, with the regenerated electricity used partly by the fleet of locomotives and, where possible, transmitted back into the Eskom grid [27]. South Africa has seen studies that address and formulate algorithms related to EVs' eco-driving and contribute to a greener future [17, 24]. One study includes improving route profile operational characteristics that involve changes to the train-driving behavior, including lowering energy usage by methods such as eco-driving [28]. For EVs, an analytical state-constrained control is implemented for eco-driving control in [29, 30]. The global optimal eco-driving of EVs is solved through sequential quadratic programming (SQP) [31, 32].

Eco-driving techniques are employed for energy saving DASs, which mainly define energy consumption as a cost function to be minimized, allowing a more considerable speed control than those commonly found in EVs. Other methods are capable of solving the problem, such as the PMP, Dynamic Programming (DP) and analytical solutions [13]. The advantages are that the PMP method will find the best possible control for taking a dynamic system from one state to another. It is computationally efficient in that the natural conditions specify a need to hold over a particular trajectory. The disadvantage of the PMP is that incorporating state constraints is not a simple task and provides the required conditions for optimality. The DP algorithm has been used to find a globally optimal solution to the end problem in [33]. DP solves a discretized version of the operational control problem. It does this by assigning independent variables to time, distance, and position, thus discretizing state and control spaces. The Hamiltonian analysis in shows that only particular types of control variables can be used in an optimal strategy [3]. The choice of control is determined by the speed and the quantity of the adjoint variables such as gradient and position. The DP method has a high computational time and creates a reference trajectory for the vehicle's driver. In contrast, a few methods have attempted to derive and use closed-form speed trajectories, which provide a Realtime route profile based on the behavior of the train at each interval or time-stamp [29].

By developing the FEDEO algorithm, this study aims to provide the South African rail industry with a method for creating and identifying energy-saving route profiles for track sections [17, 34]. The ideal option is to assist the driver through technology such as the DAS. The FEDEO algorithm will simulate the longitudinal movement of the train using the dynamic equations of motion [35, 36]. A locomotive requires tractive effort for propulsion and braking effort for slowing down. A large amount of power is necessary for propulsion, and the braking element outputs regeneration energy. This is the energy from the locomotive wheels to the overhead catenary that trailing locomotives can use. The FEDEO problem will search for the local minimum of the speed profile as the critical point [37]. FEDEO will formulate the dynamic parameters that will contribute to the train's operational savings over any route profile [38]. The FEDEO solution for the train-handling strategy consists of static and dynamic parameters where the static is the input (primary) data in the formulation. Route static parameters, in this case, would include the trip distance, speed restrictions, freight train parameters (force limits, adhesion curves), and gradient profile. The decision variables consist of acceleration (or tractive effort), train deceleration (or braking effort), and the speed profile from point A to B [39]. The principles that the FEDEO governs mainly include acceleration, speeding, deceleration, route choice, idling, external factors, and stopping [40]. The eco-driving principle outlined by the FEDEO narrows down to the driving behaviors or the control a driver has over a vehicle. FEDEO aims to minimize energy usage by obtaining optimal speed v^*_j and engine's notches.

3. Problem Formulation

The problem originates with the high energy required for freight trains to traverse the South African Ermelo-Richards Bay coal line. Electricity usage is costly in South Africa, especially for freight operation. The route profile force diagram is shown in Figure 1 below. The traction motors on board the electric locomotive is mainly responsible for the propulsion of the train, where the trailing wagons carry the freight (coal). The braking element or rheostat contributes to the energy regeneration parameter. The FEDEO algorithm optimizes the energy usage of the train using only the route elevation, gradient angle profile, and distance travelled. The FEDEO algorithm aims to lower the energy consumption J by optimizing the travel speed of the train and determining the tractive and braking effort Uj^t and Uj^b notches based on this optimal speed. The notches and power $PR_{(t)}$ represent the control variables. The state variables are the train acceleration a_j and speed v^*_j. The initial parameters refer to the train setup, such as the train mass, speed, and the resistance coefficients for the route simulation. The FEDEO problem is presented separately in both continuous time and discrete time. The main target of the eco-driving solution is to optimize the velocity profile, with the motion limited by longitudinal dynamics; lateral dynamics are not considered.

4. FEDEO Formulation for Eco-Driving Solution

This section formulates the FEDEO algorithm as outlined by [1] for the continuous optimal route profile. The energy optimization is based on J_1 and J_2 shown in Equations (1) and (2), with the critical route profile parameters described in Sections from 1 to end. A continuous-time formulation of the FEDEO algorithm is provided; the algorithm aims to minimizes the energy or the integral of the power us age optimize the tractive force $u(t)$ and velocity profile

$v(t)$ that given trajectory P by the train over a time period $t_s \in [t_0, t_f]$ over a given trajectory s(t) E [s0, sf] with known geographical parameters such as gradient profile lamda [1, 6, 41, 42]

$$\min_{v(t),\ u(t)} J_1 = \int_{t_0}^{t_f} P(v(t), u(t))\, dt. \qquad (1)$$

Equation (2) further develops Equation (1), where $f(v, s)$ is subject to the gravitational constant g from [6]. The gravity constant, g, is 9.8 m/s^2, where t_s is the length of the sampling interval. In Equations (1), (2), and (5), m represents the total mass of the train in kg; c_r represents the rolling resistance coefficient of the route section, aerodynamic drag is represented by $\sigma_d = 1/2\ C_d, p_a, A_f$ with c_d being the drag coefficient, in which ρ_a denotes the air density in kg/m^3; and A_f is the frontal area of the locomotive in m^2. The continuous-time optimal control problem is provided by (1) [1, 39, 43].

$$J_2 = \min_{v(t), u(t)} \int_{t_0}^{t_f} P(v(t), ma(t) + f(v(t), s(t)))\, dt, \quad (2)$$

Equation (2) is subject to:

$$\frac{dv}{dt} = a(t), \qquad (3)$$

$$\frac{ds}{dt} = v(t). \qquad (4)$$

Equation (5) has been simplified in reference to [1], where a, s and v are the critical parameters required for optimization.

$$\int_{t_0}^{t_f} P(v, ma + f(v, s))\, dt = \int_{t_0}^{t_f} P_R(a,\ v,\ s)\ \ dt. \quad (5)$$

$$f(v, s) = \sigma_d v(t)^2 + c_r mg cos(\alpha(s)) + mg sin(\alpha(s)). \quad (6)$$

Figure 1 describes the gradient angle derivations shown in Equation (6). The given gradient angle profile is $\alpha_{(s)}$: $[s_0, sf]$ -> $[-\Pi, \Pi]$, where $\alpha(s)$ is the gradient angle profile at position t; while being subject to longitudinal vehicle dynamics, non-negative velocity bounds of the route profile $v_{(t)\in}$ $[v_{min}, v_{max}]$ where $v_{min} = 0$ at rest and boundary conditions son the position and velocity.The inclination angle conditions $\alpha(s)$ has been based on the height difference, dh, versus the distal difference, ds. $H(s)$ is the elevation profile and s′(t) represents the horizontal projection of s(t).

Equation (7) is an approximation of Equation (6) for electric motors because friction losses, energy usage and ohmic losses are captured by the terms $\beta_0 v^2$, $\beta_1 vu$ and $\beta_2 u^2$,respectively. It has been assumed to be a quadratic function of the form [1, 6]:

$$P_R(a,\ s,\ v) = \beta_0 v^2 + \beta_1 vu + \beta_2 u^2. \qquad (7)$$

The reformulation of the problem outlined in Section 5 focuses on discrete-time approximations where the nonconvexity is introduced in Equation (7). Equation (8) for obtaining the train's energy usage is further derived from Equations (1) to (7) as:

$$\begin{aligned} P_R(a,\ s,\ v) = \beta_0 v^2 + \beta_1 \sigma_d v^3 + \\ 2\beta_2 m^2 ga(sin(\alpha(s)) + c_r cos(\alpha(s))) + \beta_2 (ma)^2 + \\ \beta_2 (mg \sin(\alpha(s)) + \sigma_d v_j^{2,*} + c_r mg cos(\alpha(s)))^2, \end{aligned} \qquad (8)$$

where:
1. $P_R(a, s, v)$ is the power requirement of the train (kW).
2. m is the mass of the train including load (kg) and g is the acceleration caused by gravity (m/s^2).
3. σ_d is the aerodynamic force constant and cr is the rolling resistance coefficient for wheel on steel.

4. a (m/s^2), s (km) and v (m/s) are the train's acceleration, distance and speed at time t_s.
5. β_0, β_1 and β_2 are the friction loss coefficient for the traction motor, braking loss coefficient and the ohmic loss coefficient of the brake resistors. respectively, of which the constants are given in Table 1.

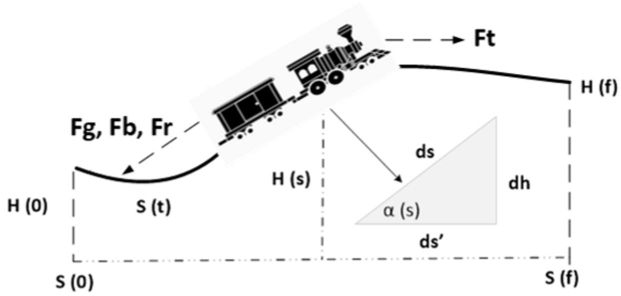

Figure 1: Locomotive force diagram against route characteristics [6]

The FEDEO algorithm solves the eco-driving problem using MINLP in the Opti-Toolbox solver from MATLAB as the problem to be optimized is non-linear owing to the use of the Davis resistance factor. sections or refer to the Section describes the train dynamics, with the aim of lowering energy consumption by optimizing the train notches.

The initial formulation of FEDEO is provided by the eco-driving algorithm in [1], which is discretized by incorporating the train notches, tractive and braking efforts, and parameter bounds. Minimum energy usage is calculated through a search of the local minima using the points where the energy usage is the lowest. This algorithm is used in the Matlab Opti-Toolbox solver to optimize the energy usage of the train.

5. FEDEO Formulation for Eco-Driving Speed Tracking Control

The eco-driving speed optimisation problem formulated in Section 4 has been discretised to make it solvable. The eco-driving speed optimisation problem does not include the tractive and braking effort notches, and the reason for discretisation is to transfer the continuous function, models, variables, and equations derived in Section 4 into discrete counterparts.

5.1 Objective function

The objective of the algorithm is to minimise the energy usage of the train within the route profile parameter bounds. In this section, vj is the train's optimal velocity, and v^*_j is the eco-driving speed of the train determined by incorporating tractive and braking effort notches and the discrete modelling approach. The results from the simulation of the parameters in the objective function shown in Equation (9) are described in Section 7.

$$J = \sum_{j=1}^{N} P_R(a_j, s_j, v_j^*) \times U_j^t \times \Delta t_s, \quad (9)$$

where:
1. J is the energy usage (kWh), also known as the objective function of the eco-driving problem.
2. Δt_s is the sampling period for the route simulation.
3. U_j^t is the tractive effort notch of the 19E locomotive respectively (0.1 to 1).
4. a_j (m/s^2), s_j (km) and v_j^* (m/s) are the acceleration, distance, and optimal speed of the train at time t_s.
5. N is the number of samples and j is the counter of sampling intervals.

5.2 FEDEO Constraints

The constraints used for FEDEO are based on the acceleration and tractive and braking decisions. The objective function in Equation (9) is subject to the constraints and bounds shown in Equations (10) to (18). The acceleration difference for the eco-driving solution has been calculated from the computed force $\sum F_j$ and vector a_j. FEDEO is formulated based on the tractive force F_j^t, braking force F_j^b, gravitational force F_j^g and the resistance force F_j^r over N sampling intervals shown in (11) with the train profile shown in Figure 3. with the train profile shown$\sum_{j=1}^{N} F_j$, while the braking force or F_j^b is dependent on two factors, based on the train dynamics suggested by [38]:

1. Adhesion between the wheel and the rail
2. Reaction force of the rail on the wheels during braking (hence on weight per braked wheel)

$$\frac{\sum_{j=1}^{N} F_j}{m} - a_j = 0, \quad (10)$$

$$\sum_{j=1}^{N} F_j = ma_j = F_j^t - F_j^g - F_j^b - F_j^r. \quad (11)$$

• $\sum N_{j=1} F_j$ is the sum of the forces in kN where equations of the forces are provided in Section 5.2.1.
The objective function in Equation (9) is subject to the following constraints:

$$U_j^t \in \{0, \ldots, 10\} \quad (1 \le j \le N), \tag{12}$$

$$U_j^b \in \{0, \ldots, 10\} \quad (1 \le j \le N), \tag{13}$$

$$U_j^t \times U_j^b = 0. \tag{14}$$

$$v_j^{*,min} \le v_j^* \le v_j^{*,max}. \tag{15}$$

$$s_j^{min} \le s_j \le s_j^{max}. \tag{16}$$

$$a_j^{min} \le a_j \le a_j^{max}. \tag{17}$$

$$h_j^{min} \le h_j \le h_j^{max}. \tag{18}$$

Where the decision 1 or 0 is the state of movement of the train, and h_j is the elevation profile. The MINLP method uses the decision variables U_j^t and U_j^b for optimization. The optimization is updated every j^{th} sampling interval [40, 44].

5.2.1. Algorithm to Solve the FEDEO Formulations

FEDEO aims to minimize $f^T X$ subject to the equality constraints ($A_{eq}X = b_{eq}$) and upper and lower boundaries of the control variables ($L_B \le X \le U_B$). The control variables are U_j^t and U_j^b, while A_{eq} and B_{eq} are the equality matrices, and L_B, U_B, and f are vectors represented below in Equations (19) and (20). The independent variables are tractive and braking effort forces F_j^t and F_j^b, dependent variables are the train acceleration a_j and continuous velocity v_j and the core state variables are the distance s_j and optimal velocity v^*_j [45]. The objective function is solved using the canonical form in Equation (9) as the vector $f^T{}_X$ [17, 46]:

$$\boldsymbol{min\ f^T X} \tag{19}$$

subject to:

$$\left\{ \begin{array}{ll} A_{eq}X = b_{eq} & (linear\ equality\ constraint) \\ L_B \le X \le U_B & (upper\ and\ lower\ bounds) \end{array} \right\}. \tag{20}$$

Vector X contains all the state and in dependent variables.
Let matrix $A_{eq}X$ and $b_{eq}X$ be:

$$\boldsymbol{A_{eq}} = \begin{bmatrix} \boldsymbol{A_{eq1}} \\ \boldsymbol{A_{eq2}} \\ \boldsymbol{A_{eq3}} \end{bmatrix}_{(2N+4)\times(5N+2)}, \tag{21}$$

$$\boldsymbol{b_{eq}} = \begin{bmatrix} \boldsymbol{b_{eq1}} \\ \boldsymbol{b_{eq2}} \\ \boldsymbol{b_{eq3}} \end{bmatrix}_{(2N+4)\times(1)}, \tag{22}$$

The vector f^T in the canonical form shown in Equation (23) can be obtained from Equation (9) to calculate the power required for every route section's sampling period.

$$f^T = [P_R(1) \quad \ldots \quad P_R(N)]_{1\times(5N+2)}. \tag{23}$$

Equation (24) is the unknown vector we are trying to solve for using the FEDEO algorithm.

$$
\boldsymbol{X} = \begin{bmatrix} U^t_{j,0} \\ \vdots \\ U^t_{j,N} \\ U^b_{j,0} \\ \vdots \\ U^b_{j,N} \\ v^*_{j,0} \\ \vdots \\ v^*_{j,N+1} \\ a_{j,0} \\ \vdots \\ a_{j,N} \\ s_{j,0} \\ \vdots \\ s_{j,N} \end{bmatrix}_{(5N+2)\times 1}, \tag{24}
$$

The linear matrix for A_{eq1} is

$$
A_{eq1} = [U^t_j,\ U^b_j,\ A_1,\ A_2,\ A_3]_{N\times(5N+2)}. \tag{25}
$$

The dynamic equation in its continuous form is discretized for use in A_{eq1} as:

$$
V^{i,*}_j - V^{f,*}_j + (a_j \times dt) = 0. \tag{26}
$$

The equality matrices required for A_{eq1} are shown in Equations (27) to (31):

$$
U^t_j = U^b_j = 0_{N\times N}. \tag{27}
$$

$$
A_1 = \begin{bmatrix} v^{i,*}_j & -v^{f,*}_j & 0 & \dots & 0 \\ 0 & v^{i,*}_j & -v^{f,*}_j & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & v^{i,*}_j & -v^{f,*}_j \end{bmatrix}_{N\times(N+1)}, \tag{28}
$$

$$
A_2 = \begin{bmatrix} a_j \times dt & 0 & \dots & 0 \\ 0 & a_j \times dt & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & a_j \times dt \end{bmatrix}_{N\times N}, \tag{29}
$$

$$
A_3 = S_j = 0_{N\times(N+1)}, \tag{30}
$$

$$
b_{eq1} = 0_{N\times 1}. \tag{31}
$$

The linear matrix for A_{eq2} is:

$$
A_{eq2} = [U^t_j,\ U^b_j,\ A_5,\ A_6,\ A_7]_{N\times(5N+2)}. \tag{32}
$$

The longitudinal movement in its continuous form must be discretized as shown below for use in A_{eq2}:

$$s_j^i - s_j^f + (\frac{1}{2} \times a_j \times dt^2) = 0. \tag{33}$$

$$(v_j^{i,*} \times dt) - s_j^i = 0. \tag{34}$$

The equality matrices required for A_{eq2} are shown in Equations (35) to (38):

$$A_5 = \begin{bmatrix} dt \times v_j^{i,*} & 0 & \dots & 0 \\ 0 & dt \times v_j^{i,*} & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & dt \times v_j^{i,*} \end{bmatrix}_{N \times (N+1)}, \tag{35}$$

$$A_6 = \begin{bmatrix} \frac{1}{2}a_j \times dt^2 & 0 & \dots & 0 \\ 0 & \frac{1}{2}a_j \times dt^2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \frac{1}{2}a_j \times dt^2 \end{bmatrix}_{N \times N}, \tag{36}$$

$$A_7 = \begin{bmatrix} s_j^i & -s_j^f & 0 & \dots & 0 \\ 0 & s_j^i & -s_j^f & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & s_j^i & -s_j^f \end{bmatrix}_{N \times (N+1)}, \tag{37}$$

$$b_{eq2} = 0_{N \times 1}. \tag{38}$$

The equality matrices required for A_{eq3} are formulated as:

$$v_j^{i,*} = v_j^{f,*}, \ s_j^i = 0km, \ s_j^f = 90.64 \ km. \tag{39}$$

The linear matrices for A_{eq3} are below in Equations (40) to (43):

$$A_{eq3} = [U^t_{j(4 \times N)}, \ U^b_{j(4 \times N)}, \ A_8, \ a_{j(4 \times N)}, \ A_9]_{4 \times (5N+2)}. \tag{40}$$

$$A_8 = \begin{bmatrix} v_j^{i,*} & 0 & \dots & 0 \\ v_j^{f,*} & 0 & \dots & -v_j^{f,*} \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{bmatrix}_{4 \times (N+1)}, \tag{41}$$

$$A_9 = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ s_j^i & 0 & \dots & 0 \\ 0 & 0 & \dots & s_j^f \end{bmatrix}_{4 \times (N+1)}, \tag{42}$$

$$b_{eq3} = \begin{bmatrix} v_j^{i,*} \\ v_j^{f,*} \\ s_j^i \\ s_j^f \end{bmatrix}_{4 \times 1}. \tag{43}$$

The lower bounds (L_B) and upper bounds (U_B) are shown in Equations (44) to (45). The simulation results shown in Section 7 using the MATLAB Opti-Toolbox are used to validate the FEDEO algorithm.

$$\begin{aligned} L_B = [0, \ldots 0,\ 0, \ldots 0, -2.87, \ldots -2.87,\ 0, \ldots \\ 0,\ 0, \ldots 0]^T_{(5N+2) \times 1}, \end{aligned} \tag{44}$$

$$\begin{aligned} U_B = [10, \ldots 10,\ 10, \ldots 10, 2.03, \ldots 2.03,\ 80, \ldots \\ 80,\ 90.64, \ldots 90.64]^T_{(5N+2) \times 1}. \end{aligned} \tag{45}$$

6. Case Study

A case study of the 19E train profile with CCR-9 wagons has been investigated to validate the optimization algorithm. The assumption is that the maximum travel time in the present work is 1.45 hours, divided into sampling period t_s or dt, of 1.05 minute, which is 62.70 seconds, yielding total samples of N is (1.45 × 60⁄1.05) = 83. The average time between data points is 1.05 minute or 62.70 seconds. This data is based on existing route profile data, where trains have traversed the Ermelo-Richards Bay section route profile. The FEDEO parameters are shown in Table 1. The route map for the Ermelo-Richard's Bay section is shown in Figure 2. The train setup between Ermelo and Richard's Bay is shown in Figure 3, and the elevation profile is shown in Figure 4 [1, 8, 12].

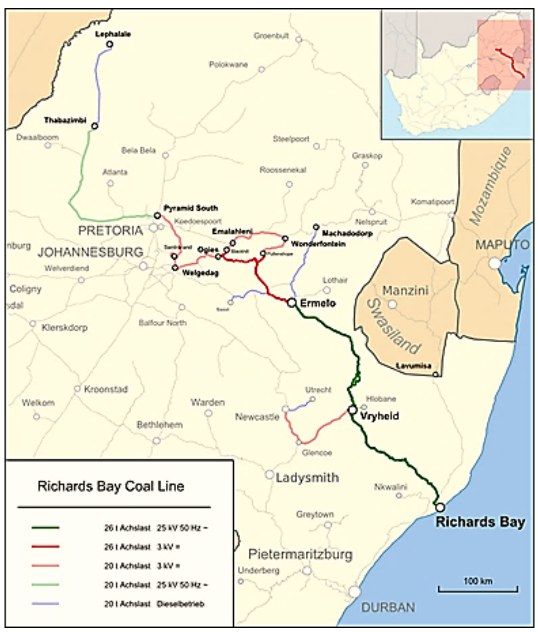

Figure 2: Richard's Bay Coal Line Route Map [2]

Table 1: FEDEO simulation route profile parameters

Parameter	Value
Minimum speed	0 km/h
Maximum speed	80 km/h
Cruising speed	70 km/h
Acceleration (maximum)	2.03 m/s^2
Acceleration (minimum)	-2.87 m/s^2
Starting point	0 km
Final distance	90.64 km
Train mass	21,742,000 kg
Acceleration due to gravity	9.81 m/s^2
Rolling resistance coefficient (Crr)	0.001
Drag coefficient (Cd)	1.8
Friction loss coefficient (β_0)	0.2
Braking loss coefficient (β_1)	0.9
Ohmic loss coefficient (β_2)	0.00602

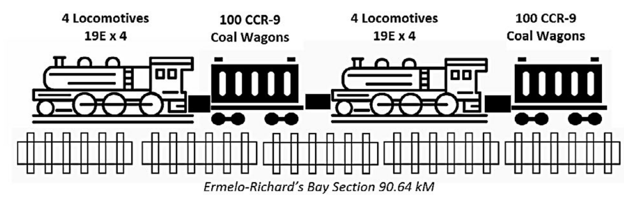

Figure 3: Test configuration for section Ermelo-Richards Bay

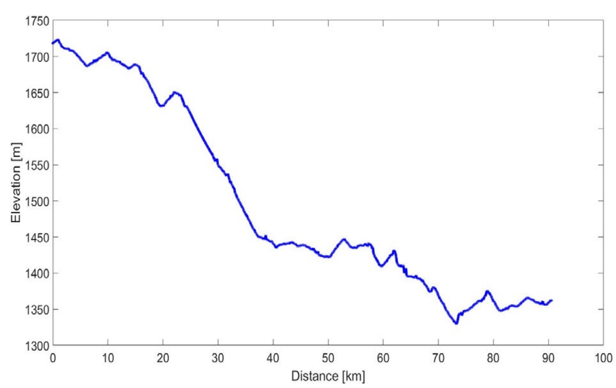

Figure 4: Graph of relative elevation profile section Ermelo-Richards Bay

6.1 Tractive and Braking Effort Data

During operation of the train, only one of either U_j^b or U_j^t can be applied. All the train efforts will be zero when the locomotive is idle or to save energy when on a decline. This means that U_j^t and U_j^b is zero. The reason usage is that the braking effort energy is fed back into the for the tractive effort U_j^t solely contributing to the energy overhead line as regeneration energy. In regeneration, the torque reduces the motor speed and generates the electrical power. The energy that is regenerative will be converted by power electronic equipment into electrical energy that is fed back into the overhead line. Globally, trains that incorporate regeneration of energy have a high capacity for eco-driving and for being economical, based on the driver behaviour. The incorporation of the tractive and braking of the tractive and braking effort requires the vehicle dynamics that are discussed in Section 3.The tractive and braking effort reference plots for the 19E locomotive fleet is shown in Figure 5 and 6. The tractive and braking effort decision regions are shown in Table 2.

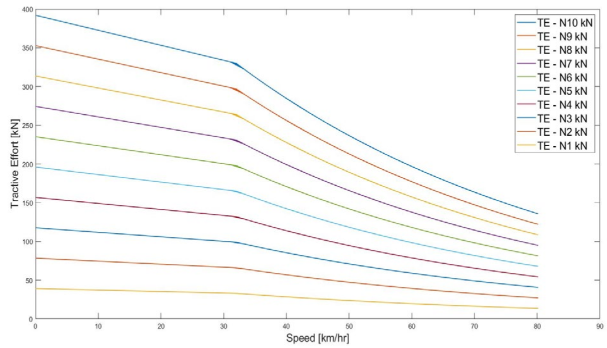

Figure 5: Notches of 19E tractive effort

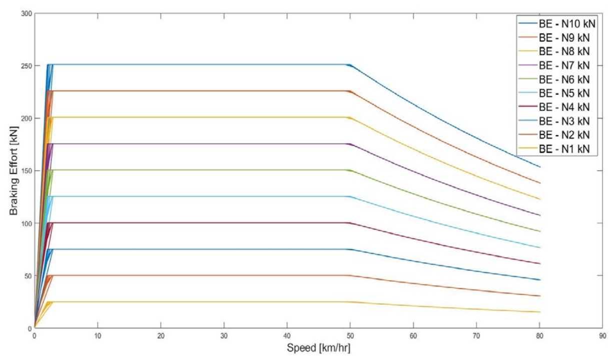

Figure 6: Notches of 19E braking effort

Table 2: Tractive and braking effort decision regions

Velocity (km/h)	Tractive effort (kN)	Braking effort (kN)
$0 \leq v_j^* < 2$	$(\frac{-31}{16} \times v_j^*) + 392$	$125.5 \times v_j^*$
$2 \leq v_j^* < 32$	$(\frac{-31}{16} \times v_j^*) + 392$	251
$32 \leq v_j^* < 50$	$595.793 \times (0.982)^{v_j^*}$	251
$50 \leq v_j^* < 80$	$595.793 \times (0.982)^{v_j^*}$	$568.036 \times (0.984)^{v_j^*}$

CCR-9 wagon), W is the axle weight in tones per axle of locomotive or car (26 tones for the 19E and 26.1375 for the CCR-9 wagon), N is the number of axles (four for the 19E); a is the frontal cross-sectional area of the locomotive in square meters (11.1484m^2 for 19E and 8.8258m^2 for the CCR9 wagon); C is the streamlining coefficient used to define train resistance (0.0024 for 19E and 0.0005 for the CCR9 wagon); and D is the resistance due to the force of wind called the aerodynamic coefficient. The force due to gravity is shown in Equation (48) and is dependent on the train mass (m) in kg, gravity (g) at 9.81m/s^2 and gradient angle profile α [38, 39].

$$A = 1.3 + \frac{29}{W}, CD = \frac{Ca}{WN}, \tag{46}$$

$$F_j^r = [A + Bv_j^* + CDv_j^{2,*}], \tag{47}$$

$$F_j^g = M \times g \times sin(\alpha(s)). \tag{48}$$

7. Simulation Results

The FEDEO algorithm performs a discrete iteration to find the global minimum, as shown in Figure 8. Figure 8 shows that the train will try to accelerate rapidly, then cruise for a distance, coast, and finally apply the brakes at the stop. Figure 10 follows a similar trajectory as the mass is used to determine the force experienced by the train. Figure 11 shows cumulative energy usage of the train. The elevation smoothing profile is used to incorporate a data set that is simplified for the FEDEO algorithm to optimize energy use.

The actual speed data was obtained from the black box located within the locomotive cab. The data was recorded during March 2015 between the stations of Ermelo and Kempton Park.

7.1. Eco-Driving Solution for [0, 90.64 km]

This section presents a simulation of the FEDEO algorithm for the entire section of the route from Ermelo to Kempton Park.

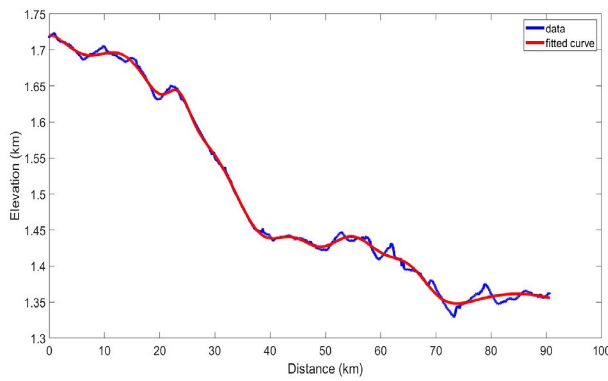

Figure 7: Elevation smoothing profile for [0, 90.64] km

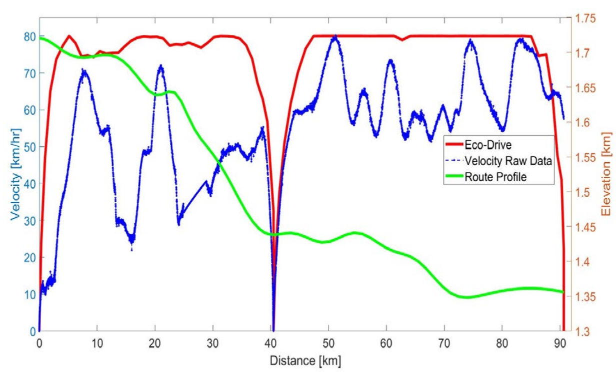

Figure 8: Speed profile for [0, 90.64] km

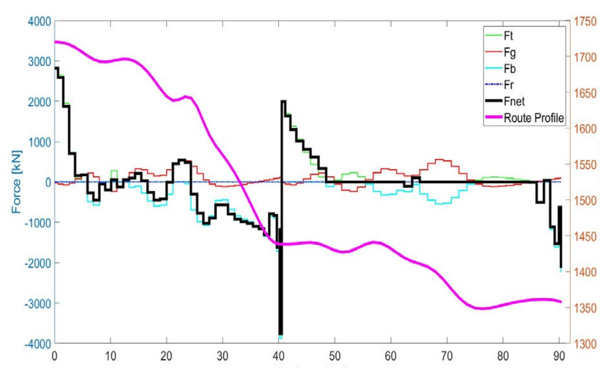

Figure 9: Route profile against forces for [0, 90.64] km

The tractive effort force of the train shown in Figure 9 indicates a maximum value of 2839 kN, while the braking effort is significant for regeneration. Ft represents the tractive force, Fg is the gravitational force, Fb represents the braking force, Fr is the rolling resistance force, $Fnet$ is the net force or summation of all the forces and Route Profile is the elevation profile. The train comes to a stop at the 40 km point as this is a security checkpoint. The power used for the 90.64 km section is 11,369 kW, where the critical parameters are shown in Table 3. The maximum acceleration is lower at 0.4739 m/s^2 for the 90.64 km section.

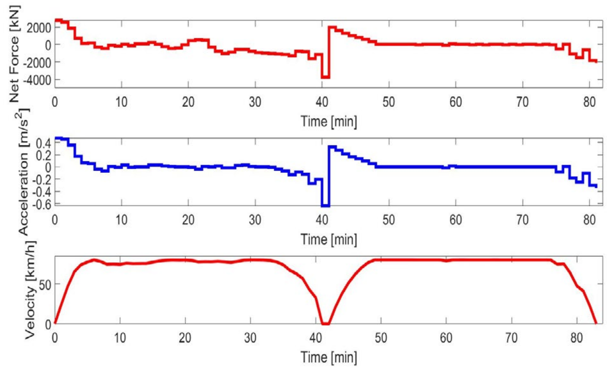

Figure 10: Net force, acceleration and velocity simulation for [0, 86.74] minutes

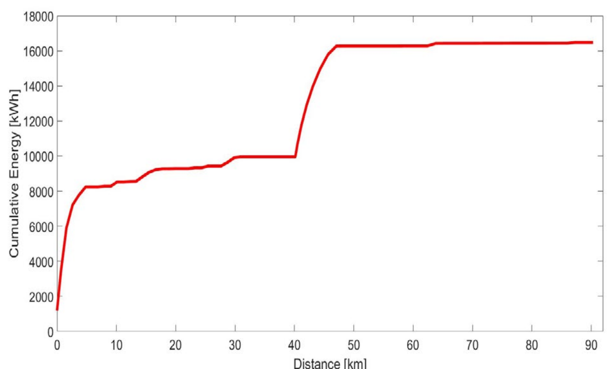

Figure 11: Cumulative energy plot for [0, 90.64] km

7.2. Eco-Driving Solution for [70, 90.64 km] with Start and End Speed at 0 km/h

This section presents the results of applying the FEDEO algorithm over a specific distance of 20.64 km, which is the last part of the route from 70 km to 90.64 km. The plots shown in Figures 12 to 16 simulate the speed at the start to be 0 km/h and the final speed to be 0 km/h.

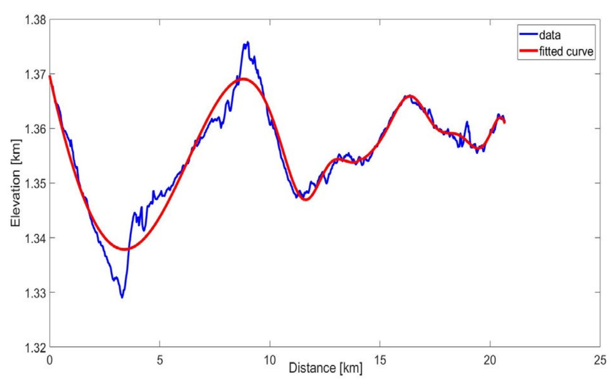

Figure 12: Elevation smoothing profile for [70, 90.64] km

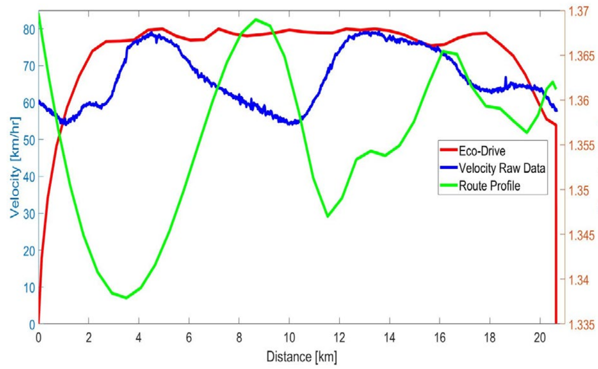

Figure 13: Speed profile for [70, 90.64] km

The energy consumption for the section [70, 90.64 km] with start and end speed at 0 km/h is higher than the measured route energy usage owing to the train speed being required to start and end at 0 km/h. The forced start and end speed simulated in MATLAB makes the energy usage 47% higher. The time taken for the optimized route profile is 6.37% less than the actual time. The optimal route profile validates that energy can be saved if there is less acceleration and braking, and more cruising and coasting. This can be seen in the speed profile shown in Figure 13.

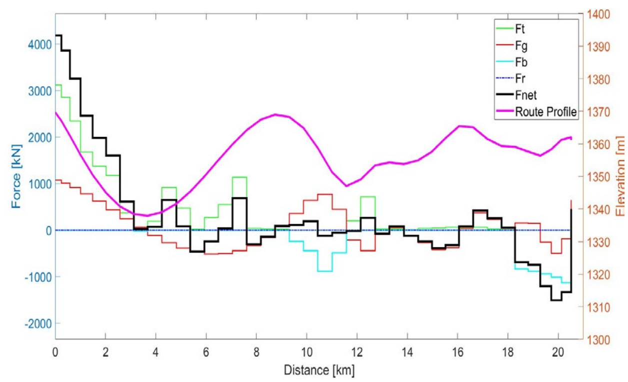

Figure 14: Route profile against forces for [70, 90.64] km

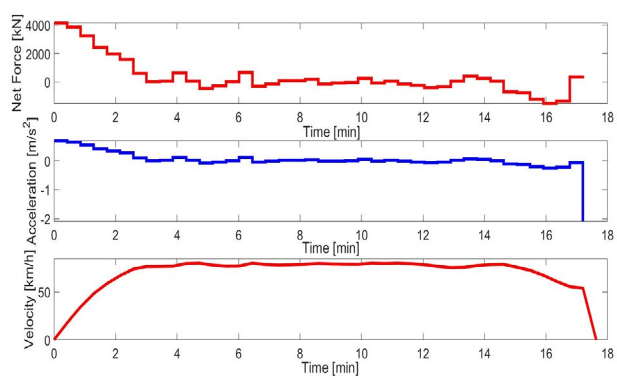

Figure 15: Net force, acceleration and velocity simulation for [0, 17.63] minutes

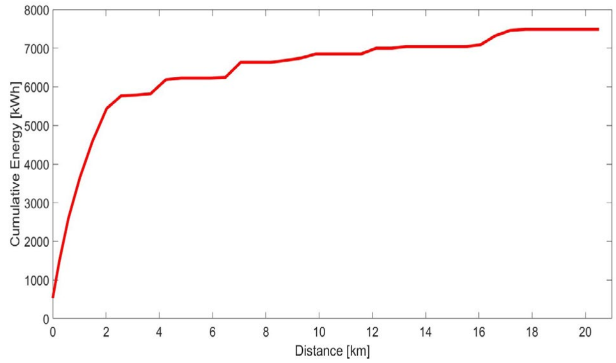

Figure 16: Cumulative energy plot for [70, 90.64] km

The force profiles shown in Figure 14 describe the adjustments to the train trajectory with regard to the tractive and braking forces as the elevation changes. The optimal energy usage of 7448.9 kWh is higher than the actual 5097 kWh used. The true representation of this optimal trajectory is presented in Section 7.3, where the FEDEO algorithm is applied. The duration of the optimal trip is lower than the actual trip because of sudden changes in acceleration and braking.

7.3. Eco-Driving Solution for [70, 90.64 km] with Start and End Speed at 60 km/h

Needs section number or reference, results for [70, 90.64 km] are presented with a simulated train speed that starts at 60 km/h and ends at 60 km/h.

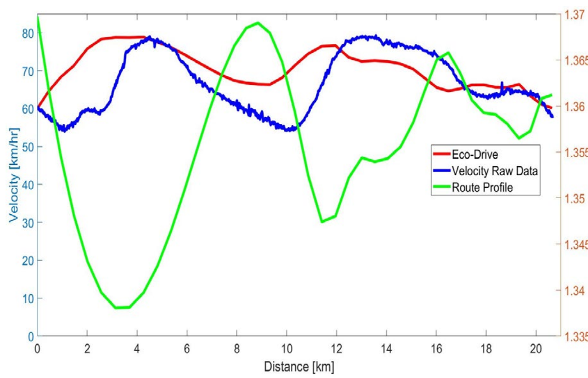

Figure 17: Speed profile for [70, 90.64] km

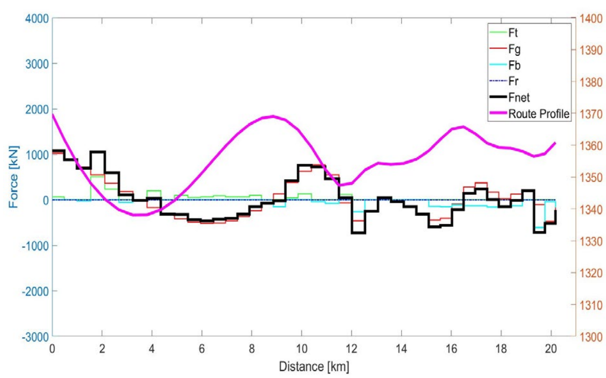

Figure 18: Route profile against forces for [70, 90.64] km

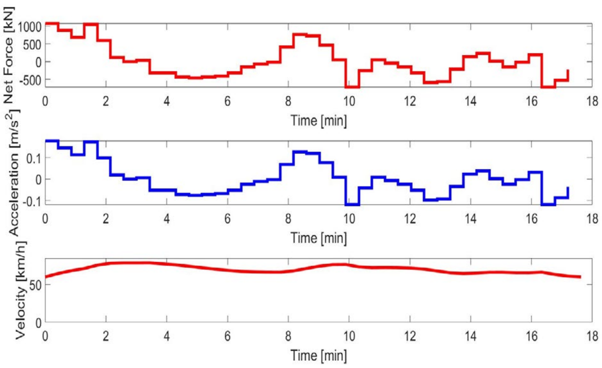

Figure 19: Net force, acceleration and velocity for [0, 17.63] minutes

The energy consumption reduces by 25.17% compared to the measured route profile. The energy usage calculated is based on the acceleration $a(t)$ in m/s^2, power $P(t)$ in Watts(W) or N.m/s and energy $E(t)$ measured in Wh. The negative acceleration is braking, and acceleration that is positive is used for traction.

$$\frac{dv}{dt} = a(t)\ ,\ Force\ F(t) = ma(t), \tag{49}$$

$$P(t) = F(t)\ x\ v(t), \tag{50}$$

$$E(t) = P(t) \times dt. \tag{51}$$

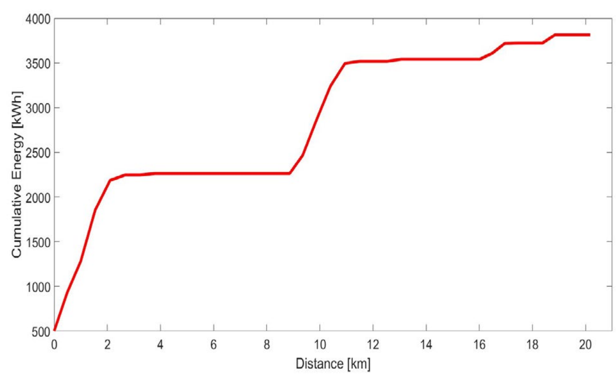

Figure 20: Cumulative energy plot for [70,90.64] km

The results were plotted according to the theoretical algorithms presented in Section 4. Figures 7 to 20 describe the results of the FEDEO optimization after the speed tracking control is applied. The results presented in this chapter are interpreted and discussed in Section 7.4, where the optimized sections of the route profile are presented and the savings achieved described.

7.4. Discussion of Results

The first finding of the study is that the FEDEO algorithm shows results with a significant reduction in energy usage over the complete journey, which is affected by train acceleration, speed, and gradient constraints. This means it has achieved energy savings using the MINLP method, which leads to improvement in overall efficiency. The profile prediction horizon of the eco-driving solution was broken down into N intervals within the discretized prediction horizon in the FEDEO algorithm. An EV will realistically follow the continuous algorithm since the route profile will follow a continuous path. However, in the case of the 19E electric locomotive with regeneration, the simulation has shown that driving can be improved at gradients requiring higher speeds and substantial energy savings can be achieved by the simple application of the train notch decisions. The braking force combines the mechanical and electrical braking, and the braking force is low when the train runs at high speed owing to the reliance on coasting for movement.

The second finding is that the FEDEO algorithm utilizes the optimal notch at each time interval of the journey. This means that the best suited algorithm has been found when distance and elevation are known. It also allows more braking as this force is responsible for regenerative energy. The typical gradient varies from -3 to

+3% for the route profile regarding freight sections globally; this is because freight trains cannot carry an excessive load over specific gradients for reasons such as possible stalling, loss of cargo, higher energy usage required to overcome higher gradients, and onboard power electronics of the locomotive not having the required traction. The algorithm developed using MINLP can allow the train control Centre to advise the driver about which route profile behavior to follow regarding the train at gradients with a high energy usage requirement [1, 44].

The third finding is that the FEDEO algorithm utilizes more coasting and cruising, and fewer changes in the acceleration and braking. The specific eco-driving solution introduced in this study uses a significant acceleration in the beginning, lower gradient resistance in the middle of the section (coasting), and fast braking at the end of the section. The FEDEO algorithm reduces journey time, finds the optimal velocity and implements speed-tracking control. The train requires the traction energy to achieve the gravitational energy on sections that are uphill and uses the gravitational energy on downhill sections. The quality of the trip is optimized by using minimal traction energy on the middle sections of the route. The eco-driving solution depends on the train mass, acceleration and speed limitations, and the gradient profile of the route.

The fourth finding is that the solution can be used for any freight train, provided that the route profile and train coefficient parameters are known. The maximum speed limits also have a critical impact on the optimal route profile and the energy savings. The static parameters such as the gradient profile, speed and acceleration limits, and train mass cannot be modified. However, dynamic parameters have been optimized using the MINLP algorithm or eco-driving. The formulation reduces the difference in the trajectory the train would follow if it were to traverse a continuous profile, compared to the discrete case, which incorporates notches as demonstrated by the MINLP algorithm. The FEDEO algorithm has shown reduced energy usage and distinct savings, as summarized in Table 3 [1, 17, 44].

The final finding is that the eco-driving solution applied to EVs can also be applied to freight trains by incorporating the train coefficient parameters. The discrete objective function obtained is for the speed greater than 0 with the train moving forward. In the ideal problem case, the tractive and braking energy both contribute to the route profile energy consumption. However, in the discrete eco-driving approach described in Section 5, the 19E locomotive only requires energy consumption from the tractive effort. The energy used during braking is fed back into the overhead line as regeneration energy. The eco-driving example is applied to a hybrid EV, as outlined by Khalik et al. [1]. In this study, this approach has been applied to the scenario of the 19E train on the specific Ermelo to Richards Bay section. A variable speed has been proposed with an eco-driving strategy where the train speed changes between the given bounds of the route profile. The comparison between the actual and optimal eco-driving energy costs and the time taken is shown in Table 3 [6, 35, 42, 44].

Table 3: Energy usage comparison of the FEDEO simulated sections

Simulated Section (FEDEO)	Actual energy usage (kWh)	Optimal energy usage (kWh)	Actual time (minutes)	Optimal time (minutes)
[0, 90.64 km]	$25,629.48$	$16,485.00$	122.97	86.74
[70, 90.64 km] $[0\ km/h, 0\ km/h]$	5097.79	7488.9	18.83	17.63
[70, 90.64 km] $[60\ km/h, 60\ km/h]$	5097.79	3814.9	18.83	17.63
Savings (%)				
[0, 90.64 km]	-	34.763	-	29.46
[70, 90.64 km] [0 km/h, 0 km/h]	-	- 46.905	-	6.373
[70, 90.64 km] [60 km/h, 60 km/h]	-	25.166	-	6.373

8. Conclusion

In conclusion, the optimization of the energy utilizing MINLP by making calculated decisions using the tractive and braking effort significantly reduces the overall energy consumption of the train by exactly 34.76% for the entire route section and 25.17% for the smaller section of 20.64 km. Non-linear vehicle dynamics formulate the general train control problem from the traction and train resistance forces as a function of speed and route elevation changes. The route is partitioned into stations of varying gradients and speed. The problem formulated is a multiple phase problem where each section of the route depends on the load being hauled, the gradient, and the train's ability to coast, cruise, brake, and accelerate during inclines and declines. Owing to the nature of the 19E train, which utilizes regenerative braking, the focus is on the train's tractive effort, acceleration, and speed for optimization. The modelling of the codriving solution has shown that intelligent driving over large gradients can significantly save cost and improve the train trajectory over the route profile. This study has reviewed the energy optimization (FEDEO) algorithm for reducing energy consumption and costs for the 19E train using CCR9 wagons on the Ermelo-Richards Bay coal line. Energy efficient train control is a requirement for the operational sector of freight transport. Any optimization model used needs to be analyzed and set out methodologically to obtain there quired performance andaccuracy. The algorithm in this study has optimized the solution of eco-driving within the freight rail sector of South Africa and globally.

Acknowledgements

This research has been supported by the University of Pretoria and has been acknowledged by the Energy Centre.

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