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Determination of Optimal Start-up Curve for Steam Turbine Unit Considering Centrifugal Stress

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Abstract

This paper concerns the optimal start-up schedule problem of steam turbine unit. Modelling of start-up process synthesizes the energy balance in turbine unit, depiction of control system, calculation of stress and damage. Quantitative analysis is made in the view of model feature, centrifugal stress's effect, correction to the theoretical schedule and feasible space. Based on the synthetic model of turbine start-up schedule, a nonlinear unitary quadratic function is proposed and solved as the trajectory to improve start-up efficiency. Finally, the cold start-up of a domestic 300MW steam turbine unit is taken as a simulation example to prove the capability of this nonlinear start-up curve. Without violating the stress limit strictly, although the desired time improvement is only 6.35%, it still has important significance in practical thermal engineering, especially in the aspect of energy saving.

Keywords: Centrifugal Stress, Start-Up Optimization, Steam Turbine, Finite Element Analysis, Effective Stress

Introduction

Presently, the start-up schedule is pre-determined by turbine design engineer and used to direct the start-up controller. However, it is usually a linear chart and some mismatch may occur in practical thermal engineering [1]. For example, it needs evaluate accurately various stress besides of the thermal stress, such as centrifugal stress and so on [2]. Due to the importance in thermal engineering, especially in aspect of energy saving, the start-up scheduling problem has attracted lots of attention from academia and industry [3-4].

Generally, academic researches concentrate on the optimization approaches of this problem, and transfer it to a mathematical model to attain the optimal resolution. Besides the objective function, the constraints should also be defined distinctly. In this way, the scheduling problem could be formulated as an additive function with certain limits to be optimized. Because of the high nonlinearity, it is difficult to apply directly the traditional particle swarm optimization method, it needs some other techniques such as support vector machines (SVM) to establish the regression model [5]. The same situation occurs in the genetic algorithm-particle swarm optimization approach [6], simulation results have proved the effect of these improvements. In order to avoid lots of calculations of low cycle fatigue, a kind of online optimal control schemes was carried out during the start-up of steam turbines [7]. Based on the Green's Function and the Pontryagin's Maximum Principle, this online optimal control scheme may shorten the time of start-up greatly without exceeding permitted fatigue damage. To characterize thermodynamic behavior under varying conditions, Luo et al decomposed the complex turbine into several simple turbines, and applied the Stodola's formulation to simulate the uncontrolled extraction steam parameters [8]. Comparison with conventional operation strategy may obtain a maximum of 5.47%of total operation cost saving. A multi-objective optimization method based on the tracking and regulation performance objectives subject to a robustness constraint was introduced to improve the SST controller parameters [9]. Comparative simulations shew the advantage of the proposed strategy, and it was confirmed by a filed test in an in-service 300MW power plant. In order to improve the accessibility to simulation software with acceptable reduction in accuracy, a framework was proposed by Rossi et al by applying used office suite-Microsoft Excel/Visual Basic [10]. Data recorded through sensors of a 390MWmulti-shaft combined cycle shew good agreement with experimental data. Artificial intelligence may be another feasible solution to this complicated optimization problem. An integrated approach using Nelder Mead optimization method coupling with artificial neural network inverse was developed by Hamzaoui et al, and the very low percentage of error and short computing time were precise and efficient [11]. It is attractive to be applied for control on line and constitutes a promising framework. Nowak et al employed two artificial neural network (ANN) working in series to control the steam turbine heating process, and the steam temperature at the turbine inlet was controlled to remain both start-up rate and safety of turbine unit at an acceptable level [12].

As above references proved that, this kind of start-up schedule optimizing problem includes active constraints, and the global optimal solution lies on hedge of the feasible space. Under the condition of active constraints, if the objection function is unimodal and differential, then conventional gradient projection based method may be effective, however this schedule optimizing problem might be missing this usually. In this way, this manuscript combines the academic view and engineering experience to explore the optimal start-up curve for steam turbine unit. Equal emphases are paid both on academic value and engineering feasibility.

Model Construction

For this scheduling problem containing active constraints, the optimal schedule lies on the edge of the feasible space. So the accuracy of edge calculation results may affect the optimal schedule seriously. In this edge determination process, there may be two main factors, one is the centrifugal stress, the other one is the parameters in the objective equation. Effect of centrifugal stress is reflected in the damage Di which may be obtained according to functions 1, 2, 3, and 4 as follows:

$$\begin{aligned}\sigma_{eq} &= \sqrt{(\sigma_{thz})^2 + (\sigma_{thy} + \sigma_t)^2 - \sigma_{thz}(\sigma_{thy} + \sigma_t)} \\ &= \sqrt{(\sigma_{thz})^2 + (\sigma_t)^2 + \sigma_{thz}\sigma_t}\end{aligned} \qquad (1)$$

Where σ_{thz} is the axial component of thermal stress, and σ_t is the mechanical centrifugal stress. Centrifugal stress σ_t may be get using following equation:

$$\sigma_t = \sigma_{eq}(\frac{N}{N_e})^2 = \sigma_{eq}(\frac{N}{3000})^2 \qquad (2)$$

Using the result of effective stress, strain ϵ could be get as follows:

$$\epsilon = \frac{\sigma_{eq}}{E} + (\frac{\sigma_{eq}}{2K'})^{\frac{1}{n'}} \qquad (3)$$

Quick damage computation equation may be obtained via the polynomial fitting method as:

$$\begin{aligned}d = \frac{1}{2N_f} &= P_1\epsilon^7 + P_2\epsilon^6 + P_3\epsilon^5 + P_4\epsilon^4 + P_5\epsilon^3 \\ &+ P_6\epsilon^2 + P_7\epsilon + P_8\end{aligned} \qquad (4)$$

where Pi is the polynomial parameter based on FEA data. Above four functions transfer the nonlinear finite element analysis data to a rapid damage computation method.

Parameters in the objective equation are composed of deterministic parameters and variable parameters. Deterministic parameters include the number of sections during start up process *K*, the final temperature requirement T_{end}, the final rotational speed *ni*, and the constants *βi*, while variable parameters include the specific damage value in every start-up section and the corresponding operation time ti. In fact, the deterministic parameters may be determined under the direct of practical engineering, and they are not in the optimization domain. If they are determined, the effects are also confirmed. Differently, the variable parameters are the objective optimization parameters in this schedule problem. Seemingly, this optimization problem need to find every optimal operation time ti to make sure the objective function minimum, however this should obey the limit condition expressed by $Di \le D_{lim}$. If the operational time t_i is long, there is difference between current value and the optimal value of the schedule function, moreover this difference may impel the operational time ti to decrease in next cyclic computing process. As long as the damage D_i does not exceed the threshold, this impelling procedure will keep going. Finally, the procedure will stop if damage D_i meet but not beyond the limit, then it enters a pre-setting minimal neighbourhood. During this iterative computation process, the start-up curve is regulated gradually near to the optimal schedule while keep the effective stress within the limits. A closed constraint loop is constructed to make sure the function of schedule problem is stable in this way, the synthetic model of start-up process is composed of a minimizing objective function and some constraints as follows:

$$\min \quad J(t_1, t_2, ...t_K|K) = \beta_1\frac{D_i}{D_{lim}} + \beta_2\frac{\sum_{i=1}^{K} t_i}{t_0} + \beta_3$$

$$s.t. \begin{cases} D_i \leqslant D_{lim}, \\ t_i > 0, \\ K \geqslant 3, \\ T_{end} \leqslant 538°C, \\ n_i \leqslant 3000. \end{cases} \qquad (5)$$

where D_i is the specific damage value in every start-up section, while D_{lim} is the limit value of damage; β_1 and β_2 are weight constants between *0* and *1*, β_3 is the constant to represent all the constant time; *K* is the number of sections during start-up process, and according to practical engineering, it is equal or greater than 3; t_i is the corresponding operation time in every section, and t_0 is the original total start-up time, it is clearly the summation of every operation time is equal or smaller than the original total start-up time. So the first term and the second time are all equal or smaller than 1 depending on the weight parameters β_i. Tend and ni are the final requirements at the end of start-up process, which are implicit in the damage computation equations. In this way, an additive form objection function is established, and all the parameters are positive, this guarantees the optimization space in the first quadrant conveniently.

Effect of Centrifugal Stress

According to the online damage calculation program for steam turbine rotor, which is combined finite element analysis technique with the polynomial fitting method, thermal stress evolution at special point of turbine rotor may be obtained as shown in figure 1. Variation of thermal stress depends on the change of temperature gradient, and the operational time for corresponding start-up stage is inversely proportional to the temperature gradient. The longer the operational time lasts, the smaller the temperature gradient emerges. In figure 1, thermal stress may reach 280MPa under cold start-up condition, and some start-up schedule optimization method adopts this thermal stress value, but there still exists certain gap for high precision computation.

Consider the rotational speed of rotor as shown in upper figure of figure 2, stages of low speed warming and medium speed warming are denoted in red arrows. All the line segments of rotation speed vary linearly in the range of 0 r/min and 3000 r/min. Low speed warming last shortly, so this stage is approximated as a linear ramp line. Correspondingly, the centrifugal stress is given in the lower figure of figure 2. What is worth mentioning is the centrifugal stress is nonlinear as the rotation speed increase, moreover three nonlinear curves are pointed out. This nonlinearity is due to the fact that the centrifugal stress is proportional to the square of rotational speed. The final centrifugal stress will remain near to 90MPa at the 3000 r/min.

According to the fourth strength theory listed in equation 1, the effective stress comprised of thermal stress and centrifugal stress is shown in figure 3. Similar to figure1, effective stress varies nonlinearly with the rotational speed increasing, but different from figure 1, the effective stress remains at the about 90MPa while the thermal stress reaches almost to vanishing point. If this 90MPa stress is ignored, there may emerge some computation errors or even some mistakes. One core recommendation of this manuscript is advising to consider this centrifugal stress seriously when draw up the turbine start-up schedule.

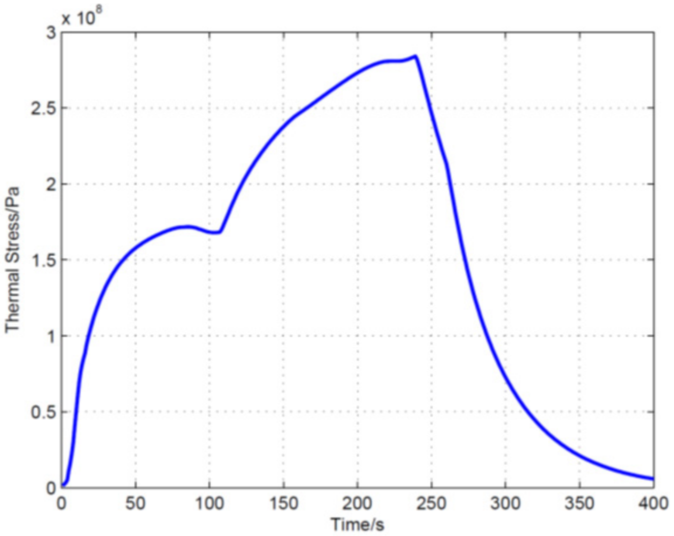

Figure 1: Thermal Stress Evolution

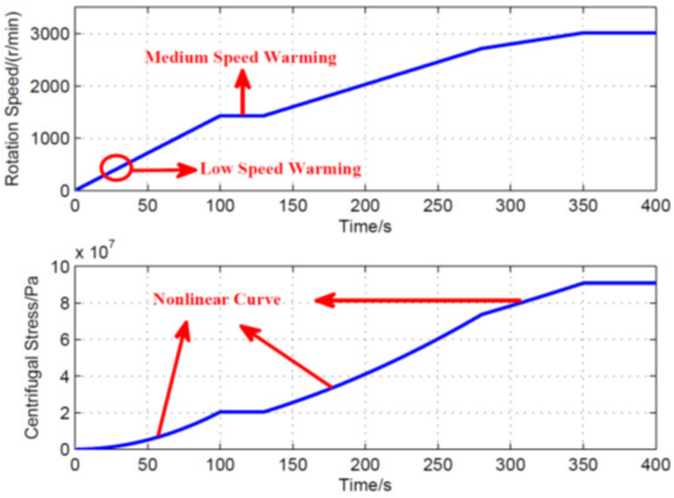

Figure 2: Sketchy diagram of the steam turbine rotational speed and the corresponding centrifugal stresses

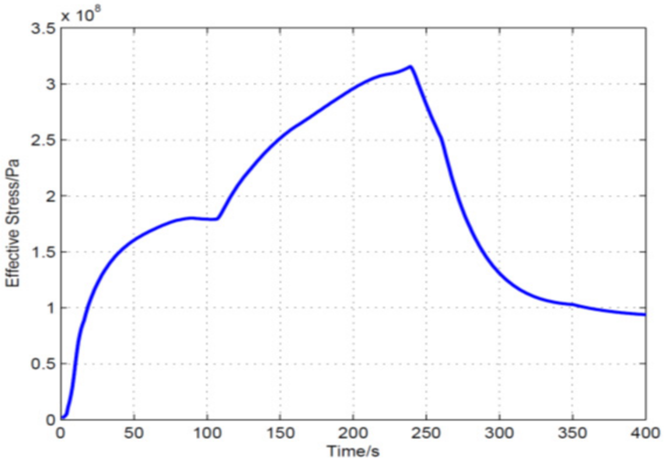

Figure 3: Effective Stress Evolution

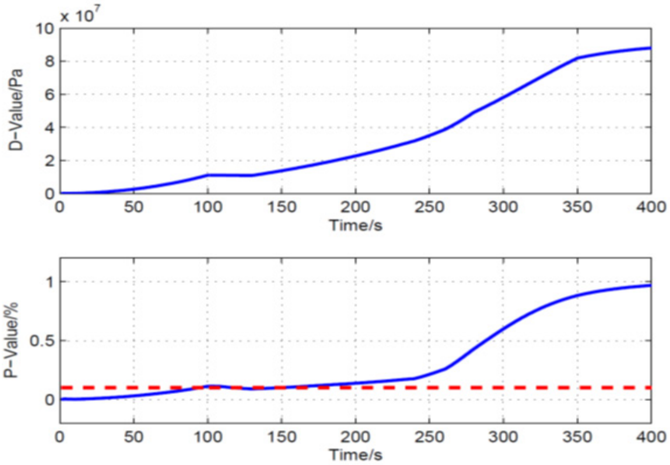

Figure 4: D-value and P-value of Centrifugal Stress

In order to evaluate quantitatively the importance of centrifugal stress, two indices are adopted. One is the difference between effective stress and thermal stress, another one is the percentage of centrifugal stress account for effective stress. Thermal stress and effective stress have been shown in figure 3 and figure 1 separately. Define the 'DValue' and 'PValue' as follows:

$$\begin{aligned} DValue &= \sigma_e - \sigma_t \\ PValue &= \frac{\sigma_c}{\sigma_e} \times 100\% \end{aligned} \tag{6}$$

There into, the 'DValue' is used to evaluate the difference variation while the 'PValue' is used to assess the percentage variation. In figure 4, it can be found the difference starts at nearly zero, and the value increase in a nonlinear projection. Beside of some short constant during the medium speed warming, this increment always remains positive gradient, and the final difference value arrives at approximate 90MPa. In the same way, the percentage value also starts at zero, but in the medium speed warming it displays negative gradient, which is labelled using red dot line. Second half variation of percentage value is similar with the difference value and it reaches near to 100% ultimately.

These simulation results indicate that during the start-up process of steam turbine, thermal stress plays a crucial role in the earlier and middle stage, however, when entering the middle and later stage, centrifugal stress becomes the main ingredient. In terms of operational mechanism, there may be two causes for these phenomena:

- in the earlier and middle stage, massive steam with very high temperature is injected on the turbine rotor, this generates large temperature gradient meanwhile the centrifugal speed is low, so thermal stress accounts the absolute majority of effective stress;
- in the middle and later stage, the temperature difference between the surface and centre of rotor vanishes gradually, and the centrifugal speed increases to a relative high level, so the centrifugal stress accounts the absolute majority of effective stress.

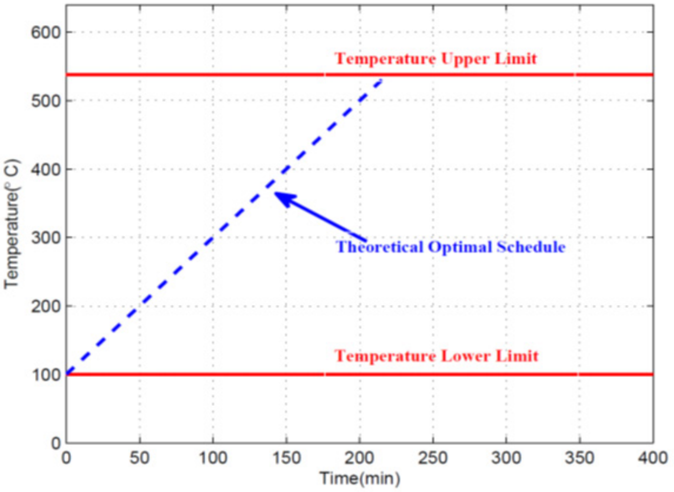

Figure 4: Theoretical optimal schedule

As result of the relatively high proportion, which may run up to 32.14% (computed by 90/280) at the peak value, this analysis reminds the effect of centrifugal stress may not be ignored in the schedule optimizing process.

Quantitative Analysis

Theoretical Optimal Schedule and Correction

According to equation 5, the problem of start-up scheduling may be transferred to addition of several stages of state transition. Every state transition is required to complete as quickly as possible, while does not violate the limit calibrated by some specific parameters. As shown in figure 5, state transition may be expressed a transition from the lower temperature to the higher temperature. Between the temperature upper limit and the temperature lower limit, the optimal schedule is apparently the dashed line with a gradient as large as possible. The larger gradient is, the less transition time lasts. However, the gradient is limited by the endurance capacity of rotor material to stress. In this simplified mathematical model, as long as the feasible space is delimited clearly, there must exists intuitively a potential optimal start-up schedule.

During the process of start-up, steam turbine needs some time to be warmed or checked. For example, in the cold start-up process, the main steam entering the turbine at least has a superheat of 50◦C, and auxiliary equipment's including control system should be thoroughly inspected when it is running. Of course these time are included in the start-up time. In fact, these warming time, checking time and other preparing time distributes in different stage of the process of start-up, and for simplicity they may be treated equivalently as a constant time named 'Medium Speed Warming' time, which is expressed by parameter β_3. As shown in figure6, the original theoretical optimal gradient line is broken and inserted the constant time. By this means, parameter β_3 is limited to a certain extent, and it should tend to the minimum when scheduling optimally.

Moreover, when consider the centrifugal stress, the theoretical optimal gradient should be further corrected, especially when the centrifugal speed reaches high value range.

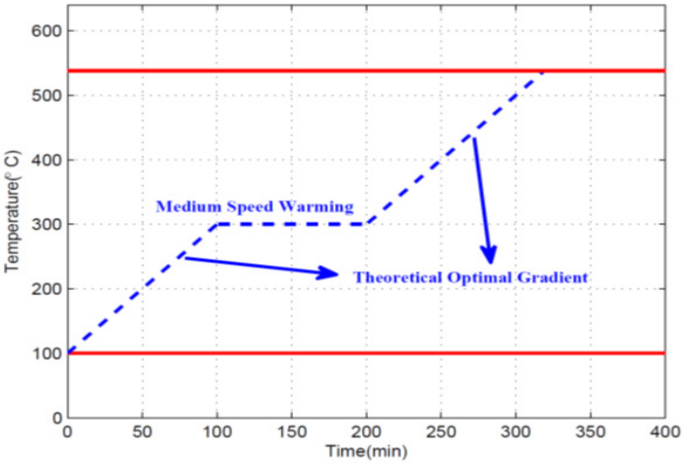

Figure 6: Theoretical optimal schedule considering the medium speed warming

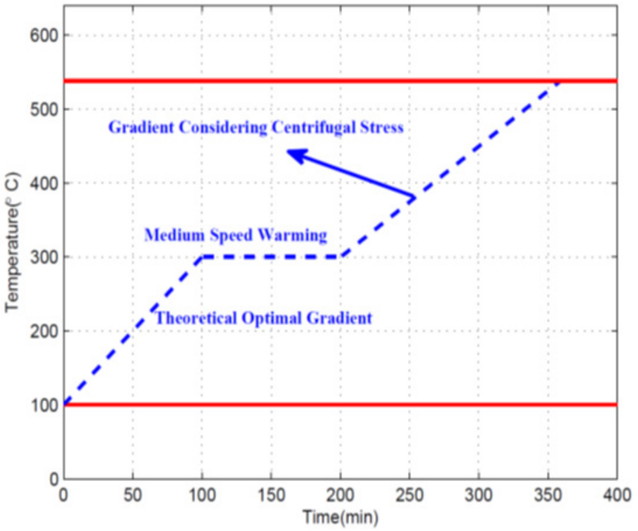

Figure 7: Modified schedule considering the centrifugal stress

In other words, the gradient of main stream temperature should be smaller than current value, as shown in figure 7. In figure 7, it is clear that the temperature gradient after medium speed warming is smaller than that before medium speed warming. Although this correction is qualitative temporarily, it points out importantly the optimizing direction. Because the possible optimal schedule lies on the edge of feasible space definitely, it is crucial to determine quantitatively the feasible edge with maximal temperature gradient.

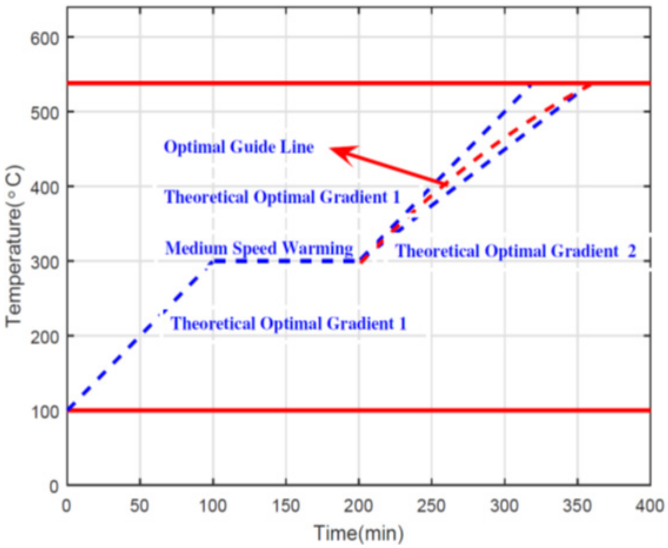

Figure 8: Optimal guide line considering the centrifugal stress

In this paper, determination of feasible space is also concentrating on the stage in which centrifugal stress plays the leading role. As shown in figure 8, this work pays main attention on the later section of schedule. Theoretical optimal gradient 1 is the original optimal schedule, while theoretical optimal gradient 2 is the corrected optimal schedule considering centrifugal stress. It is believed the optimal guide line locates between these two gradients. Assuming the red dashed line is the potential optimal schedule, it is featured with following characteristics:

- gradient at the initial point of optimal guide line is less than or equal to the theoretical optimal gradient 1, it is the maximum;
- gradient at the end point of optimal guide line is greater than or equal to the theoretical optimal gradient 2, it is the minimum;
- gradient in the other trajectory of optimal guide line is between the maximum and the minimum;
- gradient in the whole trajectory of optimal guide line, including the initial point and the end point, is decreasing monotony or remaining constant.

Simulation Example

Taking the cold start-up of a domestic 300MW steam turbine unit as an example, attention is paid on the later section as shown in figure 8. Original model (OM) of start-up is a linear schedule depicted as the theoretical optimal gradient 2, which considers the effect of centrifugal stress. Although the centrifugal stress develops in a nonlinear trajectory, it is treated as a constant with the maximum due to the linearity of theoretical optimal gradient 2. Define a nonlinear unitary quadratic function and the gradient limit as follows:

$$\begin{cases} f(x) = a_2x^2 + a_1x + a_0, \\ \frac{df(x)}{dx} \leq \alpha. \end{cases} \tag{7}$$

where αi is the corresponding coefficient, x is the time, α is the gradient limit and it changes with the start-up process. At the initial point, it is equal to the theoretical optimal gradient 1, while at the end it is equal to the theoretical optimal gradient 2. In the middle, it is determined by the effective stress limit. Detail gradient of the unitary quadratic function is listed in equation 8:

$$\begin{cases} f(x) = a_2x^2 + a_1x + a_0, \\ f'(x) = a_2x + a_1 \leq \alpha. \end{cases} \tag{8}$$

Table 1: Calculation results of parameters

Status	α2	α1	α0
OM	0	1.5	0
IM1	-0.0018	2.7292	-172.9167
IM2	-0.0021	2.8206	-182.0573
IM3	-0.0023	2.9073	-190.7292
IM4	-0.0025	2.9893	-198.9323
IM5	-0.0027	3.0667	-206.6667

Table 2: Assessing indeces

Status	Terminal Time	Maximal Stress(MPa)	Improvement
OM	360	318.2	0
IM1	337.1429	318.2	6.35%
IM2	340.1460	308.6	5.52%
IM3	343.2836	299.3	4.64%
IM4	346.5649	290.4	3.73%
IM5	350	281.1	2.78%

Besides the linear original model (OM), five other improved model (IM) are carried out to calculate the related terminal time and maximal stress. These five improved model (IM1, IM2, IM3, IM4, IM5) correspond to the five condition: none redundancy, 5% effective stress redundancy, 10% effective stress redundancy, 15% effective stress redundancy, 20% effective stress redundancy. The coefficients of respective unitary quadratic function are given in tab.1. Furthermore, calculation results indicate that with the more redundancy in different model, terminal time arrives later while maximal effective stress drops. And the most obvious time improvement presents in the improved model with none redundancy, and the improvement decreases monotonously with the redundancy increasing.

As shown in table 2, it is clear that result comparison between the linear original model (OM) and the nonlinear improved model 1(IM1) maybe more meaningful. Because the terminal gradients are equal, the maximal stress of both model all equal to 318.2

MPa. However, the terminal time of improved model 1 (IM1) is only 337.1492, less than the 360 of original model (OM). Time improvement maybe obtained as 6.35% under the condition of considering the centrifugal stress. It is believed this start-up time improvement has important significance in practical thermal engineering, especially in the aspect of energy saving.

Conclusion

This paper talks about the optimal start-up schedule problem of steam turbine unit, and detail modelling of start-up process is carried out, including the energy balance in turbine unit, depiction of control system, calculation of stress and damage. Quantitative analysis of the synthetic model of start-up process is made in the view of model feature, centrifugal stress's effect, correction to the theoretical schedule and feasible space. A nonlinear unitary quadratic function is adopted as the trajectory to obtain time improvement. Finally, taking the cold start-up of a domestic 300MW steam turbine unit as a simulation example, the capability of this nonlinear start-up curve is proved. Although the desired time improvement is only 6.35% without violating the stress limit, it is still important for the energy saving in practical thermal engineering. Some future research on the efficient method of optimal start-up schedule of steam turbine unit may consider this manuscript as a feasible reference.

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