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Demonstrating That a Particle Is a Mutual Energy Flow Rather Than a Self-Energy Flow from the K-G Equation

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Abstract

Since the author proposed the electromagnetic mutual energy theorem and its inner product concept in 1987, and after the improvement of Huygens' principle (1989), the mutual energy flow theorem was formally established in 2017, holding that the mutual energy flow is responsible for transmitting electromagnetic energy. However, this conflicts with traditional electromagnetic theory, which holds that the self-energy flow, i.e., the Poynting vector, transmits energy. From the fact that the self-energy flow transmits energy, one can derive that the mutual energy flow also transmits energy. Therefore, there must be a problem with the self-energy flow transmitting energy. To resolve this contradiction, the author conjectures that the self-energy flow does not transmit energy, and only the mutual energy flow transmits energy. For the self-energy flow not to transmit energy, there are two possibilities: (1) the self-energy flow is reactive power, but this is not supported by Maxwell's electromagnetic theory; (2) the self-energy flow undergoes reverse collapse back to its radiation source point, similar to the wave collapse process in quantum mechanics. Reverse collapse preserves Maxwell's equations but requires the addition of an unknown time-reversed electromagnetic wave. Since 2022, the author has studied a class of problems including transformer primary and secondary, and dipole transmitting and receiving antenna pairs, and found that (1) the self-energy flow being reactive power more easily makes the new theory self-consistent. The self-energy flow being reactive power requires a modification of Maxwell's electromagnetic theory. This modification preserves Maxwell's equations but gives a new interpretation to the quantities therein, namely the electric and magnetic fields. After the modification, the phase of the induced electric field of the electromagnetic wave remains unchanged, but the phase of the magnetic field must be corrected by 90 degrees. Since the self-energy flow must be reactive power and not transmit energy, the mutual energy flow theorem should be updated to the mutual energy flow law. In 2024, the author extended this idea to the Schrödinger equation, establishing a mutual energy flow with particle properties by defining a "magnetic field" and correcting its phase. This paper demonstrates the universality of this framework for the Klein-Gordon (K-G) equation. By constructing a magnetic field for the K-G equation and correcting its phase, this paper derives the corresponding mutual energy flow law. Research confirms that this mutual energy flow, formed by the coupling of retarded and advanced waves, is generated at the source and annihilated at the sink, perfectly unifying the wave and particle properties of particles and providing a solid theoretical foundation for understanding wave-particle duality.

Keywords: Klein-Gordon, Schrödinger, Maxwell, Poynting, Welch, Absorber Theory, Reactive Power, Active Power, Wave-Particle Duality, Electromagnetic Wave, Photon, Wave Collapse, Green's Function, Reciprocity Theorem, Energy Flow

1. Introduction

The author proposed the mutual energy theorem in 1987 [1-3]. This theorem is the Fourier transform form of the time-domain reciprocity theorem proposed by Welch in 1960 [4]. This theorem can also be obtained from the Lorentz (1896) reciprocity theorem via a conjugate transformation [5]. The author believes that this theorem is not merely a reciprocity theorem, but should be an energy theorem. To illustrate this, the author once attempted to prove this theorem using the complex Poynting theorem, but without success. It was not until 2017, when the author already knew Welch's time-domain reciprocity theorem, that Welch's time-domain reciprocity theorem was proved from Poynting's theorem (1884), and then the mutual energy theorem could be obtained by a Fourier transform [6]. This

shows that the author has grounds for calling it an energy theorem. However, since Welch's time-domain reciprocity theorem involves advanced waves, the mutual energy theorem does too. Advanced waves are currently not accepted by most scientists and engineers as truly physically real, and this is a controversial issue. Therefore, whether this theorem can be regarded as an energy theorem remains controversial.

Since then, the author began to pay attention to the theory of advanced waves, noting Wheeler and Feynman's absorber theory, Dirac's self-force theory, Schwarzschild et al.'s action-at-a-distance theory, Cramer's transactional interpretation of quantum mechanics, and Stephenson's theory of advanced waves [7-15]. These theories helped the author build confidence that advanced waves are indeed physically real. Since the mutual energy theorem is an energy theorem, energy should be transmitted by the mutual energy flow. In 1989, the author proposed Huygens' principle based on the mutual energy theorem. In 2017, the author found that the intermediate process formula for deriving Huygens' principle can be regarded as the mutual energy flow theorem. Thus the author proposed the concept of mutual energy flow [16]. The author believes that energy should be transmitted by the mutual energy flow. However, in Maxwell's electromagnetic theory there is the self-energy flow, i.e., the Poynting vector, which also transmits energy. The author believes that either the self-energy flow transmits energy or the mutual energy flow transmits energy; it is impossible for both the self-energy flow and the mutual energy flow to transmit energy simultaneously. Because if the self-energy flow transmits energy and the mutual energy flow also transmits energy, there would be two kinds of energy flow transmitting energy. In reality we only observe one kind of photon. To merge the mutual energy flow and the self-energy flow together is almost unimaginable. In addition, if both the self-energy flow and the mutual energy flow transmit energy, the transmitted energy would also become excessive. Therefore, the author proposed the concept of reverse collapse of the self-energy flow. After 2020, the author began to study specific examples of the mutual energy flow transmitting energy. These examples include the energy flow from the primary to the secondary coil of a transformer, the energy flow from a transmitting antenna to a receiving antenna, and the energy flow between two planar currents. These energy flows can be used to explain photons.

The author found that the reverse collapse of the self-energy flow can be replaced by the concept of the self-energy flow being reactive power. The self-energy flow being reactive power means that within one actual period, the power flow propagates forward for half the time and backward for half the time, so on average such a wave does not transmit energy. However, the wave in Maxwell's electromagnetic theory is active power. If the wave is reactive power, then Maxwell's electromagnetic theory must have a bug. Around 2022, the author found that the bug in Maxwell's electromagnetic theory is defining the curl of the magnetic vector potential as the magnetic field. This definition is conditional: $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ holds only under magneto-quasi-static conditions, and does not hold for the retarded potential (advanced potential) of a radiating electromagnetic field. For the retarded potential and advanced potential, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ calculates the phase of the magnetic field incorrectly, so a correction is needed [17-40]. The author found that after the correction, the mutual energy flow transmits energy and has the property of being generated at the source and annihilated at the sink. This property is a necessary condition for an energy flow to serve as a particle. Therefore, the author realized that the mutual energy flow is actually the particle. In electromagnetic theory, it is the photon. Saying that the electromagnetic wave is reactive power is conceptually very similar to saying that the electromagnetic wave is probabilistic. In traditional electromagnetic wave theory, because the wave carries energy and the particle also carries energy, in quantum mechanics we have no choice but to make the wave probabilistic, and it must also collapse. If the wave is reactive power, the wave need not be probabilistic. Since 2022, the author has gradually extended the achievements in electromagnetic field theory to the Schrödinger equation of quantum mechanics [33,41]. This paper further applies this theory to the Klein-Gordon (K-G) equation. Thereby a complete and correct quantum theory is established.

2. Demonstrating from Electromagnetic Theory That a Particle Is a Mutual Energy Flow Rather Than a Self-Energy Flow

2.1 Maxwell's Equations

Our discussion is restricted to vacuum, so $\mu = \mu_0, \epsilon = \epsilon_0$. Starting from Maxwell's equations, we have

$$\begin{cases} \nabla \cdot \boldsymbol{E} = \rho/\epsilon_0 \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{E} = -\dfrac{\partial}{\partial t}\mu_0 \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial}{\partial t}\epsilon_0 \boldsymbol{E} \end{cases} \tag{1}$$

2.2 The Energy Flow Problem

From Maxwell's equations (1) one can obtain Poynting's theorem [6],

$$-\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=\int_{V}\boldsymbol{E}\cdot\boldsymbol{J}dV+\frac{\partial}{\partial t}U \tag{2}$$

where

$$U=\frac{1}{2}\int_{V}(\boldsymbol{E}\cdot\epsilon_0\boldsymbol{E}+\boldsymbol{H}\cdot\mu_0\boldsymbol{H})dV \tag{3}$$

Integrating the above equation over time,

$$-\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=\int_{t=-\infty}^{\infty}dt\int_{V}\boldsymbol{E}\cdot\boldsymbol{J}dV+\int_{t=-\infty}^{\infty}dt\frac{\partial}{\partial t}U \tag{4}$$

considering:

$$\int_{t=-\infty}^{\infty}dt\frac{\partial}{\partial t}U=U(\infty)-U(-\infty)=0 \tag{5}$$

we obtain,

$$-\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=\int_{t=-\infty}^{\infty}dt\int_{V}\boldsymbol{E}\cdot\boldsymbol{J}dV \tag{6}$$

Considering the superposition principle

$$\boldsymbol{E}=\sum_{i=1}^{2}\boldsymbol{E}_i,\quad \boldsymbol{H}=\sum_{i=1}^{2}\boldsymbol{H}_i,\quad \boldsymbol{J}=\sum_{i=1}^{2}\boldsymbol{J}_i \tag{7}$$

we obtain

$$-\sum_{i=1}^{2}\sum_{j=1}^{2}\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma=\sum_{i=1}^{2}\sum_{j=1}^{2}\int_{t=-\infty}^{\infty}dt\int_{V}\boldsymbol{E}_i\cdot\boldsymbol{J}_jdV \tag{8}$$

From (6) we obtain,

$$-\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma=\int_{t=-\infty}^{\infty}dt\int_{V}\boldsymbol{E}_i\cdot\boldsymbol{J}_idV \tag{9}$$

Subtracting (9) from (8),

$$-\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma=\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\int_{t=-\infty}^{\infty}dt\int_{V}\boldsymbol{E}_i\cdot\boldsymbol{J}_jdV \tag{10}$$

or

$$-\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=\int_{t=-\infty}^{\infty}dt\int_{V}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV \tag{11}$$

The above equation is the Welch time-domain reciprocity theorem [4]. Applying a Fourier transform to equation (11) gives

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=\int_{V}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*+\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV \tag{12}$$

The above equation is the mutual energy theorem proposed by the author [1, 2, 3]. From the Silver-Muller radiation boundary condition, we obtain

$$\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=0 \tag{13}$$

where Γ is taken as a spherical surface of infinite radius. From this we obtain

$$\int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* + \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = 0 \tag{14}$$

or

$$-\int_{V_1} \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 dV = \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* dV \tag{15}$$

The above equation is also a form of the mutual energy theorem [1, 2, 3], and of course it is also a reciprocity theorem[42,43].

2.3 Mutual Energy Flow Law

Considering that $\boldsymbol{J}_2$ is outside the surface Γ, (12) can be written as,

$$\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = -\int_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{16}$$

From (15, 16) we obtain the mutual energy flow theorem,

$$-\int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*) dV \tag{17}$$

Formula (17) is used to calculate the mutual energy flow between two point sources, such as the mutual energy flow from a transmitting antenna to a receiving antenna, or the mutual energy flow from the primary to the secondary of a transformer. However, there is another case in which the absorbers are arranged over the entire 4π solid angle on a spherical surface of infinite radius. In this case the source is a point, but the sinks are in all directions. The sinks can actually be divided into two parts, one in the front and one in the back. The source actually emits the energy of one photon at a time, and this energy should be determined by formula (17). But we need to average the energy of this photon over the front and back directions, so the energy flow density is reduced. In this case the correct formula is rewritten as,

$$-\int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \frac{1}{2} \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*) dV \tag{18}$$

The above equation has an extra factor of $\frac{1}{2}$ compared with (17). This is equivalent to making the transformation in the original equation

$$\boldsymbol{J} \to 2\boldsymbol{J}, \quad \boldsymbol{E} \to \boldsymbol{E}, \quad \boldsymbol{H} \to \boldsymbol{H} \tag{19}$$

or making the transformation

$$\boldsymbol{J} \to \boldsymbol{J}, \quad \boldsymbol{E} \to \frac{1}{2}\boldsymbol{E}, \quad \boldsymbol{H} \to \frac{1}{2}\boldsymbol{H} \tag{20}$$

Doing so actually takes into account that the current produces half a retarded wave and half an advanced wave. The concept of half retarded and half advanced is involved in both Wheeler and Feynman's absorber theory [7, 8] and Dirac's self-force theory [9]. (17) is used to calculate the energy flow of a photon, and (18) is used to calculate the classical electromagnetic field radiation energy. In Maxwell's electromagnetic theory, if one calculates the radiation of an antenna, the antenna is placed in empty space and the self-energy flow radiation of the antenna is calculated. In the author's electromagnetic theory, to calculate the radiation of an antenna one must arrange absorbers all around the antenna and calculate the mutual energy flow formed by the transmitting antenna together with the absorbers.

Note that both mutual energy flow formulas (17) and (18) are correct. The two formulas are used in different situations. Formula (17) is used in the case where there is known to be one transmitting antenna and one receiving antenna, to calculate the mutual energy flow from the transmitting antenna to the receiving antenna. Formula (18) is used in another situation, namely to calculate the radiation power of the transmitting antenna, but in order to use the mutual energy flow rather than the self-energy flow (Poynting vector) to calculate the radiation power of the transmitting antenna, we arrange absorber currents on a spherical surface of infinite radius, and the absorber current depends on the far field of the transmitting antenna.

2.4 Green's Function Method

Another way to obtain the mutual energy flow theorem is to use the Green's function method.

2.4.1 First Vector Green's Function

We know that,

$$\nabla \cdot (\boldsymbol{F}_1 \times \boldsymbol{F'}_2) = \nabla \times \boldsymbol{F}_1 \cdot \boldsymbol{F'}_2 - \boldsymbol{F}_1 \nabla \times \boldsymbol{F'}_2 \tag{21}$$

Let

$$\boldsymbol{F'}_2 = \nabla \times \boldsymbol{F}_2 \tag{22}$$

Substituting into the above equation gives,

$$\nabla \cdot (\boldsymbol{F}_1 \times \nabla \times \boldsymbol{F}_2) = \nabla \times \boldsymbol{F}_1 \cdot \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_1 \cdot \nabla \times \nabla \times \boldsymbol{F}_2 \tag{23}$$

The above equation can be the first Green's function for vector functions. Similarly,

$$\nabla \cdot (\boldsymbol{F}_2 \times \nabla \times \boldsymbol{F}_1) = \nabla \times \boldsymbol{F}_2 \cdot \nabla \times \boldsymbol{F}_1 - \boldsymbol{F}_2 \cdot \nabla \times \nabla \times \boldsymbol{F}_1 \tag{24}$$

2.4.2 Second Vector Green's Function

Therefore

$$\nabla \cdot (\boldsymbol{F}_1 \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \times \nabla \times \boldsymbol{F}_1) = -(\boldsymbol{F}_1 \cdot \nabla \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \cdot \nabla \times \nabla \times \boldsymbol{F}_1) \tag{25}$$

The above equation is the second Green's function for vector functions. Performing a volume integral on the above equation gives

$$\begin{aligned} &\int_V (\boldsymbol{F}_1 \cdot \nabla \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \cdot \nabla \times \nabla \times \boldsymbol{F}_1) dV \\ &= -\oiint_\Gamma (\boldsymbol{F}_1 \times \nabla \times \boldsymbol{F}_2 - \boldsymbol{F}_2 \times \nabla \times \boldsymbol{F}_1) \cdot \hat{n} d\Gamma \end{aligned} \tag{26}$$

2.4.3 Frequency-Domain Maxwell's Equations

Consider the frequency-domain Maxwell's equations

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + j\omega\epsilon_0 \boldsymbol{E} \end{cases} \tag{27}$$

Here we have taken into account that the electric field, magnetic field, and current all have a time factor $\exp(j\omega t)$.

2.4.4 Lorentz Reciprocity Theorem

Substituting $\boldsymbol{F}_1 = \boldsymbol{E}_1$, $\boldsymbol{F}_2 = \boldsymbol{E}_2$ into (26) gives,

$$\begin{aligned} &\int_V (\boldsymbol{E}_1 \cdot \nabla \times \nabla \times \boldsymbol{E}_2 - \boldsymbol{E}_2 \cdot \nabla \times \nabla \times \boldsymbol{E}_1) dV \\ &= -\oiint_\Gamma (\boldsymbol{E}_1 \times \nabla \times \boldsymbol{E}_2 - \boldsymbol{E}_2 \times \nabla \times \boldsymbol{E}_1) \cdot \hat{n} d\Gamma \end{aligned} \tag{28}$$

Substituting Maxwell's equations (27),

$$\begin{aligned} &\int_V (\boldsymbol{E}_1 \cdot \nabla \times (-j\omega\mu_0 \boldsymbol{H}_2) - \boldsymbol{E}_2 \cdot \nabla \times (-j\omega\mu_0 \boldsymbol{H}_1)) dV \\ &= -\oiint_\Gamma (\boldsymbol{E}_1 \times (-j\omega\mu_0 \boldsymbol{H}_2) - \boldsymbol{E}_2 \times (-j\omega\mu_0 \boldsymbol{H}_1)) \cdot \hat{n} d\Gamma \end{aligned} \tag{29}$$

or

$$\int_V (\boldsymbol{E}_1 \cdot \nabla \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \cdot \nabla \times \boldsymbol{H}_1) dV$$

$$= -\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{30}$$

Substituting Ampère's circuital law from Maxwell's equations (27),

$$\int_V (\boldsymbol{E}_1 \cdot (\boldsymbol{J}_2 + j\omega\epsilon_0 \boldsymbol{E}_2) - \boldsymbol{E}_2 \cdot (\boldsymbol{J}_1 + j\omega\epsilon_0 \boldsymbol{E}_1)) dV$$

$$= -\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{31}$$

or

$$\int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 - \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV = -\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{32}$$

The above equation is the Lorentz reciprocity theorem [5,44-46,43].

2.4.5 Mutual Energy Theorem

Consider $\boldsymbol{F}_1 = \boldsymbol{E}_1$, $\boldsymbol{F}_2 = \boldsymbol{E}_2^*$, where * is complex conjugate

$$\int_V (\boldsymbol{E}_1 \cdot \nabla \times \nabla \times \boldsymbol{E}_2^* - \boldsymbol{E}_2^* \cdot \nabla \times \nabla \times \boldsymbol{E}_1) dV$$

$$= -\oiint_\Gamma (\boldsymbol{E}_1 \times \nabla \times \boldsymbol{E}_2^* - \boldsymbol{E}_2 \times \nabla \times \boldsymbol{E}_1) \cdot \hat{n} d\Gamma \tag{33}$$

Substituting Maxwell's equations,

$$\int_V (\boldsymbol{E}_1 \cdot \nabla \times (-j\omega\mu_0 \boldsymbol{H}_2)^* - \boldsymbol{E}_2^* \cdot \nabla \times (-j\omega\mu_0 \boldsymbol{H}_1)) dV$$

$$= -\oiint_\Gamma (\boldsymbol{E}_1 \times (-j\omega\mu_0 \boldsymbol{H}_2)^* - \boldsymbol{E}_2 \times (-j\omega\mu_0 \boldsymbol{H}_1)) \cdot \hat{n} d\Gamma \tag{34}$$

Hence

$$\int_V (\boldsymbol{E}_1 \cdot \nabla \times \boldsymbol{H}_2{}^* + \boldsymbol{E}_2^* \cdot \nabla \times \boldsymbol{H}_1) dV$$

$$= -\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2{}^* + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{35}$$

Hence

$$\int_V (\boldsymbol{E}_1 \cdot (\boldsymbol{J}_2 + j\omega\epsilon_0 \boldsymbol{E}_2)^* + \boldsymbol{E}_2^* \cdot (\boldsymbol{J}_1 + j\omega\epsilon_0 \boldsymbol{E}_1) dV$$

$$= -\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2{}^* + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{36}$$

Hence

$$\int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* + \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = -\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2{}^* + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{37}$$

The above equation is the mutual energy theorem [1-3]. Of course, it is also a kind of reciprocity theorem [42,43]. Since it is obtained from Green's functions, it can also be regarded as an electromagnetic field equation obtained by the Green's function method.

2.5 Conjugate Transformation

The conjugate transformation is

$$\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J} \rightarrow \boldsymbol{E}^*, -\boldsymbol{H}^*, -\boldsymbol{J}^* \tag{38}$$

The conjugate transformation has the following theorem: in a frequency-domain electromagnetic field equation, performing the above conjugate transformation, if a quantity $\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J}$ satisfied Maxwell's equations before the transformation, it still satisfies Maxwell's equations after the transformation. More specifically, if it was a retarded wave before the transformation, it becomes an advanced wave after the transformation, and vice versa.

Applying the conjugate transformation to the quantities with subscript 2, $\boldsymbol{E}_2, \boldsymbol{H}_2, \boldsymbol{J}_2$, in the mutual energy flow theorem (37) gives the reciprocity theorem (32); conversely, applying the conjugate transformation to the quantities with subscript 2, $\boldsymbol{E}_2, \boldsymbol{H}_2, \boldsymbol{J}_2$, in the reciprocity theorem (32) gives the mutual energy theorem (37). We know that generally in the reciprocity theorem (32) both quantities $\boldsymbol{E}_1, \boldsymbol{H}_1, \boldsymbol{J}_1$ and $\boldsymbol{E}_2, \boldsymbol{H}_2, \boldsymbol{J}_2$ are retarded waves, whereas in the mutual energy theorem $\boldsymbol{E}_1, \boldsymbol{H}_1, \boldsymbol{J}_1$ remain retarded waves, but $\boldsymbol{E}_2, \boldsymbol{H}_2, \boldsymbol{J}_2$ are advanced waves.

2.6 The Mutual Energy Flow and Self-Energy Flow Paradox

We know Poynting's theorem (9). According to traditional electromagnetic theory, energy is transmitted by the Poynting vector, i.e.,

$$Q_{ii} = \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma, \quad i = 1,2 \tag{39}$$

Q_{ii} can transmit energy. In the frequency domain the above equation becomes

$$Q_{ii} = \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i^*) \cdot \hat{n} d\Gamma = \text{real} \quad i = 1,2 \tag{40}$$

But according to the mutual energy flow theory (17), the mutual energy flow

$$Q_{12} = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{41}$$

also transmits energy. Thus both the mutual energy flow and the self-energy flow transmit energy. In traditional electromagnetic field theory, the self-energy flow has already transmitted sufficient energy; if the energy transmitted by the mutual energy flow is added, the transmitted energy would exceed expectations. Therefore, it is impossible for both the mutual energy flow and the self-energy flow to transmit energy.

However, as long as the self-energy flow transmits energy, from the superposition principle we can obtain that the mutual energy flow transmits energy, see Section 2.2, thus the mutual energy flow transmits energy. This constitutes the paradox that both the mutual energy flow and the self-energy flow transmit energy. To eliminate the paradox, the self-energy flow cannot transmit energy. That is, traditional electromagnetic theory is wrong on this point: the self-energy flow must not transmit energy.

2.7 Radiation Does Not Overflow the Universe

The author summarizes the fact that the self-energy flow does not transmit energy as the law that radiation should not overflow the universe, i.e., the electromagnetic field satisfies the following axiom:

$$\text{Re} \oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{n} d\Gamma = 0 \tag{42}$$

Γ is a spherical surface of infinite radius. Re is taking the real part. The above equation means that,

$$\text{Re} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_1^*) \cdot \hat{n} d\Gamma = 0, \quad \text{Re} \oiint_{\Gamma} (\boldsymbol{E}_2 \times \boldsymbol{H}_2^*) \cdot \hat{n} d\Gamma = 0 \tag{43}$$

and

$$\text{Re} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = 0 \tag{44}$$

Γ is a spherical surface of radius $R = \infty$. It is worth mentioning that equation (43) cannot be satisfied within the framework of Maxwell's electromagnetic theory; only the author's electromagnetic field theory requires formula (43) to be satisfied.

2.8 The Bug in Electromagnetic Theory Related to Maxwell's Equations

The author has already proved that Maxwell's electromagnetic theory has a bug. To correct this bug, Maxwell's equations should be written as

$$\begin{cases} \nabla \cdot \boldsymbol{E}_M = \rho/\epsilon_0 \\ \nabla \cdot \boldsymbol{B}_M = 0 \\ \nabla \times \boldsymbol{E}_M = -\dfrac{\partial}{\partial t}\mu_0 \boldsymbol{H}_M \\ \nabla \times \boldsymbol{H}_M = \boldsymbol{J} + \dfrac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}_M \end{cases} \tag{45}$$

$\boldsymbol{E}_M$ is the electric field defined by Maxwell, and $\boldsymbol{H}_M$ is the magnetic field defined by Maxwell. The author does not consider $\boldsymbol{E}_M$ to be the correct electric field, nor $\boldsymbol{H}_M$ to be the correct magnetic field, so a subscript M is added to these electric and magnetic fields. But under Maxwell's equations

$$\begin{cases} \boldsymbol{E}_{MI} \triangleq -\dfrac{\partial}{\partial t}\boldsymbol{A}^{(r)} \\ \boldsymbol{E}_{MS} \triangleq -\nabla \phi^{(r)} \\ \boldsymbol{B}_M \triangleq \nabla \times \boldsymbol{A}^{(r)} \end{cases} \tag{46}$$

The superscript (r) denotes retarded. $\boldsymbol{E}_{MI}$ is the induced electric field defined according to Maxwell's electromagnetic theory. $\boldsymbol{E}_{MS}$ is the static electric field defined according to Maxwell's electromagnetic theory. $\boldsymbol{B}_M$ is the magnetic field defined according to Maxwell's electromagnetic theory. ≜ means definition. The retarded potentials are,

$$\begin{cases} \boldsymbol{A}^{(r)} = \dfrac{\mu_0}{4\pi}\displaystyle\int_V \frac{\boldsymbol{J}(\boldsymbol{x}\prime, t - r/c)}{r} dV \\ \phi^{(r)} = \dfrac{1}{4\pi\epsilon_0}\displaystyle\int_V \frac{\rho(\boldsymbol{x}\prime, t - r/c)}{r} dV \end{cases} \tag{48}$$

$$r = ||\boldsymbol{x} - \boldsymbol{x}'|| \tag{49}$$

c is the speed of light. Similarly, besides the retarded potentials, there are also advanced potentials,

$$\begin{cases} \boldsymbol{A}^{(a)} = \dfrac{\mu_0}{4\pi}\displaystyle\int_V \frac{\boldsymbol{J}(\boldsymbol{x}\prime, t + r/c)}{r} dV \\ \phi^{(a)} = \dfrac{1}{4\pi\epsilon_0}\displaystyle\int_V \frac{\rho(\boldsymbol{x}\prime, t + r/c)}{r} dV \end{cases} \tag{50}$$

Note that although the author does not consider $\boldsymbol{E}_{MI}$ to be the induced electric field, does not consider E_{MS} to be the static electric field, and does not consider $\boldsymbol{B}_M$ to be the magnetic field, the author still acknowledges that Maxwell's equations (45) are correct equations, only that the variables therein are not the electric and magnetic fields.

It is worth noting that the author is actually not the originator of this approach. Contemporaries of Maxwell, mainly Kirchhoff and Lorenz, did not regard

$$-\nabla \phi^{(r)} - \frac{\partial}{\partial t}\boldsymbol{A}^{(r)} \tag{51}$$

as the electric field, nor did they regard

$$\nabla \times \boldsymbol{A}^{(r)} \tag{52}$$

as the magnetic field. Kirchhoff's electromagnetic theory predates Maxwell's, completed in 1857, and he obtained the telegrapher's equation. Lorenz's theory postdates Maxwell's, completed in 1866, but his theory went further than Maxwell's electromagnetic theory and obtained general solutions for the vector potential and scalar potential [47,48]. They created an electromagnetic theory completely equivalent to Maxwell's electromagnetic theory. When Kirchhoff encountered Ohm's law, he wrote

$$\boldsymbol{J} = \sigma(-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}) \tag{53}$$

rather than

$$\boldsymbol{J} = \sigma\boldsymbol{E}, \quad \boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{54}$$

When encountering Ampère's circuital theorem, he wrote

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0 \boldsymbol{J} \tag{55}$$

rather than

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}, \quad \boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{56}$$

The reader may be curious that formulas (53) and (54) seem to have no difference, and formulas (55) and (56) also seem to have no difference. In fact, they have a principled difference. Formula (54) requires you to know that $-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}$ must be the electric field, and (56) also requires you to know that $\nabla \times \boldsymbol{A}$ is the magnetic field. But formula (53) does not require $-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}$ to be the electric field, and (55) does not require $\nabla \times \boldsymbol{A}$ to be the magnetic field!

2.9 Electromagnetic Field Theory Under Quasi-Static Conditions

The electromagnetic field theory under quasi-static conditions is

$$\begin{cases} \nabla \cdot \boldsymbol{E}_S = \rho/\epsilon_0 \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\mu_0 \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}_S \end{cases} \tag{57}$$

where $\boldsymbol{E} = \boldsymbol{E}_I + \boldsymbol{E}_S$, $\boldsymbol{B} = \mu_0 \boldsymbol{H}$. The above equations can be solved to give,

$$\begin{cases} \boldsymbol{E}_I \triangleq -\frac{\partial}{\partial t}\boldsymbol{A} \\ \boldsymbol{E}_S \triangleq -\nabla\phi \\ \boldsymbol{B} \triangleq \nabla \times \boldsymbol{A} \end{cases} \tag{58}$$

and

$$\begin{cases} \boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r} dV \\ \phi = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{r} dV \end{cases} \tag{59}$$

In the quasi-static equations we have not added a subscript *M* to the electric and magnetic fields, because the author considers that the electric and magnetic fields obtained from the quasi-static equations are still the correct electric and magnetic fields. Quasi-static

electromagnetic theory is an electromagnetic theory based on Ampère's three null experiments, and this theory is correct within its own scope of application.

2.10 Electromagnetic Fields Under Quasi-Static Conditions and Under Radiation Conditions

The author believes that $\boldsymbol{E}_{MI}$, $\boldsymbol{E}_{MS}$, $\boldsymbol{B}_{M}$ under radiation conditions are not necessarily the same concepts as $\boldsymbol{E}_{I}$, $\boldsymbol{E}_{S}$, $\boldsymbol{B}$ under quasi-static conditions. To verify this, we must be clear about what quasi-static means. Quasi-static means satisfying the condition

$$r = |\boldsymbol{x} - \boldsymbol{x}'| \ll \lambda \tag{60}$$

$\boldsymbol{x}'$ is the position of the source current $\boldsymbol{J}(\boldsymbol{x}')$, and $\boldsymbol{x}$ is the position of the field point $\boldsymbol{E}(x)$. The above equation means that the distance between the field point and the source point being much smaller than the wavelength means the quasi-static condition is satisfied. The above condition means

$$kr \to 0 \tag{61}$$

where

$$k = \frac{\omega}{c} \tag{62}$$

or

$$\frac{2\pi}{\lambda} r \to 0 \tag{63}$$

or

$$\frac{2\pi}{\lambda} r \ll 1 \tag{64}$$

or

$$r \ll \frac{\lambda}{2\pi} < \lambda \tag{65}$$

Consider

$$\boldsymbol{J}(\boldsymbol{x}', t) = \boldsymbol{J}_0(\boldsymbol{x}')\exp(j\omega t) \tag{66}$$

$$\boldsymbol{J}(\boldsymbol{x}', t - r/c) = \boldsymbol{J}_0(\boldsymbol{x}')\exp(j\omega(t - r/c))$$

$$= \boldsymbol{J}_0(\boldsymbol{x}')\exp(j\omega t)\exp(-j\frac{\omega}{c}r))$$

$$= \boldsymbol{J}(\boldsymbol{x}', t)\exp(-jkr) \tag{67}$$

So the retarded potential (48)

$$\boldsymbol{A}^{(r)} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{x}\prime,t)\exp(-jkr)}{r} dV \tag{68}$$

The author believes that $\boldsymbol{E}_{MI}$, $\boldsymbol{E}_{MS}$, $\boldsymbol{B}_{M}$ are not necessarily the same concepts as $\boldsymbol{E}_{I}$, $\boldsymbol{E}_{S}$, $\boldsymbol{B}$. If they were the same concept, then at least we should have

$$\lim_{kr\to 0} \boldsymbol{E}_{MI} = \boldsymbol{E}_{I}, \quad \lim_{kr\to 0} \boldsymbol{E}_{MS} = \boldsymbol{E}_{S}, \quad \lim_{kr\to 0} \boldsymbol{B}_{M} = \boldsymbol{B} \tag{69}$$

Let us verify (69) below,

$$\lim_{kr\to 0} \boldsymbol{E}_{MI}^{(r)} = \lim_{kr\to 0} (-\frac{\partial}{\partial t}\boldsymbol{A}^{(r)})$$

$$= -j\omega \lim_{kr\to 0} (\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{x}\prime,t)\exp(-jkr)}{r} dV)$$

$$= -j\omega (\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{x}\prime,t)}{r} dV)$$

$$= -\frac{\partial}{\partial t} (\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{x}\prime,t)}{r} dV)$$

$$= -\frac{\partial}{\partial t}\boldsymbol{A} = \boldsymbol{E}_I \tag{70}$$

The superscript (r) denotes retarded, and $\boldsymbol{A}^{(r)}$ is the retarded vector potential. Therefore,

$$\lim_{kr\to 0} \boldsymbol{E}_{MI}^{(r)} = \boldsymbol{E}_I \tag{71}$$

So we can regard $\boldsymbol{E}_{MI}$ as the induced electric field, i.e., consider the following equation

$$\boldsymbol{E}_{MI} = \boldsymbol{E}_I \tag{72}$$

to hold. Consider

$$\lim_{kr\to 0} \boldsymbol{B}_M^{(r)} = \lim_{kr\to 0} \nabla \times \boldsymbol{A}^{(r)}$$

$$= \lim_{kr\to 0} \nabla \times (\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{x}\prime,t)\exp(-jkr)}{r} dV)$$

$$= \lim_{kr\to 0} (\frac{\mu_0}{4\pi}\int_V \nabla \frac{\exp(-jkr)}{r} \times \boldsymbol{J}(\boldsymbol{x}', t) dV)$$

$$= \lim_{kr\to 0} (\frac{\mu_0}{4\pi}\int_V (-\frac{\boldsymbol{r}}{r^3} - jk\frac{\hat{r}}{r})\exp(-jkr) \times \boldsymbol{J}(\boldsymbol{x}', t) dV)$$

$$= \lim_{kr\to 0} (\frac{\mu_0}{4\pi}\int_V \boldsymbol{J}(\boldsymbol{x}', t) \times (\frac{\boldsymbol{r}}{r^3} + jk\frac{\hat{r}}{r})\exp(-jkr) dV)$$

$$= \frac{\mu_0}{4\pi}\int_V \boldsymbol{J}(\boldsymbol{x}', t) \times (\frac{\boldsymbol{r}}{r^3} + jk\frac{\hat{r}}{r}) dV$$

$$= \frac{\mu_0}{4\pi}\int_V \boldsymbol{J}(\boldsymbol{x}', t) \times \frac{\boldsymbol{r}}{r^3} + jk\frac{\mu_0}{4\pi}\int_V \frac{\hat{r}}{r} dV$$

$$= \boldsymbol{B} + jk\frac{\mu_0}{4\pi}\int_V \frac{\hat{r}}{r} dV \tag{73}$$

We find that

$$\lim_{kr\to 0} \boldsymbol{B}_M^{(r)} \neq \boldsymbol{B} \tag{74}$$

So we cannot regard $\boldsymbol{B}_M^{(r)}$ as the magnetic field $\boldsymbol{B}$.

$$\lim_{kr\to 0}\boldsymbol{E}_{MS}^{(r)} = \lim_{kr\to 0}(-\nabla\phi^{(r)})$$

$$= \lim_{kr\to 0}(-\nabla(\frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\boldsymbol{x}\prime,t)\exp(-jkr)}{r}dV))$$

$$= \lim_{kr\to 0}(-(\frac{1}{4\pi\epsilon_0}\int_V \rho(\boldsymbol{x}',t)\nabla\frac{\exp(-jkr)}{r}dV))$$

$$= \lim_{kr\to 0}(-(\frac{1}{4\pi\epsilon_0}\int_V \rho(\boldsymbol{x}',t)(-\frac{\boldsymbol{r}}{r^3}-jk\frac{\hat{r}}{r})\exp(-jkr)dV))$$

$$= \frac{1}{4\pi\epsilon_0}\int_V \rho(\boldsymbol{x}',t)(\frac{\boldsymbol{r}}{r^3}+jk\frac{\hat{r}}{r})dV$$

$$= \frac{1}{4\pi\epsilon_0}\int_V \rho(\boldsymbol{x}',t)\frac{\boldsymbol{r}}{r^3}dV + jk\frac{1}{4\pi\epsilon_0}\int_V \frac{\hat{r}}{r}dV$$

$$= \boldsymbol{E}_S + jk\frac{1}{4\pi\epsilon_0}\int_V \frac{\hat{r}}{r}dV \tag{75}$$

Therefore

$$\lim_{kr\to 0}\boldsymbol{E}_{MS}^{(r)} \neq \boldsymbol{E}_S \tag{76}$$

Thus we also cannot regard $\boldsymbol{E}_{MS}^{(r)}$ as the same static electric field as $\boldsymbol{E}_S$! Consider the far-field component of the magnetic field

$$\boldsymbol{B}_{Mf}^{(r)} = jk\frac{\mu_0}{4\pi}\int_V \frac{\hat{r}}{r}\exp(-jkr)dV \tag{77}$$

The subscript *f* denotes the far field, and the subscript *n* denotes the near field. Consider

$$-j\boldsymbol{B}_{Mf}^{(r)} = k\frac{\mu_0}{4\pi}\int_V \frac{\hat{r}}{r}\exp(-jkr)dV \tag{78}$$

The author takes the right-hand side of the above equation as the corrected correct magnetic field $\boldsymbol{B}_f^{(r)}$, i.e.,

$$\boldsymbol{B}_f^{(r)} = -j\boldsymbol{B}_{Mf}^{(r)} \tag{79}$$

$$\boldsymbol{E}_{SMf}^{(r)} = jk\frac{1}{4\pi\epsilon_0}\int_V \frac{\hat{r}}{r}\exp(-jkr)dV \tag{80}$$

Consider

$$-j\boldsymbol{E}_{SMf}^{(r)} = k\frac{1}{4\pi\epsilon_0}\int_V \frac{\hat{r}}{r}\exp(-jkr)dV \tag{81}$$

The author takes the right-hand side of the above equation as the corrected correct static electric field $\boldsymbol{E}_{Sf}^{(r)}$, i.e.,

$$\boldsymbol{E}_{Sf}^{(r)} = -j\boldsymbol{E}_{SMf}^{(r)} \tag{82}$$

Finally we obtain,

$$\boxed{\boldsymbol{E}_{MI} = \boldsymbol{E}_I} \tag{83}$$

$$\boxed{\boldsymbol{B}_n^{(r)} = \boldsymbol{B}_{Mn}^{(r)}, \quad \boldsymbol{B}_f^{(r)} = -j\boldsymbol{B}_{Mf}^{(r)}} \tag{84}$$

For advanced waves, similarly,

$$\boxed{\boldsymbol{B}_n^{(a)} = \boldsymbol{B}_{Mn}^{(a)}, \quad \boldsymbol{B}_f^{(a)} = j\boldsymbol{B}_{Mf}^{(a)}} \tag{85}$$

The auther will not give the derivation. For the static electric field,

$$\boxed{\boldsymbol{E}_{Sn}^{(r)} = \boldsymbol{E}_{SMn}^{(r)}, \quad \boldsymbol{E}_{Sf}^{(r)} = -j\boldsymbol{E}_{SMf}^{(r)}} \tag{86}$$

Similarly,

$$\boxed{\boldsymbol{E}_{Sn}^{(a)} = \boldsymbol{E}_{SMn}^{(a)}, \quad \boldsymbol{E}_{Sf}^{(a)} = j\boldsymbol{E}_{SMf}^{(r)}} \tag{87}$$

In (83–87), the quantities with subscript *M* are the fields defined by Maxwell's equations, and the quantities without subscript are the corrected correct electromagnetic fields of the author.

The phase of the corrected magnetic field $\boldsymbol{B}_f^{(r)}$, $\boldsymbol{B}_f^{(a)}$ remains consistent with $\boldsymbol{B}_n^{(r)}, \boldsymbol{B}_n^{(a)}$. Although after this correction we still cannot guarantee that

$$\lim_{kr\to 0} \boldsymbol{B}^{(r)} = \lim_{kr\to 0} \boldsymbol{B}^{(a)} = \boldsymbol{B} \tag{88}$$

$$\lim_{kr\to 0} \boldsymbol{E}_S^{(r)} = \lim_{kr\to 0} \boldsymbol{E}_S^{(a)} = \boldsymbol{E}_S \tag{89}$$

at least we can guarantee that

$$\lim_{kr\to 0} \text{phase}(\boldsymbol{B}^{(r)}) = \lim_{kr\to 0} \text{phase}(\boldsymbol{B}^{(a)}) = \text{phase}(\boldsymbol{B}) \tag{90}$$

$$\lim_{kr\to 0} \text{phase}(\boldsymbol{E}_S^{(r)}) = \lim_{kr\to 0} \text{phase}(\boldsymbol{E}_S^{(a)}) = \text{phase}(\boldsymbol{E}_S) \tag{91}$$

phase(·) means taking the phase.

2.11 The Author's Theory of Electromagnetic Wave Radiation

1. The author believes that the source randomly produces retarded waves, and the sink randomly produces advanced waves. These electromagnetic waves are spherical waves. Such waves decay with a $\frac{1}{r}$ factor. We say that both the source and the sink are in broadcast mode. The retarded and advanced waves emitted by the source and the sink are all reactive power waves; the electric and magnetic fields of such waves maintain a 90-degree phase difference, so the average radiation power is zero. Therefore, radiating such electromagnetic waves does not lose energy. The source and the sink can continuously and randomly emit such waves.

2. If the retarded wave emitted by the source reaches the sink and the sink happens to emit an advanced wave at that moment, we say the retarded and advanced waves are synchronized. At this time the retarded and advanced waves interfere, and this interference causes the electromagnetic field on the line connecting the sink and the source to continuously strengthen. In other directions the amplitude of the electromagnetic field continuously decreases. Eventually both the retarded and advanced waves become quasi-plane waves on the line connecting the source and the sink. A quasi-plane wave is like the wave emitted by a laser: it is a beam of light waves, but it is also a plane wave. The retarded wave emitted by the source and the advanced wave emitted by the sink form the mutual energy flow. The mutual energy flow transmits energy. Therefore, this mutual energy flow is actually the energy flow of a photon. The author calls this mode the photon mode. In this case the mutual energy flow transmits energy, and the corresponding self-energy flow, because the electric and magnetic fields maintain a 90-degree phase difference, is reactive power, so on average the self-energy flow does not transmit energy. Therefore all energy is transmitted by the mutual energy flow. The mutual energy flow is active power. The mutual energy flow is narrow at the source and the sink, like a particle; the mutual energy flow is thick between the source and the sink, like a wave. Therefore the mutual energy flow can well explain the duality of wave and particle.
3. The total electromagnetic field formed by the superposition of the electromagnetic fields of a large number of particles should approximately satisfy Maxwell's equations. The particle mode has been described in detail above. That the electromagnetic wave formed by a large number of photons approximately satisfies Maxwell's equations needs to be demonstrated. We give a demonstration here.

We first assume that if sinks are arranged on a spherical surface Γ_{sink} of radius $R = \infty$, the electromagnetic fields $\boldsymbol{E}_2, \boldsymbol{H}_2$ produced by these sinks satisfy

$$|E_2| = |E_1|, \quad |H_2| = |H_1| \tag{92}$$

Therefore,

$$\boldsymbol{H}_2 = \boldsymbol{H}_{1M}, \quad \boldsymbol{H}_1 = \boldsymbol{H}_{2M} \tag{93}$$

We know the mutual energy flow theorem. The scenario in which this mutual energy flow theorem is applied is shown in Fig. 1,

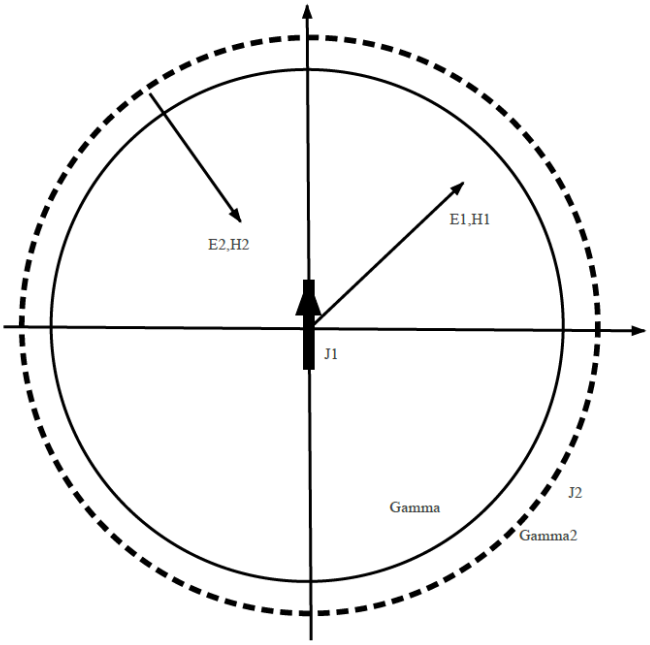


Figure 1: Calculating the Electromagnetic radiation of current $\boldsymbol{J}_1$ according to the author's electromagnetic theory. Absorber material needs to be arranged on a spherical surface of infinite radius. The absorber material is arranged on the spherical surface Γ_2 and produces an induced current $\boldsymbol{J}_2 \sim \boldsymbol{E}_1$ ($r = R$). R is the radius of the spherical surface Γ_2.

$$-\int_{V_1}(\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=\frac{1}{2}\oiint_\Gamma(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=\int_{V_2}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*)dV \tag{94}$$

Thus the mutual energy flow density is

$$\begin{aligned}\boldsymbol{S}_m&=\frac{1}{2}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\\&=\frac{1}{2}(E_1H_2^*+E_2^*H_1)\hat{R}\\&=\frac{1}{2}(E_1H_{1M}^*+E_2^*H_{2M})\hat{R}\\&=\mathrm{Re}(E_1H_{1M}^*)\hat{R}\\&=\mathrm{Re}\boldsymbol{E}_1\times\boldsymbol{H}_{1M}\\&=\mathrm{Re}\boldsymbol{S}_{1\ M}\end{aligned} \tag{95}$$

$$\boldsymbol{S}_m=\mathrm{Re}\boldsymbol{S}_{1\ M} \tag{96}$$

$\boldsymbol{S}_{1\ M}=\boldsymbol{E}_1\times\boldsymbol{H}_{1M}^*$ is the Poynting vector calculated from Maxwell's electromagnetic theory. From the left-hand side of (94) we obtain

$$-\int_{V_1}(\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=\oiint_\Gamma\boldsymbol{S}_{1\ M}\cdot\hat{n}d\Gamma \tag{97}$$

We know Poynting's theorem is,

$$-\int_{V_1}(\boldsymbol{E}_1\cdot\boldsymbol{J}_1)dV=\oiint_\Gamma(\boldsymbol{E}_1\times\boldsymbol{H}_{1M}+\boldsymbol{E}_1\cdot\frac{\partial}{\partial t}\boldsymbol{E}_1+\boldsymbol{H}_1\cdot\frac{\partial}{\partial t}\boldsymbol{H}_1)\cdot\hat{n}d\Gamma \tag{98}$$

Integrating the above equation over time,

$$-\int_{t=-\infty}^{\infty}dt\int_{V_1}(\boldsymbol{E}_1\cdot\boldsymbol{J}_1)dV=\int_{t=-\infty}^{\infty}dt\oiint_\Gamma(\boldsymbol{E}_1\times\boldsymbol{H}_{1M})\cdot\hat{n}d\Gamma \tag{99}$$

Applying a Fourier transform,

$$-\int_{V_1}(\boldsymbol{E}_1\cdot\boldsymbol{J}_1^*)dV=\oiint_\Gamma\boldsymbol{S}_{1\ M}\cdot\hat{n}d\Gamma \tag{100}$$

The above equation is the radiation formula obtained by calculating according to the Poynting theorem of Maxwell's electromagnetic theory. The radiation mechanism is shown in Fig. 2. Here we have not arranged any absorber on the spherical surface of infinite radius. This case assumes that the electromagnetic field can keep traveling in vacuum forever, and no absorber material is needed to absorb these electromagnetic waves. We say that if an aether exists, electromagnetic waves traveling in the aether can of course keep traveling forever. But today we have already negated the existence of the aether, yet we still let electromagnetic waves keep traveling in vacuum forever and transport energy flow out of our cosmic boundary. This is quite absurd! However, traditional electromagnetic theory has not noticed this absurdity. Nevertheless, the right-hand side of (100) does give the correct radiation of the electromagnetic wave.

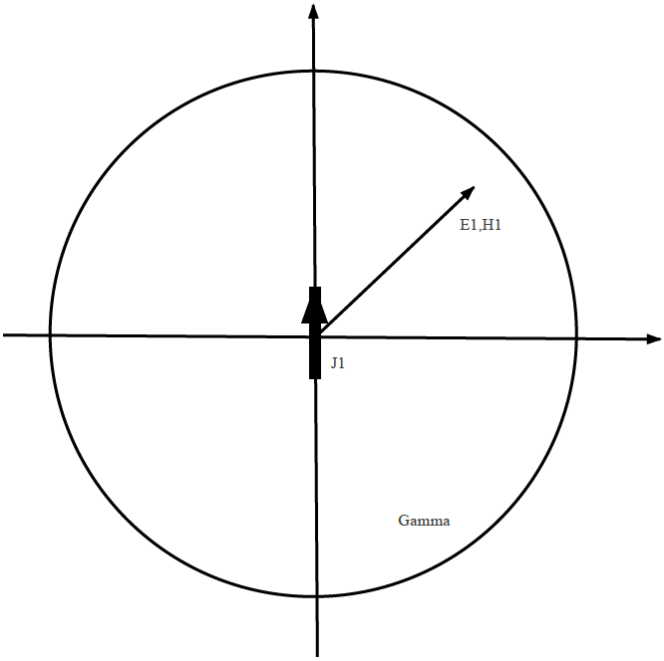


Figure 2: Calculating the electromagnetic radiation of current $\boldsymbol{J}_1$ according to Maxwell's electromagnetic theory. The electromagnetic radiation satisfies the Silver-Müller radiation condition. It is assumed that the electromagnetic wave can keep traveling in vacuum forever. Here no absorber material is needed to absorb the energy of these electromagnetic waves.

Comparing formulas (97) and (100), we find that the right-hand sides are the same. The left-hand sides of the two equations are different. The left-hand side is the radiation power of antenna $\boldsymbol{J}_1$. The right-hand side is the far-field electromagnetic radiation energy. From the perspective of far-field radiation power, the far-field radiation power according to the author's mutual energy flow theory and according to Poynting's theorem are the same. This is also the reason why Maxwell's electromagnetic theory can successfully calculate antenna radiation energy.

But the left-hand sides of the two theories are different. We know that

$$\boldsymbol{E}_1 \sim -j\boldsymbol{J}_1 \tag{101}$$

Therefore

$$\boldsymbol{E}_1 \cdot \boldsymbol{J}_1^* = \text{Imaginary and divergent} \tag{102}$$

According to Maxwell's electromagnetic theory, considering $\boldsymbol{J}_1$ to be a dipole $\delta(\boldsymbol{x})$, then

$$-\int_{V_1} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_1^*) dV = \text{Imaginary and divergent} \tag{103}$$

So the current output obtained by calculating according to Maxwell's electromagnetic theory is reactive power, which is obviously wrong! But

$$\boldsymbol{E}_2 \sim -j\boldsymbol{J}_2 \tag{104}$$

$$\boldsymbol{J}_2 \sim \boldsymbol{E}_1 = -j\boldsymbol{J}_1 \tag{105}$$

$$\boldsymbol{E}_2 \sim -j(-j\boldsymbol{J}_1) = -\boldsymbol{J}_1 \tag{106}$$

So

$$-\int_{V_1}(\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=\text{real and finite}>0 \tag{107}$$

This shows that according to the author's electromagnetic theory, the output power of $\boldsymbol{J}_1$ is real and greater than 0, which is consistent with the far-field radiation! This shows that if we only care about the far-field radiation, we can calculate it according to the Poynting vector in Maxwell's electromagnetic theory, or we can calculate the radiation power of many photons according to the author's electromagnetic theory; the two are the same. Although the two are the same, the radiation models are completely different. In Maxwell's electromagnetic theory, radiation is the self-radiation of the radiation source in vacuum, whereas in the author's electromagnetic theory, absorbers are added at the cosmic boundary, and radiation is the mutual energy flow radiation formed by the source together with the environmental absorbers. The physical concepts of the two are completely different.

The author's electromagnetic theory provides the correct current power output (107), while the current power output provided by Maxwell's electromagnetic theory (103) is divergent and imaginary, which is obviously wrong.

2.12 Dirac's Self-Force Theory

It is worth mentioning that Dirac, Wheeler, and Feynman all knew about this problem of Maxwell's electromagnetic theory. They knew that (103) would diverge, so the two sides of Poynting's theorem are not equal. Of course, what they discussed was the radiation of an accelerated electron. Dirac's self-force theory assumes that the electron radiates

$$\boldsymbol{E}_{1D}=\frac{1}{2}(\boldsymbol{E}_{1M}^{(r)}-\boldsymbol{E}_{1M}^{(a)}) \tag{108}$$

i.e., the field of the source $\boldsymbol{E}_{1D}$ (the subscript D indicates it is Dirac's), but this field equals the difference between half the retarded wave $\frac{1}{2}\boldsymbol{E}_{1M}^{(r)}$ and half the advanced wave $\frac{1}{2}(\boldsymbol{E}_{1M}^{(a)})$ the source [49]. The fields in the above equation all carry the subscript M, because Dirac discussed the problem within the framework of Maxwell's equations. The reason Dirac used the difference between the retarded and advanced waves is that they have the same divergent quantity; if subtracted, the divergent quantity is eliminated. This is the earliest so-called "renormalization" method for dealing with integral divergence to appear in classical electromagnetic theory.

But people are very puzzled as to why the retarded and advanced waves can be subtracted. Dirac adopted the following time-domain Poynting theorem,

$$-\int_{t=-\infty}^{\infty}dt\int_{V_1}(\boldsymbol{E}_{1D}\cdot\boldsymbol{J}_1^*)dV=\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}\boldsymbol{S}_{1\,M}\cdot\hat{n}d\Gamma \tag{109}$$

What people find puzzling is why the field on the left-hand side of the equation is $\boldsymbol{E}_{1D}$ while the field on the right-hand side is $\boldsymbol{E}_{1M}$ ($\boldsymbol{S}_{1\,M}=\boldsymbol{E}_{1M}\times\boldsymbol{H}_{1M}$).

2.13 Absorber Theory

In their absorber theory [7, 8], Wheeler and Feynman held that the above electric field is not emitted by the source, but by the sink, i.e.,

$$\boldsymbol{E}_{1D}=\frac{1}{2}(\boldsymbol{E}_{1M}^{(r)}-\boldsymbol{E}_{1M}^{(a)})\rightarrow\boldsymbol{E}_{2W} \tag{110}$$

The subscript W indicates it is Wheeler's. Therefore Wheeler and Feynman actually adopted the formula,

$$-\int_{t=-\infty}^{\infty}dt\int_{V_1}(\boldsymbol{E}_{2W}\cdot\boldsymbol{J}_{1M}^*)dV=\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}\boldsymbol{S}_{1\,M}\cdot\hat{n}d\Gamma \tag{111}$$

Wheeler and Feynman considered the radiation of $\boldsymbol{J}_1$ to be

$$\boldsymbol{E}_{1W}=\frac{1}{2}(\boldsymbol{E}_{1M}^{(r)}+\boldsymbol{E}_{1M}^{(a)}) \tag{112}$$

The field returned by the environment is (110). So the total electric field is

$$\boldsymbol{E}_{total}=\boldsymbol{E}_{1W}+\boldsymbol{E}_{2W} \tag{113}$$

$$\boldsymbol{E}_{total} = \frac{1}{2}(\boldsymbol{E}_{1M}^{(r)} + \boldsymbol{E}_{1M}^{(a)}) + \frac{1}{2}(\boldsymbol{E}_{1M}^{(r)} - \boldsymbol{E}_{1M}^{(a)}) = \boldsymbol{E}_{1M}^{(r)} \tag{114}$$

The total field is equivalent to the retarded electric field obtained in Maxwell's electromagnetic theory.

Dirac, Wheeler, and Feynman discovered the flaw in classical electromagnetic theory, and they patched the theory, but this patch is very hard to understand. Their specific operations, namely formulas (108, 109) and (110, 111), are considered by the author to be very valuable. Their idea of introducing advanced waves into electromagnetic theory is correct!

The formula (97) obtained by the author through the idea of mutual energy flow. In this formula, $\boldsymbol{E}_2$ is the electric field of the environmental absorbers on a spherical surface of infinite radius, and this electric field focuses from all directions onto the point $r = 0$. Focusing from all directions onto this point, the electric field

$$\boldsymbol{E}_2(r \to 0) \sim \frac{1}{r}\exp(-jkr) \sim \frac{1}{r} \tag{115}$$

Therefore the integral $-\int_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1)dV$ does not diverge, and is real. This completely avoids the divergence problem of Maxwell's electromagnetic theory. The author's solution supports Wheeler and Feynman's absorber theory. Likewise, Wheeler and Feynman's absorber theory is also an affirmation of the author's electromagnetic theory. It can be proved that

$$\boldsymbol{E}_2 = \boldsymbol{E}_W \tag{116}$$

2.14 Wave Collapse

In Maxwell's electromagnetic theory the mutual energy flow theorem still holds, namely (17). We rewrite the formula as,

$$-\int_{V_1} (\boldsymbol{E}_{M2}^* \cdot \boldsymbol{J}_1)dV = \oiint_\Gamma (\boldsymbol{E}_{M1} \times \boldsymbol{H}_{M2}^* + \boldsymbol{E}_{M2}^* \times \boldsymbol{H}_{M1}) \cdot \hat{n}d\Gamma = \int_{V_2} (\boldsymbol{E}_{M1} \cdot \boldsymbol{J}_2^*)dV \tag{117}$$

The reader should note that we have added the subscript *M* below the electric and magnetic fields to emphasize that this is the theorem obtained under Maxwell's electromagnetic theory.

Suppose there is a dipole transmitting antenna at $x = 0, y = 0, z = 0$, with dipole current

$$\boldsymbol{J}_1 = \boldsymbol{J}_{10}\exp(j\omega t)\delta(x-0)\delta(y-0)\delta(z-0)\Delta l\hat{z}, \tag{118}$$

Suppose there is a dipole receiving antenna at $x = L, y = 0, z = 0$, with dipole current

$$\boldsymbol{J}_2 = \boldsymbol{J}_{20}\exp(j\omega t)\delta(x-L)\delta(y-0)\delta(z-0)\Delta l\hat{z}, \tag{119}$$

We take Γ to be an infinite plane between the two dipoles, and this plane is

$$z = \frac{L}{2} \tag{120}$$

The formula (117) is correct. $-\int_{V_1} (\boldsymbol{E}_{M2}^* \cdot \boldsymbol{J}_1)dV$ is the work done by the transmitting antenna current $\boldsymbol{J}_1$ against the advanced wave fed back by the second dipole of the universe, $\int_{V_2} (\boldsymbol{E}_{M1} \cdot \boldsymbol{J}_2^*)dV$ is the work received by $\boldsymbol{J}_2$ from the electric field $\boldsymbol{E}_{M1}$, and $\oiint_\Gamma (\boldsymbol{E}_{M1} \times \boldsymbol{H}_{M2}^* + \boldsymbol{E}_{M2}^* \times \boldsymbol{H}_{M1}) \cdot \hat{n}d\Gamma$ is the energy carried by the energy flow passing through the surface Γ. These three are exactly equal. See Figure 3.

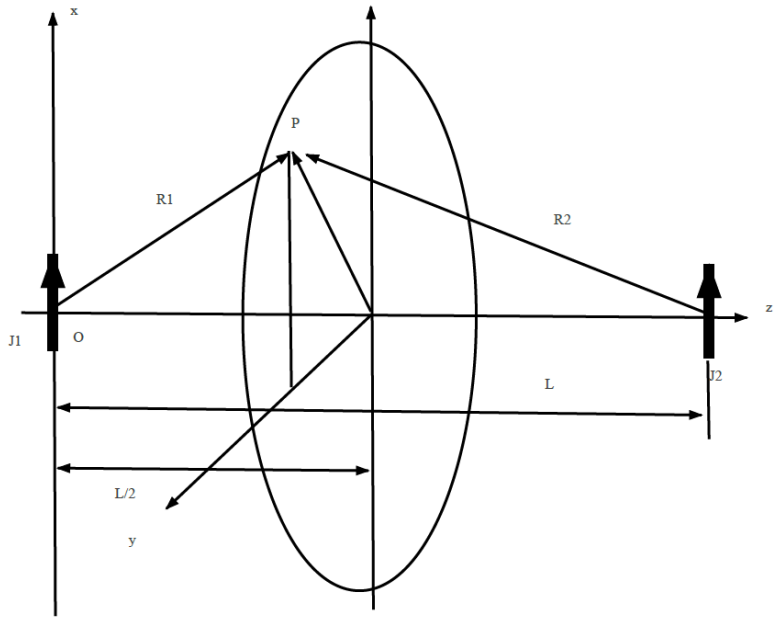


Figure 3: Calculating the mutual energy flow according to Maxwell's equations. Γ is an infinite plane, and the distance from Γ to both dipoles is *L*/2.

If both $-\int_{V_1}(\boldsymbol{E}_{M2}^{*}\cdot\boldsymbol{J}_1)dV$ and $\int_{V_2}(\boldsymbol{E}_{M1}\cdot\boldsymbol{J}_2^{*})dV$ are real, then $\oiint_{\Gamma}(\boldsymbol{E}_{M1}\times\boldsymbol{H}_{M2}^{*}+\boldsymbol{E}_{M2}^{*}\times\boldsymbol{H}_{M1})\cdot\hat{n}d\Gamma$ is also real. We can divide the mutual energy flow density into two parts, real and imaginary:

$$\mathrm{Im}(\boldsymbol{E}_{M1}\times\boldsymbol{H}_{M2}^{*}+\boldsymbol{E}_{M2}^{*}\times\boldsymbol{H}_{M1}) \tag{121}$$

$$\mathrm{Re}(\boldsymbol{E}_{M1}\times\boldsymbol{H}_{M2}^{*}+\boldsymbol{E}_{M2}^{*}\times\boldsymbol{H}_{M1}) \tag{122}$$

Since the path at the edge of the above surface integral is longer than at the center, the edge integral contributes a exp(-*ja*), where a is a constant. In actual specific calculation, the contribution of the entire surface integral is an imaginary number -*j*, see [50]. That is,

$$\oiint_{\Gamma}\cdot\hat{n}d\Gamma\sim -j \tag{123}$$

Therefore,

$$\oiint_{\Gamma}j\mathrm{Im}(\boldsymbol{E}_{M1}\times\boldsymbol{H}_{M2}^{*}+\boldsymbol{E}_{M2}^{*}\times\boldsymbol{H}_{M1})\cdot\hat{n}d\Gamma=\mathrm{real} \tag{124}$$

That is, the imaginary part of the mutual energy flow density contributes the real part of the mutual energy flow!

$$\begin{aligned}&\boldsymbol{E}_{M1}\times\boldsymbol{H}_{M2}^{*}+\boldsymbol{E}_{M2}^{*}\times\boldsymbol{H}_{M1}\\&=\boldsymbol{E}_{1}\times\boldsymbol{H}_{M2}^{*}+\boldsymbol{E}_{2}^{*}\times\boldsymbol{H}_{M1}\\&=\boldsymbol{E}_{1}\times(-j\boldsymbol{H}_{2})^{*}+\boldsymbol{E}_{2}^{*}\times(j\boldsymbol{H}_{1})\\&=j(\boldsymbol{E}_{1}\times\boldsymbol{H}_{2}{}^{*}+\boldsymbol{E}_{2}^{*}\times\boldsymbol{H}_{1})\end{aligned} \tag{125}$$

where the previously obtained correction formulas are taken into account: $\boldsymbol{H}_1=-j\boldsymbol{H}_{M1}$, $\boldsymbol{H}_2=j\boldsymbol{H}_{M2}$, so $\boldsymbol{H}_{M1}=j\boldsymbol{H}_1$, $\boldsymbol{H}_{M2}=-j\boldsymbol{H}_2$. The above equation can be written as

$$\boldsymbol{S}_{Mm} = j\boldsymbol{S}_m, \qquad \boldsymbol{S}_m = -j\boldsymbol{S}_{Mm} \tag{126}$$

$$\mathrm{Re}\boldsymbol{S}_m = -j(j\mathrm{Im}(\boldsymbol{S}_{Mm})) = \mathrm{Im}(\boldsymbol{S}_{Mm}) \tag{127}$$

From this we see that the imaginary part of the mutual energy flow calculated according to Maxwell's electromagnetic theory exactly corresponds to the real part calculated according to the author's electromagnetic theory.

$$\oiint_\Gamma \mathrm{Im}(\boldsymbol{E}_{M1} \times \boldsymbol{H}^*_{M2} + \boldsymbol{E}^*_{M2} \times \boldsymbol{H}_{M1}) \cdot \hat{n} d\Gamma \sim -j = \text{imaginary} \tag{128}$$

or

$$\oiint_\Gamma \mathrm{Re}(\boldsymbol{E}_1 \times \boldsymbol{H}^*_2 + \boldsymbol{E}^*_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \sim -j = \text{imaginary} \tag{129}$$

Thus, in the mutual energy flow density calculated according to the author's electromagnetic theory, the mutual energy flow corresponding to the real part is imaginary. Therefore, in the case where both $-\int_{V_1} (\boldsymbol{E}^*_2 \cdot \boldsymbol{J}_1) dV$ and $\int_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}^*_2) dV$ are real, the mutual energy flow $\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}^*_2 + \boldsymbol{E}^*_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$ calculated according to the author's electromagnetic theory is imaginary.

However, if we take into account that the retarded wave $\boldsymbol{E}_1, \boldsymbol{H}_1$ emitted by $\boldsymbol{J}_1$ and the advanced wave $\boldsymbol{E}_2, \boldsymbol{H}_2$ emitted by $\boldsymbol{J}_2$ will interfere. This interference will cause both the retarded and advanced waves to be strengthened on the line connecting the source and the sink, and weakened in other directions. Eventually both the retarded and advanced waves become quasi-plane waves. Here a quasi-plane wave means that they become a beam of plane waves directed from the source to the sink. In this way the aperture of the beam is finite, so it is as if the electromagnetic wave is transmitted through a small hole in a baffle. Therefore $\Gamma \to \Gamma_{hole}$. Thus the mutual energy flow becomes,

$$\oiint_{\Gamma_{hole}} \mathrm{Re}(\boldsymbol{E}_1 \times \boldsymbol{H}^*_2 + \boldsymbol{E}^*_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \text{real} \tag{130}$$

Since the aperture Γ_{hole} is finite, the contribution of the baffle edge is completely removed, and thus we have the above equation. We can also understand the above interference mechanism of retarded and advanced waves as wave collapse. Both the retarded and advanced waves collapse into quasi-plane waves. A quasi-plane wave means it is a beam of plane waves. After the wave collapses, the real part of the mutual energy flow density also contributes the real part of the mutual energy flow!

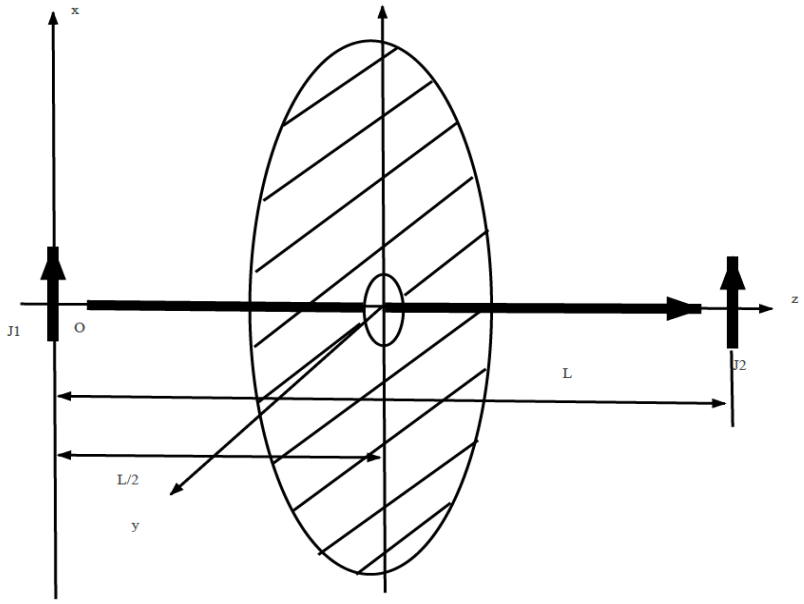


Figure 4: The mutual energy flow calculated according to the author's electromagnetic theory. Γ is an infinite plane, and Γ_{hole} is a small hole in Γ. In the mutual energy flow calculated by the author, because of wave collapse, the mutual energy flow collapses onto the line connecting the two dipoles, as if passing through a hole.

2.15 Example of Two Dipole Antennas: Calculating the Mutual Energy Flow After Collapse

Below we calculate the mutual energy flow density according to the electromagnetic theory provided by the author, taking into account wave collapse.

Suppose there is a dipole transmitting antenna at $x = 0, y = 0, z = 0$, with dipole current

$$\boldsymbol{J}_1 = \boldsymbol{J}_{10}\exp(j\omega t)\delta(x-0)\delta(y-0)\delta(z-0)\Delta l\hat{z}, \tag{131}$$

Suppose there is a dipole receiving antenna at $x = L, y = 0, z = 0$, with dipole current

$$\boldsymbol{J}_2 = \boldsymbol{J}_{20}\exp(j\omega t)\delta(x-L)\delta(y-0)\delta(z-0)\Delta l\hat{z}, \tag{132}$$

$\boldsymbol{J}_2$ is not arbitrary; it is influenced by the electric field of $\boldsymbol{J}_1$. The retarded potential produced by $\boldsymbol{J}_1$ is,

$$\boldsymbol{E}^{(r)} = -j\omega\boldsymbol{A}^{(r)}$$

$$= -j\omega\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}_1}{r}\exp(-jkr)dV$$

$$= -j\omega\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}_{10}\exp(j\omega t)\delta(x-0)\delta(y-0)\delta(z-0)\Delta l\hat{z}}{r}\exp(-jkr)dV$$

$$= -j\omega\frac{\mu_0}{4\pi}\frac{\boldsymbol{J}_{10}\exp(j\omega t)\Delta l\hat{z}}{r}\exp(-jkr) \tag{133}$$

This electric field is a spherical wave with an attenuation factor $\frac{1}{r}$. But considering that when the retarded wave emitted by the source and the advanced wave of the sink are synchronized, the advanced wave becomes the waveguide of the retarded wave, and the retarded wave becomes the waveguide of the advanced wave, so that both the retarded and advanced waves become waves traveling in a waveguide, and such waves become quasi-plane waves. Therefore the factor $\frac{1}{r}$ can be removed, and the propagation direction is also fixed, in the x direction.

$$\boldsymbol{E}_1^{(r)} \sim -jJ_{10}\exp(j\omega t)\exp(-jkx)\hat{z} = -jJ_1\exp(-jkx)\hat{z} \tag{134}$$

Considering that the direction of $\boldsymbol{E}_1^{(r)}$ is in the $-\hat{z}$ direction,

$$E_1 \sim jJ_1\exp(-jkx) \tag{135}$$

We assume that the load of the receiving dipole antenna is resistive, so the current $\boldsymbol{J}_2$ on the receiving antenna remains consistent with the electric field $\boldsymbol{E}_1^{(r)}$,

$$\boldsymbol{J}_2 \sim \boldsymbol{E}_1^{(r)} \sim -jJ_1\exp(-jkx)\hat{z}|_{x=L} = -jJ_1\exp(-jkL)\hat{z} \tag{136}$$

At $x < L$, $\boldsymbol{E}_2$ is an advanced wave,

$$\boldsymbol{E}_2 \sim -j\boldsymbol{J}_2\exp(+jk|x-L|)$$

$$\sim -j(-jJ_1\exp(-jkL)\hat{z})\exp(+jk|x-L|)$$

$$\sim -j(-jJ_1\exp(-jkL)\hat{z})\exp(-jk(x-L))$$

$$\sim jjJ_1\exp(-jkx)\hat{z} \tag{137}$$

If the direction of $\boldsymbol{E}_2$ is set to $-\hat{z}$,

$$E_2 \sim -jjJ_1\exp(-jkx) = J_1\exp(-jkx) \tag{138}$$

From Faraday's law of Maxwell's equations,

$$\nabla \times \boldsymbol{E} = -j\omega\mu_0\boldsymbol{H}_M \tag{139}$$

In the above equation $\boldsymbol{E} = \boldsymbol{E}_I$. We neglect the static electric field $\boldsymbol{E}_S$. This is because we consider the far field to be close to a plane wave, and in a plane wave $\boldsymbol{E}_S$ can be taken as 0. The above equation becomes,

$$\nabla \times \boldsymbol{E} \sim -j\boldsymbol{H}_M \tag{140}$$

or

$$\boldsymbol{H}_M \sim j\nabla \times \boldsymbol{E} \tag{141}$$

Thus,

$$\boldsymbol{H}_{M1} \sim j\nabla \times \boldsymbol{E}_1 = j\nabla \times (E_1(-\hat{z})) = j\nabla E_1 \times (-\hat{z}) \tag{142}$$

$$\boldsymbol{H}_{M2} \sim j\nabla \times \boldsymbol{E}_2 = j\nabla \times (E_2(-\hat{z})) = j\nabla E_2 \times (-\hat{z}) \tag{143}$$

$\boldsymbol{H}_1$ is an advanced wave at $x < 0$ and a retarded wave at $x \geq 0$,

$$\boldsymbol{H}_1 = -j\boldsymbol{H}_{M1}\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases} = -j(j\nabla E_1 \times (-\hat{z}))\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases}$$

$$= \nabla E_1 \times (-\hat{z}))\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases} = \frac{d}{dx}E_1\hat{x} \times (-\hat{z}))\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases}$$

$$= \frac{d}{dx}E_1\hat{y}\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases} \tag{144}$$

$$\boxed{H_1 = \frac{d}{dx}E_1\begin{cases}-1 & x < 0\\ 1 & x \geq 0\end{cases}} \tag{145}$$

$\boldsymbol{H}_2$ is an advanced wave at $x < L$ and a retarded wave at $x \geq L$,

$$\boldsymbol{H}_2 = -j\boldsymbol{H}_{M2}\begin{cases}-1 & x \leq L\\ 1 & x > L\end{cases} = -j(j\nabla E_2 \times (-\hat{z}))\begin{cases}-1 & x \leq L\\ 1 & x > L\end{cases}$$

$$= \nabla E_2 \times (-\hat{z})\begin{cases}-1 & x \leq L\\ 1 & x > L\end{cases} = \frac{d}{dx}E_2\hat{x} \times (-\hat{z})\begin{cases}-1 & x \leq L\\ 1 & x > L\end{cases}$$

$$= \frac{d}{dx}E_2\hat{y}\begin{cases}-1 & x \leq L\\ 1 & x > L\end{cases} \tag{146}$$

$$\boxed{H_2 = \frac{d}{dx}E_2\begin{cases}-1 & x \le L\\ 1 & x > L\end{cases}} \tag{147}$$

The mutual energy flow density is

$$\boldsymbol{S}_m = (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) = \hat{x}(E_1 H_2^* + E_2^* H_1)$$

$$= \hat{x}\begin{cases}E_1(-\frac{d}{dx}E_2)^* + E_2^*(-\frac{d}{dx}E_1) & x < 0\\ E_1(-\frac{d}{dx}E_2)^* + E_2^*(\frac{d}{dx}E_1) & 0 \le x \le L\\ E_1(\frac{d}{dx}E_2)^* + E_2^*(\frac{d}{dx}E_1) & L < x\end{cases}$$

$$= \hat{x}\begin{cases}jJ_1\alpha(-\frac{d}{dx}(J_1\alpha)^* + (J_1\alpha)^*(-\frac{d}{dx}jJ_1\alpha) & x < 0\\ jJ_1\alpha(-\frac{d}{dx}J_1\alpha)^* + (J_1\alpha)^*(\frac{d}{dx}jJ_1\alpha) & 0 \le x \le L\\ jJ_1\alpha(\frac{d}{dx}J_1\alpha)^* + (J_1\alpha)^*(\frac{d}{dx}jJ_1\alpha)) & L < x\end{cases}$$

$$= \hat{x}\begin{cases}jJ_1\alpha(jk(J_1\alpha)^* + (J_1\alpha)^*(jkjJ_1\alpha) & x < 0\\ jJ_1\alpha(jkJ_1\alpha)^* + (J_1\alpha)^*(-jkjJ_1\alpha) & 0 \le x \le L\\ jJ_1\alpha(-jkJ_1\alpha)^* + (J_1\alpha)^*(-jkjJ_1\alpha) & L < x\end{cases}$$

$$= k\hat{x}J_1J_1^*\begin{cases}j(j)^* + (1)^*(-1) & x < 0\\ j(j)^* + (1)^*(1) & 0 \le x \le L\\ j(-j)^* + (1)^*(1) & L < x\end{cases}$$

$$= 2k\hat{x}J_1J_1^*\begin{cases}0 & x < 0\\ 1 & 0 \le x \le L\\ 0 & L < x\end{cases} \tag{148}$$

Where $\alpha = \exp(-jkx)$.Therefore

$$\boxed{\boldsymbol{S}_m \sim k\hat{x}J_1J_1^*\begin{cases}0 & x < 0\\ 1 & 0 \le x \le L\\ 0 & L < x\end{cases}} \tag{149}$$

This shows that the mutual energy flow density is generated at the source $x = 0$ and annihilated at the sink $x = L$. Additionally,

$$\boldsymbol{S}_{11} \sim \hat{x}E_1H_1^* = \hat{x}E_1\frac{d}{dx}E_1\begin{cases}-1 & x < 0\\ 1 & x \ge 0\end{cases}$$

$$\sim k\hat{x}jE_1E_1^*\begin{cases}-1 & x < 0\\ 1 & x \ge 0\end{cases} \tag{150}$$

$\boldsymbol{S}_{11}$ is imaginary. It can be seen that the self-energy flow $\boldsymbol{S}_{11}$ is reactive power. Similarly, $\boldsymbol{S}_{22}$ is also reactive power. Neither transmits energy. The only thing that transmits energy is the mutual energy flow density $\boldsymbol{S}_m$. Therefore the author says that the photon is the mutual energy flow, not the self-energy flow. Here self-energyflow corresponds to the Poynting vector.

2.16 Summary of Electromagnetic Wave Theory

Initially, calculating the radiation energy flow from a dipole transmitting antenna to a dipole receiving antenna according to Maxwell's electromagnetic theory, we found that both the self-energy flow and the mutual energy flow participate in the transmission and radiation of energy flow. Fig. 5 below shows this situation. We assume there is only one transmitting antenna a and one receiving antenna b in vacuum. We assume that the transmitting antenna radiates a retarded wave and the receiving antenna radiates an advanced wave. According to Maxwell's electromagnetic theory, the retarded wave and the advanced wave can form a mutual energy flow, and this mutual energy flow is exactly equal to the energy radiated by the transmitting antenna to the receiving antenna, and also equal to the energy received by the receiving antenna from the electromagnetic field of the transmitting antenna. Therefore it seems that the mutual energy flow transmits energy from *a* to *b*, as shown in Figure 5.

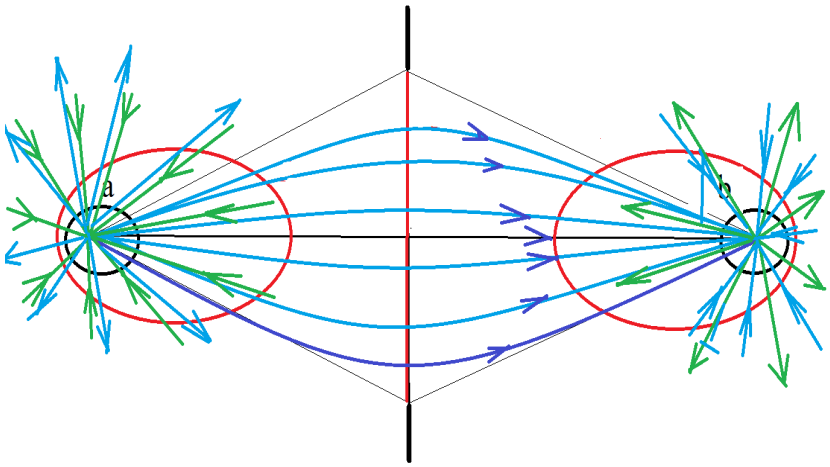


Figure 5: The transmitting antenna *a* radiates a retarded wave. The receiving antenna *b* radiates an advanced wave.

However, in Maxwell's electromagnetic theory there is not only the mutual energy flow: the transmitting antenna radiates a retarded wave into space, and the receiving antenna radiates an advanced wave into space. If these radiations into space are included in the calculation, the radiated energy would exceed expectations. In fact, either the self-energy flow radiates or the mutual energy flow radiates; it is impossible for both the mutual energy flow and the self-energy flow to radiate. Initially [16], the author assumed that the self-energy flow underwent reverse collapse. We see that from point *a* and point *b* there are electromagnetic waves emitted, and also electromagnetic waves returning to point *a* and point *b*.

Later, the author realized that reverse collapse can be replaced by reactive power radiation. If the self-energy flow is reactive power, then on average over a period, the self-energy flow does not radiate energy outward. In this way only the mutual energy flow radiates. Removing the self-energy flow and keeping only the mutual energy flow, the energy flow from the transmitting antenna to the receiving antenna is as shown in Fig. 6 below. We see that if only the mutual energy flow remains, the energy flow is generated at the source *a* and annihilated at *b*.

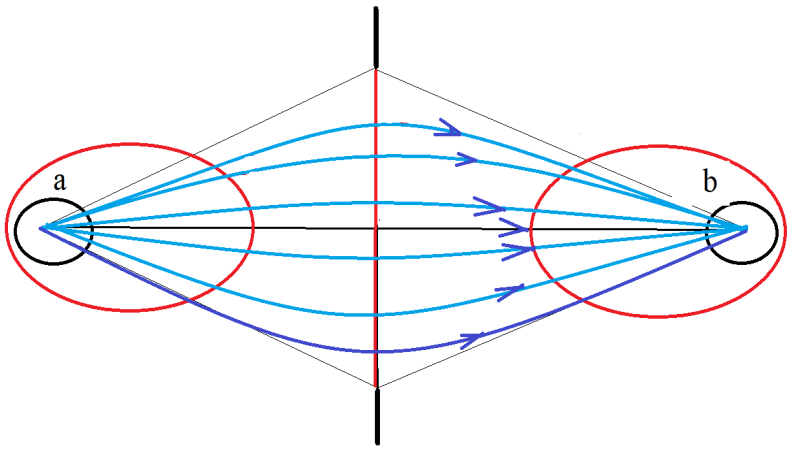


Figure 6: The transmitting antenna a radiates a retarded wave. The receiving antenna *b* radiates an advanced wave. Assuming the self-energy flow of the electromagnetic wave is reactive power, only the mutual energy flow remains.

In Section 2.14 we discussed that because the retarded and advanced waves interfere, the interference causes the wave to collapse. After collapse the mutual energy flow is concentrated on the line connecting the source and the sink, and at this time both the retarded and advanced waves are a beam of quasi-plane waves. See Fig. 7

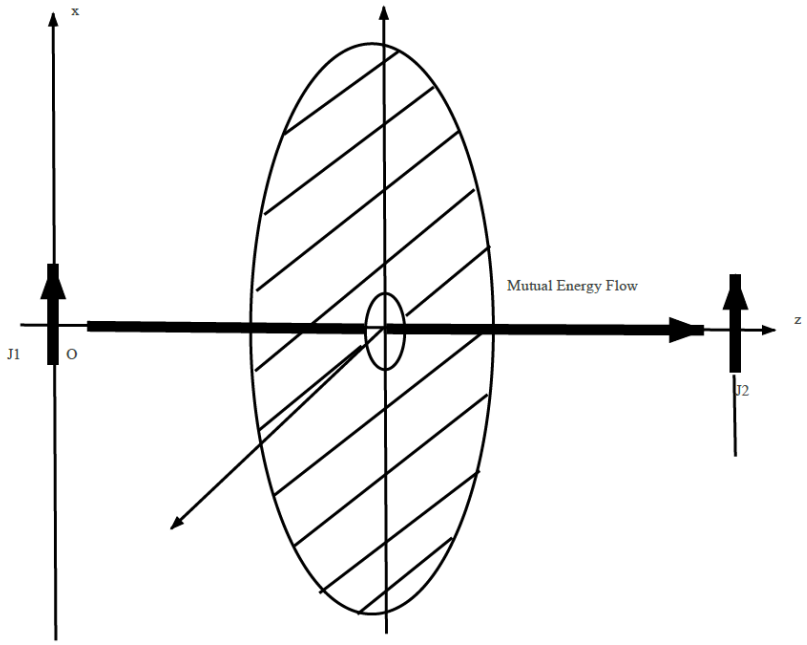


Figure 7: After wave collapse, the mutual energy flow becomes a ray of light from *a* to *b*, i.e., a beam of light. At this time both the retarded and advanced waves become plane waves.

So we say that the mutual energy flow is the photon, and the photon is the mutual energy flow formed by the retarded wave emitted by the source and the advanced wave emitted by the sink. We see that the mutual energy flow is generated at the source and annihilated at the sink, which is completely consistent with the properties of a particle.

We see that both the retarded and advanced waves in the photon are plane waves and do not decay with distance. But we usually say that the radiation electromagnetic field of a dipole antenna decays with distance. The reason the macroscopic electromagnetic field decays with distance is the averaging effect. Looking at Fig. 1, although the photon energy flow emitted from the transmitting antenna is a non-decaying quasi-plane wave, if a mutual energy flow (photon) is averaged over a spherical surface, eventually the density of the energy

flow will decay with the square of the distance, i.e., the attenuation factor is $\frac{1}{r^2}$. Then the attenuation factor of the electric and magnetic fields is $\frac{1}{r}$. In this way the macroscopic electromagnetic wave formed by many photons together can still be described by Maxwell's electromagnetic theory. Thus both the electric and magnetic fields decay with a $\frac{1}{r}$ factor.

The idea of the author's mutual energy flow theory can be further developed in quantum theory. We know that there is no magnetic field in quantum mechanics. We can establish a new magnetic field for it and then correct its magnetic field. In this way the achievements we have made in electromagnetic field theory can be completely transferred to quantum mechanics and quantum field theory.

3. Demonstrating from the Schrödinger Equation That a Particle Is a Mutual Energy Flow Rather Than a Self-Energy Flow

3.1 The Schrödinger Equation

The Schrödinger equation is one of the core equations of quantum mechanics, used to describe the behavior of microscopic particles (such as electrons) in space and time. Depending on the situation, it has two basic forms: the time-independent Schrödinger equation and the time-dependent Schrödinger equation.

Time-dependent Schrödinger Equation:

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t)=\hat{H}\psi(\boldsymbol{r},t) \tag{151}$$

where: $\psi(\boldsymbol{r},t)$ is the wave function of the particle, describing the state of the particle at position $\boldsymbol{r}$ and time t. $\hat{H}$ is the Hamiltonian operator, representing the total energy operator of the system. $\hbar$ is the reduced Planck constant. For a single-particle system, the Hamiltonian is generally:

$$\hat{H}=-\frac{\hbar^2}{2m}\nabla^2+V(\boldsymbol{r}) \tag{152}$$

3.2 The Self-Energy Flow Theorem of the Schrödinger Equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\boldsymbol{r},t)+V(\boldsymbol{r})\psi(\boldsymbol{r},t)=i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) \tag{153}$$

or

$$-\frac{\hbar}{2im}\nabla^2\psi(\boldsymbol{r},t)+\frac{1}{i\hbar}V(\boldsymbol{r})\psi(\boldsymbol{r},t)=\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) \tag{154}$$

or

$$-\frac{\hbar}{2im}\nabla^2\psi(\boldsymbol{r},t)+\frac{1}{i\hbar}V(\boldsymbol{r})\psi(\boldsymbol{r},t)-\frac{\partial}{\partial t}\psi(\boldsymbol{r},t)=0 \tag{155}$$

Adding a source τ,

$$-\frac{\hbar}{2im}\nabla^2\psi+\frac{1}{i\hbar}V(\boldsymbol{r})\psi-\frac{\partial}{\partial t}\psi=\tau \tag{156}$$

Multiplying (156) by ψ^*,

$$-\frac{\hbar}{2im}\nabla^2\psi\psi^*+\frac{1}{i\hbar}V(\boldsymbol{r})\psi\psi^*-\frac{\partial}{\partial t}\psi\psi^*=\tau\psi^* \tag{157}$$

Taking the complex conjugate of (156) and multiplying by $\psi(\boldsymbol{r},t)$,

$$\frac{\hbar}{2im}\psi\nabla^2\psi^*-\frac{1}{i\hbar}V(\boldsymbol{r})\psi\psi^*-\psi\frac{\partial}{\partial t}\psi^*=\tau^*\psi \tag{158}$$

Adding the above two equations,

$$\frac{\hbar}{2im}(\psi\nabla^2\psi^* - \nabla^2\psi\psi^*) - \psi\frac{\partial}{\partial t}\psi^* - \frac{\partial}{\partial t}\psi\psi^* = \tau^*\psi + \tau\psi^* \tag{159}$$

Consider

$$\int_{t=-\infty}^{\infty} dt(\psi\frac{\partial}{\partial t}\psi^* + \frac{\partial}{\partial t}\psi\psi^*)$$

$$= \int_{t=-\infty}^{\infty} dt\frac{\partial}{\partial t}(\psi(\boldsymbol{r},t)\psi(\boldsymbol{r},t)^*) = \int_{t=-\infty}^{\infty} dt\frac{\partial}{\partial t}(\rho) = \rho(\infty) - \rho(-\infty) = 0 \tag{160}$$

Therefore

$$\int_{t=-\infty}^{\infty} dt\int_V dV(\frac{\hbar}{2im}(\psi\nabla^2\psi^* - \nabla^2\psi\psi^*) = \int_{t=-\infty}^{\infty} dt\int_V dV(\tau^*\psi + \tau\psi^*) \tag{161}$$

where

$$\nabla\cdot(\psi\nabla\psi^* - \nabla\psi\psi^*)$$

$$= (\nabla\psi\cdot\nabla\psi^* + \psi\nabla^2\psi^* - \nabla^2\psi\psi^* + \nabla\psi\cdot\nabla\psi^*)$$

$$= \psi\nabla^2\psi^* - \nabla^2\psi\psi^* \tag{162}$$

Therefore,

$$\psi\nabla^2\psi^* - \nabla^2\psi\psi^* = \nabla\cdot(\psi\nabla\psi^* - \nabla\psi\psi^*) \tag{163}$$

Formula (161) becomes

$$\int_{t=-\infty}^{\infty} dt\int_V dV(\frac{\hbar}{2im}\nabla\cdot(\psi\nabla\psi^* - \nabla\psi\psi^*) = \int_{t=-\infty}^{\infty} dt\int_V dV(\tau^*\psi + \tau\psi^*) \tag{164}$$

or

$$\int_{t=-\infty}^{\infty} dt\int_V dV(\frac{\hbar}{2im}\nabla\cdot(\psi\nabla\psi^* - \nabla\psi\psi^*) = 2\mathrm{Re}\int_{t=-\infty}^{\infty} dt\int_V dV(\tau^*\psi) \tag{165}$$

Consider

$$\nabla\psi = i\boldsymbol{h}_S \tag{166}$$

Therefore

$$\frac{1}{2i}(\psi\nabla\psi^* - \nabla\psi\psi^*)$$

$$= \frac{1}{2}(-\psi(\frac{1}{i}\nabla\psi)^* - \frac{1}{i}\nabla\psi\psi^*)$$

$$= -\frac{1}{2}(\psi\boldsymbol{h}_S^* + \boldsymbol{h}_S\psi^*)$$

$$= -\mathrm{Re}(\psi\boldsymbol{h}_S^*) \tag{167}$$

Therefore

$$-\int_{t=-\infty}^{\infty} dt \int_V dV\left(\frac{\hbar}{m}\nabla\cdot(\mathrm{Re}(\psi \boldsymbol{h}_S{}^*)\right) = 2\mathrm{Re}\int_{t=-\infty}^{\infty} dt \int_V dV(\tau^*\psi) \tag{168}$$

Keeping Re in mind,

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma \left(\frac{\hbar}{2m}(\psi \boldsymbol{h}_S{}^*)\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty} dt \int_V dV(\tau^*\psi)\right. \tag{169}$$

The above equation is the self-energy flow theorem of the Schrödinger equation, equivalent to (the time integral of) Poynting's theorem in electromagnetic field theory.

3.3 The Mutual Energy Flow Theorem of the Schrödinger Equation

From the above equation (169) we obtain

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma \left(\frac{\hbar}{2m}(\psi_i \boldsymbol{h}_{Si}{}^*)\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty} dt \int_V dV(\tau_i^*\psi_i)\right. \tag{170}$$

Considering the superposition principle,

$$\psi = \sum_{i=1}^{2}\psi_i, \quad \boldsymbol{h}_S = \sum_{i=1}^{2}\boldsymbol{h}_{Si}, \quad \tau = \sum_{i=1}^{2}\tau_i \tag{171}$$

Equation (169) gives,

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma \sum_{i=1}^{2}\sum_{j=1}^{2}\left(\frac{\hbar}{2m}(\psi_i \boldsymbol{h}_{Sj}{}^*)\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty} dt \int_V dV \sum_{i=1}^{2}\sum_{j=1}^{2}(\tau_i^*\psi_j)\right. \tag{172}$$

Subtracting (170) from (172),

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma \sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\left(\frac{\hbar}{2m}(\psi_i \boldsymbol{h}_{Sj}{}^*)\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty} dt \int_V dV \sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}(\tau_i^*\psi_j)\right. \tag{173}$$

or

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma \frac{\hbar}{2m}(\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2 \boldsymbol{h}_{S1}^*)\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty} dt \int_V dV(\tau_1^*\psi_2 + \tau_2^*\psi_1) \tag{174}$$

Considering that the Fourier transform can remove one time integral,

$$-\frac{\hbar}{2m}\cdot\oiint_\Gamma(\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2 \boldsymbol{h}_{S1}^*)\cdot\hat{n}d\Gamma = \int_V dV(\tau_1^*\psi_2 + \tau_2^*\psi_1) \tag{175}$$

The above equation actually needs to take the real part. Considering $\mathrm{Re}(\psi_2 \boldsymbol{h}_{S1}^*) = \mathrm{Re}(\psi_2^* \boldsymbol{h}_{S1})$ and $\mathrm{Re}(\tau_1^*\psi_2) = \mathrm{Re}(\tau_1\psi_2^*)$, the above equation can be rewritten as,

$$-\frac{\hbar}{2m}\cdot\oiint_\Gamma(\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2^* \boldsymbol{h}_{S1})\cdot\hat{n}d\Gamma = \int_V dV(\tau_1\psi_2^* + \tau_2^*\psi_1) \tag{176}$$

Note that in the above formulas, after the Fourier transform the same symbols are used as before the transform. Considering that the mutual energy flow does not overflow the universe,

$$\oiint_\Gamma(\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2^* \boldsymbol{h}_{S1})\cdot\hat{n}d\Gamma = 0 \tag{177}$$

Γ is a spherical surface of radius $R = \infty$. From (176, 177) we obtain,

$$\int_V dV(\tau_1\psi_2^* + \tau_2^*\psi_1) = 0 \tag{178}$$

or

$$-\int_{V_1} (\tau_1\psi_2^*)dV = \int_{V_2} (\tau_2^*\psi_1)dV \tag{179}$$

Considering formula (176), moving τ_2 from inside Γ to outside Γ. Here Γ is a finite surface. We obtain,

$$\frac{\hbar}{2m} \cdot \oiint_\Gamma (\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2^* \boldsymbol{h}_{S1}) \cdot \hat{n} d\Gamma = -\int_V (\tau_1\psi_2^*)dV \tag{180}$$

From the above two equations we obtain,

$$-\int_V (\tau_1\psi_2^*)dV = \frac{\hbar}{2m} \cdot \oiint_\Gamma (\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2^* \boldsymbol{h}_{S1}) \cdot \hat{n} d\Gamma = \int_V (\tau_2^*\psi_1)dV \tag{181}$$

The above equation is the mutual energy flow theorem of the Schrödinger equation. The above equation is a mutual energy flow theorem rigorously derived from the Schrödinger equation, and like the mutual energy flow theorem of electromagnetic theory corresponding to Maxwell's equations, it suffers from the problem that both the mutual energy flow and the self-energy flow transmit energy.

In this case the mutual energy flow density transmits reactive power,

$$\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2^* \boldsymbol{h}_{S1} = \text{imaginary} \tag{182}$$

The surface integral $\oiint_\Gamma$ contributes a *-j = i*, see [50]. This makes the mutual energy flow

$$\frac{\hbar}{2m} \cdot \oiint_\Gamma (\psi_1 \boldsymbol{h}_{S2}{}^* + \psi_2^* \boldsymbol{h}_{S1}) \cdot \hat{n} d\Gamma = \text{real} \tag{183}$$

still real.

On the contrary, in the framework of the Schrödinger equation, what transmits energy is the self-energy flow density

$$\psi_1 \boldsymbol{h}_{S1} = \text{real} \tag{184}$$

For the self-energy flow the surface integral $\oiint_\Gamma$ is still real, so

$$\oiint_\Gamma \psi_1 \boldsymbol{h}_{S1} \cdot \hat{n} d\Gamma = \text{real} \tag{185}$$

If the self-energy flow transmits energy, the mutual energy flow (169) also transmits energy, and the transmitted energy would exceed the expected value, violating energy conservation. Therefore the author modifies the mutual energy flow theorem to,

$$\boxed{-\int_V (\tau_1\psi_2^*)dV = \frac{\hbar}{2m} \cdot \oiint_\Gamma (\psi_1 \boldsymbol{h}_2^* + \psi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma = \int_V (\tau_2^*\psi_1)dV} \tag{186}$$

As for the correction to the magnetic field $\boldsymbol{h}$, considering the difference between retarded and advanced waves, we have

$$\boldsymbol{h}^{(r)} = i\boldsymbol{h}_S^{(r)}, \quad \boldsymbol{h}^{(a)} = -i\boldsymbol{h}_S^{(a)} \tag{187}$$

After this correction the self-energy flow

$$\psi_1 \boldsymbol{h}_1{}^* = \text{imaginary}, \qquad \oiint \psi_1 \boldsymbol{h}_1{}^* \cdot \hat{n} d\Gamma = \text{imaginary} \tag{188}$$

The self-energy flow no longer transmits energy. The mutual energy flow density transmits energy,

$$\psi_1 \boldsymbol{h}_2{}^* + \psi_2^* \boldsymbol{h}_1 = \text{real} \tag{189}$$

The mutual energy flow density is real. Considering that the surface integral $\oiint$ of the mutual energy flow contributes $-j = i$, this makes the mutual energy flow

$$\frac{\hbar}{2m} \cdot \oiint_\Gamma (\psi_1 \boldsymbol{h}_2^* + \psi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma \sim i = \text{imaginary} \tag{190}$$

This seems to be a problem: the mutual energy flow also does not transmit energy! But the wave will collapse, and the collapse is due to the interference of ψ_1 and ψ_2. The interference causes the field to be strengthened on the line from the source to the sink and weakened in other directions. After collapse ψ_1 and ψ_2 become quasi-plane waves. A so-called quasi-plane wave is a plane wave like a laser beam. Such a wave can pass through a small hole, and the contribution of $\oiint_{\Gamma_{hole}}$ is no longer imaginary. Γ_{hole} is a small hole in the surface Γ. The small hole is on the line connecting the source and the sink. Therefore, if wave collapse is taken into account,

$$\frac{\hbar}{2m} \cdot \oiint_{\Gamma_{hole}} (\psi_1 \boldsymbol{h}_2^* + \psi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma \sim \text{real} \tag{191}$$

The mutual energy flow theorem is rewritten as,

$$\boxed{-\int_V (\tau_1 \psi_2^*) dV = \frac{\hbar}{2m} \cdot \oiint_{\Gamma_{hole}} (\psi_1 \boldsymbol{h}_2^* + \psi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma = \int_V (\tau_2^* \psi_1) dV}$$

3.4 Calculation of the Mutual Energy Flow Density After Collapse

τ_1 is at $x = 0$, and τ_2 is at $x = L$. Energy flows from τ_1 to τ_2. Energy propagates to the right, so we assume the wave also propagates to the right. This requires that the waves produced by both sources are advanced on the left and retarded on the right. Therefore for $\boldsymbol{h}_1$, it is an advanced wave at $x < 0$ and a retarded wave at $x \geq 0$, so,

$$\boldsymbol{h}_1 = \begin{cases} -i\boldsymbol{h}_{S1} & x < 0 \\ i\boldsymbol{h}_{S1} & 0 \leq x \end{cases} = \begin{cases} -i\frac{1}{i}\nabla\psi_1 & x < 0 \\ i\frac{1}{i}\nabla\psi_1 & 0 \leq x \end{cases} = \nabla\psi_1 \begin{cases} -1 & x < 0 \\ 1 & 0 \leq x \end{cases} \tag{192}$$

τ_2 is at $x = L$. For $\boldsymbol{h}_2$, it is an advanced wave at $x \leq L$ and a retarded wave at $x > L$,

$$\boldsymbol{h}_2 = \begin{cases} -i\boldsymbol{h}_{S2} & x < L \\ i\boldsymbol{h}_{S2} & L \leq x \end{cases} = \begin{cases} -i\frac{1}{i}\nabla\psi_2 & x < L \\ i\frac{1}{i}\nabla\psi_2 & L \leq x \end{cases} = \nabla\psi_2 \begin{cases} -1 & x < L \\ 1 & L \leq x \end{cases} \tag{193}$$

$$\boldsymbol{J}_m = (\psi_1 \boldsymbol{h}_2{}^* + \psi_2^* \boldsymbol{h}_1) \tag{194}$$

$$= \begin{cases} (\psi_1(-\nabla\psi_2)^* + \psi_2^*(-\nabla\psi_1) & x < 0 \\ (\psi_1(-\nabla\psi_2)^* + \psi_2^*(\nabla\psi_1) & 0 \leq x \leq L \\ (\psi_1(\nabla\psi_2)^* + \psi_2^*(\nabla\psi_1) & L < x \end{cases} \tag{195}$$

Assume ψ_1 is a plane wave,

$$\psi_1 = \exp(ikx) \tag{196}$$

ψ_1 is continuous at the source and the sink.

$$\boldsymbol{h}_{1S} = \frac{1}{i}\nabla\psi_1 = \frac{1}{i}\nabla\exp(ikx) = \frac{1}{i}ik\hat{x}\exp(ikx) = k\hat{x}\exp(ikx) \tag{197}$$

Between $0 < x < L$, $\boldsymbol{h}_{1S}$ is a retarded wave,

$$\boldsymbol{h}_1 = i\boldsymbol{h}_{S1} = ik\hat{x}\exp(ikx) \tag{198}$$

$$\psi_2^*\boldsymbol{h}_1 \tag{199}$$

should be real

$$\psi_2 \sim \boldsymbol{h}_1 \tag{200}$$

$$\psi_2 = i\exp(ikx) \tag{201}$$

$$\boldsymbol{J}_m = \begin{cases} (\psi_1(-\nabla\psi_2)^* + \psi_2^*(-\nabla\psi_1) & x < 0 \\ (\psi_1(-\nabla\psi_2)^* + \psi_2^*(\nabla\psi_1) & 0 \le x \le L \\ (\psi_1(\nabla\psi_2)^* + \psi_2^*(\nabla\psi_1) & L < x \end{cases}$$

$$= \begin{cases} (\exp(ikx)(-\nabla(i\exp(ikx)))^* + (i\exp(ikx))^*(-\nabla\exp(ikx)) & x < 0 \\ (\exp(ikx)(-\nabla(i\exp(ikx)))^* + (i\exp(ikx))^*(\nabla\exp(ikx)) & 0 \le x \le L \\ (\exp(ikx)(\nabla(i\exp(ikx)))^* + (i\exp(ikx))^*(\nabla\exp(ikx)) & L < x \end{cases}$$

$$= \hat{x}k \begin{cases} (\exp(ikx)(-i(i\exp(ikx)))^* + (i\exp(ikx))^*(-i\exp(ikx)) & x < 0 \\ (\exp(ikx)(-i(i\exp(ikx)))^* + (i\exp(ikx))^*(i\exp(ikx)) & 0 \le x \le L \\ (\exp(ikx)(i(i\exp(ikx)))^* + (i\exp(ikx))^*(i\exp(ikx)) & L < x \end{cases}$$

$$= \hat{x}k \begin{cases} ((-ii)^* + (i)^*(-i) & x < 0 \\ ((-ii)^* + (i)^*(i) & 0 \le x \le L \\ ((ii)^* + (i)^*(i) & L < x \end{cases}$$

$$= 2k\hat{x} \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le L \\ 0 & L < x \end{cases} \tag{202}$$

$$\boxed{\boldsymbol{J}_m = 2k\hat{x} \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le L \\ 0 & L < x \end{cases}} \tag{203}$$

It can be seen that the mutual energy flow $\boldsymbol{J}_m = (\psi_1\boldsymbol{h}_2{}^* + \psi_2^*\boldsymbol{h}_1)$ is generated at the source $x = 0$ and annihilated at the sink $x = L$! This mutual energy flow density resembles a quasi-plane wave, because ψ_1 and ψ_2 are both quasi-plane waves.

4. Demonstrating from the Klein-Gordon (K-G) Equation That a Particle Is a Mutual Energy Flow Rather Than a Self-Energy Flow

4.1 The Klein-Gordon (K-G) Equation

If the Klein-Gordon (K-G) equation has a source term, i.e.:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\phi(x) = \tau(x), \tag{204}$$

where $\tau(x)$ is an external source term, then the probability flow equation needs to be re-derived, because the source term affects the conservation law.

Complex scalar field case: Assume $\phi(x)$ is a complex scalar field, and its conjugate equation is:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\phi^*(x) = \tau^*(x). \tag{205}$$

4.2 The Flow of the Klein-Gordon (K-G) Equation

Constructing a conserved flow: We still hope to construct a four-dimensional flow j^μ, but now, due to the presence of the source term, the continuity equation will change. The steps are as follows:

Multiply the K-G equation by ϕ^*:

$$\phi^*(\partial^2 + m^2)\phi = \phi^*\tau. \tag{206}$$

Multiply the conjugate K-G equation by ϕ:

$$\phi(\partial^2 + m^2)\phi^* = \phi\tau^*. \tag{207}$$

Subtracting the two equations:

$$\phi^*\partial^2\phi - \phi\partial^2\phi^* = \phi^*\tau - \phi\tau^*. \tag{208}$$

The left-hand side can be written as:

$$\partial_\mu(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) = \phi^*\tau - \phi\tau^*. \tag{209}$$

where we considered

$$\partial_\mu(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) = (\partial_\mu\phi^*)\partial^\mu\phi - (\partial_\mu\phi)\partial^\mu\phi^* + \phi^*\partial_\mu\partial^\mu\phi - \phi\partial_\mu\partial^\mu\phi^*$$

$$= \phi^*\partial_\mu\partial^\mu\phi - \phi\partial_\mu\partial^\mu\phi^* = \phi^*\partial^2\phi - \phi\partial^2\phi^* \tag{210}$$

where we considered

$$(\partial_\mu\phi^*)\partial^\mu\phi - (\partial_\mu\phi)\partial^\mu\phi^* = g^{\mu\nu}(\partial_\mu\phi^*)\partial_\nu\phi - g^{\mu\nu}(\partial_\mu\phi)\partial_\nu\phi^*$$

$$= g^{\mu\nu}(\partial_\mu\phi^*)\partial_\nu\phi - g^{\nu\mu}\partial_\mu\phi^*(\partial_\nu\phi)$$

$$= 0 \tag{211}$$

Defining the probability flow: We still define the probability flow:

$$j^{\mu} = i(\phi^{*}\partial^{\mu}\phi - \phi\partial^{\mu}\phi^{*}), \tag{212}$$

but now its four-dimensional divergence is:

$$\partial_{\mu}j^{\mu} = i(\phi^{*}\tau - \phi\tau^{*}). \tag{213}$$

Continuity equation: If $\tau \neq 0$, the probability flow is no longer conserved, but satisfies:

$$\frac{\partial j^{0}}{\partial t} + \nabla \cdot \mathbf{j} = i(\phi^{*}\tau - \phi\tau^{*}), \tag{214}$$

where:

$$j^{0} = i\left(\phi^{*}\frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^{*}}{\partial t}\right) \tag{215}$$

is the probability density, and

$$\mathbf{j} = i(\phi^{*}\nabla\phi - \phi\nabla\phi^{*}) \tag{216}$$

is the probability flow density.

If $\tau = 0$ (no source), then $\partial_{\mu}j^{\mu} = 0$, probability is conserved. - If $\tau \neq 0$ (with source), the probability flow is no longer conserved, because the external source can inject or extract probability density.

Final conclusion: For the K-G equation with a source:

$$\left(\frac{\partial^{2}}{\partial t^{2}} - \nabla^{2} + m^{2}\right)\phi(x) = \tau(x), \tag{217}$$

the probability flow equation is:

$$j^{\mu} = i(\phi^{*}\partial^{\mu}\phi - \phi\partial^{\mu}\phi^{*}), \tag{218}$$

and its continuity equation is:

$$\partial_{\mu}j^{\mu} = i(\phi^{*}\tau - \phi\tau^{*}). \tag{219}$$

This indicates that in the presence of an external source, the probability flow is no longer conserved, but is affected by the source term.

4.3 The Mutual Energy Flow Law

Since $-j^{\mu}$ is also a flow, we take $-j^{\mu} \to j^{\mu}$. This is done for later convenience. Therefore,

$$j^{\mu} = \frac{1}{i}(\phi^{*}\partial^{\mu}\phi - \phi\partial^{\mu}\phi^{*}), \tag{220}$$

so we have

$$\partial_{\mu}j^{\mu} = \frac{1}{i}(\phi^{*}\tau - \phi\tau^{*})$$

$$= \left(\phi^{*}\frac{1}{i}\tau + \phi(\frac{1}{i}\tau)^{*}\right)$$

$$= -(\phi^* i\tau + \phi(i\tau)^*) \tag{221}$$

Let

$$\tau' = i\tau \tag{222}$$

Then we have

$$\partial_\mu j^\mu = -(\phi^* \tau' + \phi \tau'^*)$$

$$= -2\frac{1}{2}(\phi^* \tau' + \phi \tau'^*)$$

$$= -2\mathrm{Re}(\phi^* \tau') \tag{223}$$

i.e.,

$$\partial_\mu j^\mu = -2\mathrm{Re}(\phi^* \tau') \tag{224}$$

or

$$\partial_t j^0 + \nabla \cdot \mathbf{j} = -2\mathrm{Re}(\phi^* \tau') \tag{225}$$

Integrating over time, this is a relaxation process; after relaxation, ϕ may deviate from the K-G equation.

$$\int_{t=-\infty}^{\infty} dt \int_V dV(\partial_t j^0 + \nabla \cdot \mathbf{j}) = -2\mathrm{Re} \int_{t=-\infty}^{\infty} dt \int_V dV(\phi^* \tau') \tag{226}$$

Considering

$$\int_{t=-\infty}^{\infty} dt \int_V dV(\partial_t j^0) = Q(\infty) - Q(-\infty) = 0 \tag{227}$$

we obtain

$$-\int_{t=-\infty}^{\infty} dt \int_V dV(\nabla \cdot \mathbf{j}) = 2\mathrm{Re} \int_{t=-\infty}^{\infty} dt \int_V dV(\phi^* \tau') \tag{228}$$

or

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma \mathbf{j} \cdot \hat{n} d\Gamma = 2\mathrm{Re} \int_{t=-\infty}^{\infty} dt \int_V dV(\phi^* \tau') \tag{229}$$

The above equation corresponds to the time-integrated Poynting theorem in electromagnetic field theory. Where

$$\mathbf{j} = \frac{1}{i}(\phi^* \nabla \phi - \phi \nabla \phi^*) \tag{230}$$

Define

$$\nabla \phi = i\boldsymbol{h}_K \tag{231}$$

This definition is consistent with the approach for the Schrödinger equation (166). $\boldsymbol{h}_K$ is the magnetic field under the K-G equation, equivalent to the magnetic field in Maxwell's electromagnetic theory. However, later the author still needs to correct this magnetic field, just as we treated Maxwell's electromagnetic theory. Consider,

$$\mathbf{j} = 2\frac{1}{2i}(\phi^*(i\boldsymbol{h}_K) - \phi(i\boldsymbol{h}_K)^*)$$

$$= 2\frac{1}{2}(\phi^*(\boldsymbol{h}_K) + \phi(\boldsymbol{h}_K)^*)$$

$$= 2\mathrm{Re}(\phi^*\boldsymbol{h}_K) \tag{232}$$

Substituting (232) into (229) gives,

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma 2\mathrm{Re}(\phi^*\boldsymbol{h}_K)\cdot\hat{n}d\Gamma = 2\mathrm{Re}\int_{t=-\infty}^{\infty} dt\int_V dV(\phi^*\tau') \tag{233}$$

Keeping Re in mind, we have,

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\phi^*\boldsymbol{h}_K)\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty} dt\int_V dV(\phi^*\tau') \tag{234}$$

The above equation corresponds to the time integral of Poynting's theorem in Maxwell's electromagnetic theory. Considering the superposition principle,

$$\phi = \sum_{i=1}^{2}\phi_i, \quad \boldsymbol{h}_K = \sum_{i=1}^{2}\boldsymbol{h}_{Ki}, \quad \tau' = \sum_{i=1}^{2}\tau'_i \tag{235}$$

$$-\sum_{i=1}^{2}\sum_{j=1}^{2}\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\phi_i^*\boldsymbol{h}_{Kj})\cdot\hat{n}d\Gamma = \sum_{i=1}^{2}\sum_{j=1}^{2}\int_{t=-\infty}^{\infty} dt\int_V dV(\phi_i^*\tau'_j) \tag{236}$$

Considering that from formula (234) we can directly obtain,

$$-\sum_{i=1}^{2}\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\phi_i^*(\boldsymbol{h}_{Ki}))\cdot\hat{n}d\Gamma = \sum_{i=1}^{2}\int_{t=-\infty}^{\infty} dt\int_V dV(\phi_i^*\tau'_i) \tag{237}$$

Subtracting formula (237) from formula (236),

$$-\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\phi_i^*(\boldsymbol{h}_{Kj}))\cdot\hat{n}d\Gamma = \sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\int_{t=-\infty}^{\infty} dt\int_V dV(\phi_i^*\tau'_j) \tag{238}$$

or

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\phi_1^*\boldsymbol{h}_{K2} + \phi_2^*\boldsymbol{h}_{K1})\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty} dt\int_V dV(\phi_1^*\tau'_2 + \phi_2^*\tau'_1) \tag{239}$$

where we considered

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\phi_i^*\boldsymbol{h}_{Kj} = (\phi_1^* + \phi_2^*)(\boldsymbol{h}_{K1} + \boldsymbol{h}_{K2}) = \phi_1^*\boldsymbol{h}_{K1} + \phi_1^*\boldsymbol{h}_{K2} + \phi_2^*\boldsymbol{h}_{K1} + \phi_2^*\boldsymbol{h}_{K2} \tag{240}$$

$$\sum_{i=1}^{2}\phi_i^*(\boldsymbol{h}_{Ki}) = \phi_1^*\boldsymbol{h}_{K1} + \phi_2^*\boldsymbol{h}_{K2} \tag{241}$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\phi_i^*\boldsymbol{h}_{Kj} - \sum_{i=1}^{2}\phi_i^*(\boldsymbol{h}_{Ki}) = \phi_1^*\boldsymbol{h}_{K2} + \phi_2^*\boldsymbol{h}_{K1} \tag{242}$$

It must be noted here that since the above equation (239) actually needs to take the real part, considering

$$\mathrm{Re}(\phi_1^* \boldsymbol{h}_{K2}) = \mathrm{Re}(\phi_1 \boldsymbol{h}_{K2}^*) \tag{243}$$

$$\mathrm{Re}(\phi_1^* \tau'_2) = \mathrm{Re}(\phi_1 (\tau'_2)^*) \tag{244}$$

formula (239) becomes,

$$-\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\phi_1 \boldsymbol{h}_{K2}^* + \phi_2^* \boldsymbol{h}_{K1}) \cdot \hat{n} d\Gamma = \int_{t=-\infty}^{\infty} dt \int_V dV (\phi_1 (\tau'_2)^* + \phi_2^* \tau'_1) \tag{245}$$

Applying a Fourier transform to the above equation can remove one time integral, giving,

$$-\oiint_\Gamma (\phi_1 \boldsymbol{h}_{K2}^* + \phi_2^* \boldsymbol{h}_{K1}) \cdot \hat{n} d\Gamma = \int_V dV (\phi_1 (\tau'_2)^* + \phi_2^* \tau'_1) \tag{246}$$

The above equation is the mutual energy theorem of the K-G equation. Considering that the mutual energy flow cannot overflow the universe, when Γ is a spherical surface with radius $R = \infty$, we have,

$$\oiint_\Gamma (\phi_1^* \boldsymbol{h}_{K2} + \phi_2^* \boldsymbol{h}_{K1}) \cdot \hat{n} d\Gamma = 0 \tag{247}$$

The reason the mutual energy flow cannot overflow the universe is the same as the reason the mutual energy flow in the author's electromagnetic field theory does not overflow the universe. Readers with questions can find it there [33, 41]. Therefore,

$$\int_V dV (\phi_1 (\tau'_2)^* + \phi_2^* \tau'_1) = 0 \tag{248}$$

or

$$-\int_{V_1} dV (\phi_2^* \tau'_1) = \int_{V_2} dV (\phi_1 (\tau'_2)^*) \tag{249}$$

The above equation can also be called the mutual energy theorem. In (245) consider moving τ'_2 out of the surface Γ.

$$\oiint_\Gamma (\phi_1 \boldsymbol{h}_{K2}^* + \phi_2^* \boldsymbol{h}_{K1}) \cdot \hat{n} d\Gamma = -\int_{V_1} dV (\phi_2^* \tau'_1) \tag{250}$$

From (249, 250) we obtain

$$-\int_{V_1} dV (\phi_2^* \tau'_1) = \oiint_\Gamma (\phi_1 \boldsymbol{h}_{K2}^* + \phi_2^* \boldsymbol{h}_{K1}) \cdot \hat{n} d\Gamma = \int_{V_2} dV (\phi_1 (\tau'_2)^*) \tag{251}$$

The above equation is the mutual energy flow theorem corresponding to the K-G equation.

4.4 Correction to the Theory

We mentioned in (226) that the time integral plays the role of a relaxation process. Because of this relaxation process, $\phi, \boldsymbol{h}_K$ may deviate from the solution of the K-G equation. In fact, we replace $\phi_1 \boldsymbol{h}_{K2}^* + \phi_2^* \boldsymbol{h}_{K1}$ with $\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1$:

$$-\int_{V_1} dV (\phi_2^* \tau'_1) = \oiint_\Gamma (\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma = \int_{V_2} dV (\phi_1 (\tau'_2)^*) \tag{252}$$

The author considers the above equation no longer to be the mutual energy flow theorem, but the mutual energy flow law. Formula (251) is the mutual energy flow theorem formula rigorously obtained from the K-G equation. The problem with this formula is the same as the problem encountered in electromagnetic theory. That is, the self-energy flow (234)

$$j_{K1} = \phi_1^* \boldsymbol{h}_{K1} = \text{real} \tag{253}$$

For the self-energy flow the contribution of the surface integral $\oiint_\Gamma$ is real, so

$$Q_{K11} = \oiint_{\Gamma}(\phi_1^* \boldsymbol{h}_{K1}) \cdot \hat{n} d\Gamma = \text{real} \tag{254}$$

$$\boldsymbol{j}_{K12} = \phi_1 \boldsymbol{h}_{K2}^* + \phi_2^* \boldsymbol{h}_{K1} = \text{imaginary} \tag{255}$$

For the mutual energy flow the contribution of the surface integral $\oiint_{\Gamma}$ is imaginary *-j = i*, so

$$Q_{K12} = \oiint_{\Gamma}(\phi_1 \boldsymbol{h}_{K2}^* + \phi_2^* \boldsymbol{h}_{K1}) \cdot \hat{n} d\Gamma = \text{real} \tag{256}$$

Therefore both Q_{K11} and Q_{K12} transmit energy, and the transmitted energy would exceed expectations. If the self-energy flow Q_{K11} transmits energy, the mutual energy flow Q_{K12} cannot transmit energy. If the mutual energy flow Q_{K12} transmits energy, the self-energy flow Q_{K11} cannot transmit energy! According to the calculation, if the self-energy flow transmits energy, then the mutual energy flow certainly also transmits energy. But if the self-energy flow does not transmit energy, it does not affect the mutual energy flow transmitting energy. Therefore the author assumes that it is the mutual energy flow, not the self-energy flow, that transmits energy. The correction in (252) is precisely to solve this problem: the author makes the corrected self-energy flow Q_{11} reactive power. In this way Q_{11}, i.e., the corrected self-energy flow, no longer transmits energy.

The following example will verify this. The correction formula from the magnetic field $\boldsymbol{h}_K$ to the magnetic field $\boldsymbol{h}$ is,

$$\boldsymbol{h}^{(r)} = i\boldsymbol{h}_K^{(r)}, \quad \boldsymbol{h}^{(a)} = -i\boldsymbol{h}_K^{(a)} \tag{257}$$

where the superscript (*r*) denotes retarded and (*a*) denotes advanced. The above correction formula is obtained correspondingly according to (84, 85). We have taken into account that the time factor of electromagnetic field theory is $\exp(j\omega t)$, while the time factor of quantum theory is $\exp(-i\omega t)$, so

$$i = -j \tag{258}$$

After this correction,

$$\boldsymbol{j}_{11} = \phi_1^* \boldsymbol{h}_1 = \text{imaginary}, \tag{259}$$

$$Q_{11} = \oiint_{\Gamma}(\phi_1^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma = \text{imaginary} \tag{260}$$

After this correction the self-energy flow Q_{11} becomes reactive power, so it no longer participates in energy transmission. At this time the mutual energy flow

$$\boldsymbol{j}_{12} = \phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1 = \text{real} \tag{261}$$

Since the contribution of the surface integral $\oiint_{\Gamma}$ to the mutual energy flow is imaginary *-j = i*, the mutual energy flow

$$Q_{12} = \oiint_{\Gamma}(\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma = \text{imaginary} \tag{262}$$

This seems to mean that the mutual energy flow also does not transmit energy. But this is not so, because for the mutual energy flow, the wave will collapse. This is because the retarded wave emitted by the source and the advanced wave emitted by the sink will interfere. The result of the interference is that the mutual energy flow density is strengthened on the line connecting the source and the sink, and weakened in other directions. Eventually the mutual energy flow density all converges onto the line connecting the source and the sink. This is equivalent to only a small hole at the center of the surface Γ allowing the mutual energy flow to pass through. Thus we have

$$Q_{12} = \oiint_{\Gamma_{hole}} (\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma = \text{real} \tag{263}$$

The mutual energy flow law is modified to

$$-\int_{V_1} dV(\phi_2^* \tau'_1) = \oiint_{\Gamma_{hole}} (\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1) \cdot \hat{n} d\Gamma = \int_{V_2} dV(\phi_1 (\tau'_2)^*) \tag{264}$$

4.5 Calculating the Mutual Energy Flow Density After Considering Wave Collapse

Considering that after wave collapse both ϕ_1 and ϕ_2 become quasi-plane waves, below the author will calculate them as plane waves. The source and the sink are to emit retarded waves and also advanced waves. But since these retarded and advanced waves are all reactive power, they can be ineffective. A so-called ineffective wave means that although these waves are emitted, no energy is lost. The only situation that truly causes energy transfer is when the retarded and advanced waves form the mutual energy flow; then these waves are effective.

We assume that to the right of the source $\boldsymbol{h}_1$ is a retarded wave, and to the left of the source $\boldsymbol{h}_1$ is an advanced wave. (The advanced wave to the right of the source is ineffective, and the retarded wave to the left of the source is also ineffective.) The source τ'_1 is at position $x = 0$. Therefore,

$$\boldsymbol{h}_1 = \begin{cases} (-i)\boldsymbol{h}_{1K} & x < 0 \\ (i)\boldsymbol{h}_{1K} & 0 \le x \end{cases} = \begin{cases} (-i)\frac{1}{i}\nabla\phi_1 & x < 0 \\ (i)\frac{1}{i}\nabla\phi_1 & 0 \le x \end{cases} = \nabla\phi_1 \begin{cases} -1 & x < 0 \\ 1 & 0 \le x \end{cases} \tag{265}$$

We assume that to the right of the sink τ'_2, $\boldsymbol{h}_2$ is a retarded wave, and to the left of the sink, $\boldsymbol{h}_2$ is an advanced wave. The sink is at position $x = L$. Consistent with the source, the left side is an advanced wave and the right side is a retarded wave; other waves are ineffective. Therefore,

$$\boldsymbol{h}_2 = \begin{cases} (-i)\boldsymbol{h}_{2K} & x < L \\ (i)\boldsymbol{h}_{2K} & L \le x \end{cases} = \begin{cases} (-i)\frac{1}{i}\nabla\phi_2 & x < L \\ (i)\frac{1}{i}\nabla\phi_2 & L \le x \end{cases} = \nabla\phi_2 \begin{cases} -1 & x < L \\ 1 & L \le x \end{cases} \tag{266}$$

The mutual energy flow density is

$$\boldsymbol{j}_{12} = (\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1) = \begin{cases} \phi_1(-\nabla\phi_2)^* + \phi_2^*(-\nabla\phi_1) & x < 0 \\ \phi_1(-\nabla\phi_2)^* + \phi_2^*(\nabla\phi_1) & 0 \le x \le L \\ \phi_1(\nabla\phi_2)^* + \phi_2^*(\nabla\phi_1) & L < x \end{cases} \tag{267}$$

Assume that for $0 \le x \le L$

$$\phi_1 = \exp(ikx) \tag{268}$$

$$\boldsymbol{h}_1 = \nabla\phi_1 = ik\hat{x}\exp(ikx) \tag{269}$$

Formula

$$\boldsymbol{j}_{12} = (\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1) \tag{270}$$

the author assume that both terms of the above equation are real numbers, so the second term means

$$\phi_2 \sim \boldsymbol{h}_1 \tag{271}$$

When this holds, ϕ_1, ϕ_2 are synchronized. From this we obtain,

$$\phi_2 = i\exp(ikx) \tag{272}$$

We assume throughout space

$$\phi_1 = \exp(ikx), \quad \phi_2 = i\exp(ikx) \tag{273}$$

Calculate the mutual energy flow density

$$\boldsymbol{j}_{12} = \begin{cases} \phi_1(-\nabla\phi_2)^* + \phi_2^*(-\nabla\phi_1) & x < 0 \\ \phi_1(-\nabla\phi_2)^* + \phi_2^*(\nabla\phi_1) & 0 \le x \le L \\ \phi_1(\nabla\phi_2)^* + \phi_2^*(\nabla\phi_1) & L < x \end{cases}$$

$$= \begin{cases} \exp(ikx)(-\nabla i\exp(ikx))^* + (i\exp(ikx))^*(-\nabla\exp(ikx)) & x < 0 \\ \exp(ikx)(-\nabla i\exp(ikx))^* + (i\exp(ikx))^*(\nabla\exp(ikx)) & 0 \le x \le L \\ \exp(ikx)(\nabla i\exp(ikx))^* + (i\exp(ikx))^*(\nabla\exp(ikx)) & L < x \end{cases}$$

$$= \hat{x}_1 \begin{cases} \exp(ikx)(-iik\exp(ikx))^* + (i\exp(ikx))^*(-ik\exp(ikx)) & x < 0 \\ \exp(ikx)(-iik\exp(ikx))^* + (i\exp(ikx))^*(ik\exp(ikx)) & 0 \le x \le L \\ \exp(ikx)(iik\exp(ikx))^* + (i\exp(ikx))^*(ik\exp(ikx)) & L < x \end{cases}$$

$$= \hat{x}_1 \begin{cases} (-iik)^* + (i)^*(-ik) & x < 0 \\ (-iik)^* + (i)^*(ik) & 0 \le x \le L \\ (iik)^* + (i)^*(ik) & L < x \end{cases}$$

$$= k\hat{x}_1 \begin{cases} (-ii)^* + (i)^*(-i) & x < 0 \\ (-ii)^* + (i)^*(i) & 0 \le x \le L \\ (ii)^* + (i)^*(i) & L < x \end{cases}$$

$$= k\hat{x}_1 \begin{cases} 1 - 1 & x < 0 \\ 1 + 1 & 0 \le x \le L \\ -1 + 1 & L < x \end{cases}$$

$$= 2k\hat{x} \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le L \\ 0 & L < x \end{cases} \tag{274}$$

Therefore

$$\boxed{\frac{1}{2}\hbar\boldsymbol{j}_{12} = \boldsymbol{p}_1 \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le L \\ 0 & L < x \end{cases}} \tag{275}$$

where

$$\boldsymbol{p}_1 = \hbar k\hat{x} \tag{276}$$

Formula (275) indicates that the mutual energy flow is generated at the source $x = 0$ and annihilated at the sink $x = L$. $\frac{1}{2}\hbar \boldsymbol{j}_{12}$ transmits momentum $\boldsymbol{p}_1 = \hbar k\hat{x}$.

The reader may verify for themselves the self-energy flow before correction,

$$\boxed{\boldsymbol{j}_{K11} = \phi_1 \boldsymbol{h}_{1K}^* = \text{real}, \quad \boldsymbol{j}_{K22} = \phi_2 \boldsymbol{h}_{2K}^* = \text{real}} \tag{277}$$

The mutual energy flow before correction

$$\boxed{\boldsymbol{j}_{K12} = (\phi_1 \boldsymbol{h}_{2K}^* + \phi_2^* \boldsymbol{h}_{1K}) = \text{imaginary}} \tag{278}$$

$$\boxed{Q_{K12} = \oiint_{\Gamma_{hole}} \boldsymbol{j}_{K12} \cdot \hat{n} d\Gamma = \text{real}} \tag{279}$$

After correction

$$\boxed{\boldsymbol{j}_{11} = \phi_1 \boldsymbol{h}_1^* = \text{imaginary}, \quad \boldsymbol{J}_{22} = \phi_2 \boldsymbol{h}_2^* = \text{imaginary}} \tag{280}$$

$$\boxed{\boldsymbol{j}_{12} = (\phi_1 \boldsymbol{h}_2^* + \phi_2^* \boldsymbol{h}_1) = \text{real}} \tag{281}$$

$$\boxed{Q_{12} = \oiint_{\Gamma_{hole}} \boldsymbol{j}_{12} \cdot \hat{n} d\Gamma = \text{real}} \tag{282}$$

Therefore, if one does not correct the magnetic field entirely within the framework of the K-G equation, there is the problem that both the self-energy flow and the mutual energy flow transmit energy. After correcting the magnetic field of the K-G equation, the self-energy flow becomes reactive power, and the mutual energy flow, after considering wave collapse, still remains active power.

Therefore, like the Schrödinger equation, the K-G equation can directly yield the mutual energy flow theorem, but this mutual energy flow theorem is realized together with the self-energy flow theorem (similar to Poynting's theorem in electromagnetic field theory). This leads to both the self-energy flow and the mutual energy flow transmitting energy, so the transmitted energy exceeds expectations. The way to overcome this contradiction is to modify the theoretical model so that the self-energy flow becomes reactive power, and thus the self-energy flow no longer transmits energy. The transmission of energy is uniquely realized by the mutual energy flow density. This is not all: the mutual energy flow must also collapse, collapsing onto the line from the source to the sink. If the mutual energy flow does not collapse, when the mutual energy flow density is real, the mutual energy flow is imaginary. Therefore collapse of the mutual energy flow is necessary. In this respect we explain the wave collapse in quantum mechanics. Both the retarded and advanced waves in the mutual energy flow collapse into quasi- (a beam of) plane waves. At this point, and readers may still be confused. They may ask, what is the basis for making revisions to their K-G equation? This is because the author believes that for quantum phenomena such as the K-G equation, the physical laws are not only that the solution satisfies the K-G equation, but more importantly, that the self energy flow is reactive power and the mutual energy flow is active power, and the law of mutual energy flow must be satisfied. The law of mutual energy flow (264) is the most important among them. To meet these two requirements, we need to establish a magnetic field for the K-G equation and correct the phase of this magnetic field. All the reasons for doing this are completely similar to what the author has done in electromagnetic theory.

5. Conclusion

For a long time, the K-G equation, the Schrödinger equation, and Maxwell's equations all regarded the self-energy flow as the carrier of energy transfer. The author points out that this traditional understanding has limitations. The author inherits Wheeler and Feynman's absorber theory, holding that the sink receives the energy of the wave by emitting an advanced wave. And the author maintains that the mutual energy flow is the essence of energy flow. Within the existing K-G equation theoretical system, the result derived is that both the self-energy flow and the mutual energy flow transmit energy. This is incorrect, because if the energy transmitted by the self-energy flow has already transmitted sufficient energy, then if the mutual energy flow also transmits energy, the transmitted energy would exceed

expectations.

This paper achieves a theoretical breakthrough by introducing a magnetic field for the K-G equation and correcting the phase of its magnetic field. After the correction, the self-energy flow exhibits the property of reactive power, and the mutual energy flow density is real, but the mutual energy flow becomes reactive power. However, after we introduce the concept of wave collapse, the mutual energy flow exactly remains consistent with the mutual energy flow density, so the mutual energy flow is then active power. Therefore the mutual energy flow transmits energy. The mutual energy flow is composed of retarded and advanced waves, generated at the source and annihilated at the sink, with obvious particle properties, and the mutual energy flow is composed of retarded and advanced waves, so it also has wave properties. Therefore the mutual energy flow can perfectly explain wave-particle duality.

This paper also verifies this through practical calculation examples. At this point, the mutual energy flow theory proposed by the author has completely covered the broad field from electromagnetic fields, the Schrödinger equation, to the K-G equation. This theory not only corrects the deviation of classical flow theory, but also lays a solid physical foundation for perfectly interpreting the wave-particle duality of particles.

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