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Review Article

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Computing Sticks against Random Walk

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Abstract

A new deterministic model with the help of geometric constructions and computing sticks (not related to trajectories) is proposed for the new justification of consistency of the probabilistic approach to explain the random walk on a plane. A new, stepped form of the arithmetic triangle of Pascal based on the construction of horizontal and vertical lines (arrows) is suggested, a comparison is made with Pascal's triangle of the usual form. A two-sided generalization of Pascal's triangle is proposed. Geometric constructions and formulas for calculating the coefficients that fill in these new geometric (arithmetic) figures are given. Further types of generalization of the step-shaped Pascal triangle are proposed. Examples of generalized initial conditions and generalized recursive formulas for constructing various types of a generalized Pascal triangle are given.

Keywords: Stochastics, Probabilistic Model, Deterministic Model, Pascal's Triangle, Geometrization of Physics

Introduction

The arithmetic triangle of Pascal has been known since ancient times. Figure 1 shows the image of this triangle taken from the ancient Chinese manuscript [1]:

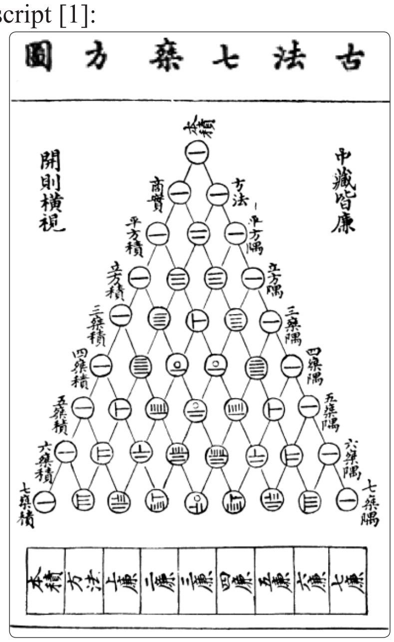

Figure 1: The Pascal triangle from the ancient Chinese manuscript

In [2-9], we carried out studies of Pascal's triangle, its analogues, generalizations, and possible applications of this visual geometric model. In the paper we proposed a new, stepped form for Pascal's triangle and its bilateral generalization [10].

In the proposed paper we suggest various recursive formulas for calculating the step-shaped Pascal triangle and various initial conditions for describing different processes. We compare our geometric constructions with the well-known random walk problem. However, our geometric constructions cannot be considered as a random or stochastic process. We confirm this graphically and by directly computing the number of countering sticks on which students begin to study mathematics in the first grade of elementary school.

Building Pascal's triangle in the form of oblique lines

Usually the arithmetic triangle of Pascal [1, 11, and 12] is depicted in the form of oblique intersecting lines (arrows). At the intersection of the lines are numbers arranged in rows. The number in the next row is the sum of the two numbers of the previous row. Rows of numbers are usually located in one direction from top to bottom [1, 11] and Figure 1 or from left to right and Figure 2 [12].

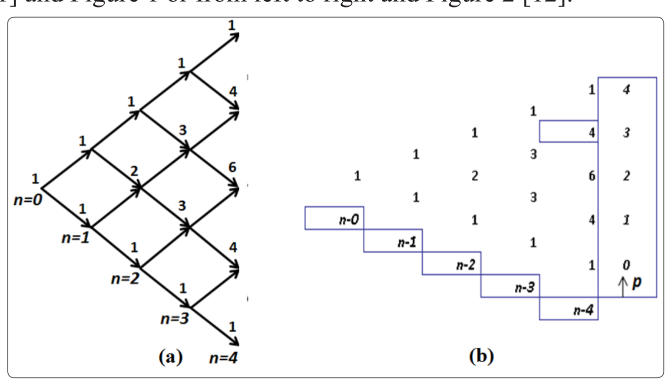

Figure 2: The first four rows of the Pascal triangle constructed with the help of oblique lines (arrows): General view (a); separately written numbers of Pascal's triangle (b).

The rows in the triangle in this example are denoted by n: n=0,1,2,… , the numbers in the row are denoted by p: p=0,1,2,…,n.

The numbers in the Pascal triangle (Figure 2b) are binomial coefficients $\binom{n}{p}$ or combinations C_n^p; they can be found [1, 11, and 12] using the recursive expression:

$$C_n^p = \binom{n}{p} = \binom{n-1}{p-1} + \binom{n-1}{p}. \qquad (1)$$

It is necessary to specify the numbers of the zero rows (n=0) or in other words the initial conditions:

$$C_0^p = \binom{0}{p} = 1 \text{ for } p = 0 \text{ and } C_0^p = \binom{0}{p} = 0 \qquad (2)$$

for other values of p.

To find the binomial coefficients [1, 11, 12], you can also use the Newton formula:

$$C_n^p = \binom{n}{p} = \frac{n!}{p!(n-p)!}. \qquad (3)$$

For example for n=4 and p=3 we have: $\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{24}{6*1} = 4.$

(In Figure 2b $\boxed{4}$ is at the intersection of n=4 and p=3.)

The sum of coefficients in the row n is 2^n.

Stepped Pascal's triangle

Figure 3.1 shows the sequential (row by row) construction of Pascal's stepped triangle using ordinary computing sticks from which students begin to study mathematics in elementary school:

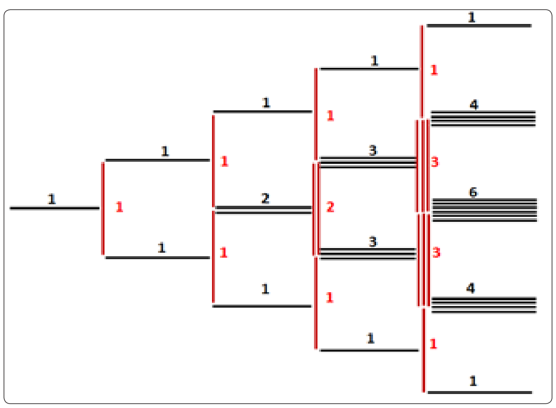

Figure 3.1: The first rows of a stepped Pascal triangle constructed with the help of computing sticks

To build better it is possible to use the computing sticks of two colors. In Figure 3.1 the black sticks are horizontal (or along the meridian), and the red ones are vertically (or along the parallel). Black and red sticks are arranged in vertical alternating rows. Instead of computing sticks you can use ordinary sticks of the same length or pencils. The ends of the sticks do not differ from each other.

For the case shown in Figure 3.1 only black sticks are successively summed. The number of black sticks in a row is summed up according to a simple law: if the ends of red sticks look at each other; then in the next row the number of black sticks is summed up; if the ends of red sticks do not look at each other then the number of black sticks repeats the number of red sticks of the previous row. The number of red sticks is not summarized but repeats the number of black sticks of the previous row.

In the ancient manuscript in Figure 1 the numbers of an arithmetic triangle are also indicated with sticks (horizontal dashes) in circles. The quantity of sticks in our construction and in the manuscript is the same (especially in the initial stage of construction, with further construction in the manuscript a combination of horizontal, vertical dashes and circles denote Chinese numbers).

In our constructions in Figure 3.1 using computing sticks we used a simple but strict pattern (construction algorithm) so the random walk [12, 13], fluctuations, stochastic and probabilistic processes were not observed. At the same time with the help of our new deterministic model of Pascal's triangle and computing sticks not associated with trajectories and one can geometrically substantiate the consistency of the probabilistic description of a random walk along a straight line [12]. On the contrary by considering the random walk it is impossible to substantiate the geometric construction of the Pascal triangle.

Figure 3.2 shows a new, stepped form of Pascal's triangle (the same as in Figure 3.1) built with the help of horizontal and vertical lines (arrows):

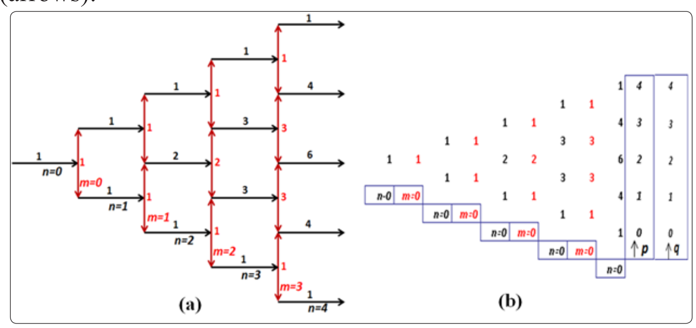

Figure 3.2: The first four whole rows (consist of five half rows n and four half rows m) of a stepped Pascal triangle constructed with the help of horizontal and vertical lines (arrows): general view (a); separately written numbers of stepped triangle (b).

For clarity, the horizontal lines (arrows) are colored (Figures 3.1) black and have only one direction: from left to right, and the vertical lines (arrows) are colored red and have two directions: top to bottom and bottom to top. The black and red arrows are arranged one after the other and form the step-shaped Pascal triangle.

Rows in our model are of two kinds. Let's call them "whole rows" and "half rows". Whole rows consist of two half rows. Whole rows in the stepped triangle form both by black and red arrows. The rows of black arrows are half rows, denoted by n: n=0,1,2,…. The numbers in this half row are indicated by p: p=0,1,2,…,n. The rows of red arrows are also half rows, denoted by m: m=0,1,2,…. The numbers in this half row are indicated by q: q=0,1,2,…,m. Numbers in Pascal's stepped triangle (Figure 3.2b) are also like in the usual Pascal triangle (Figure 2b); binomial coefficients can be found by using the expressions (1) and (3) for n and using the expressions (4) and (5) for m:

$$\binom{m}{q} = \binom{m-1}{q-1} + \binom{m-1}{q}, \quad (4)$$

$$C_m^q = \binom{m}{q} = \frac{m!}{q!(m-q)!}. \quad (5)$$

In principle, expressions (1, 3) and (4, 5) are identical and binomial coefficients characterized by black and red arrows coincide (Figure 3.2).

Bilateral generalization of stepped Pascal's triangle

Geometric construction

In the previous paragraphs we considered the construction of one-sided forms of the Pascal triangle. Figure 4.1 shows the sequential construction of the stepped generalized two-sided form of Pascal's triangle.

Building as in Figure 3.1 carried out using horizontal and vertical computing sticks:

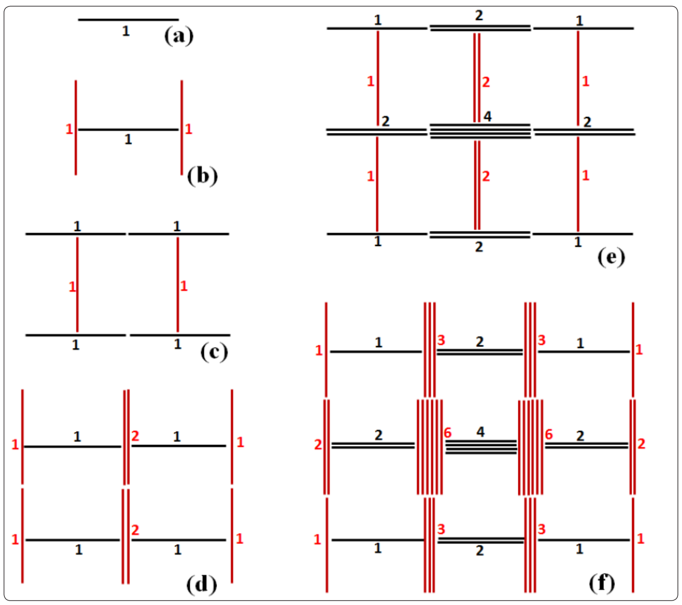

Figure 4.1: The first rows of a stepped generalized two-sided Pascal triangle constructed using horizontal and vertical computing sticks. For clarity each fragment of the picture (a - f) contains two rows; repeating rows of black horizontal or red vertical sticks.

For the case presented in Figure 4.1 both black and red sticks are successively summed. The number of black (or red) sticks in a row matrix is summed up according to a simple law: if the ends of black (or red) sticks look at each other than in the next row the number of red (or respectively black) sticks is added up, if the ends of black (or respectively red) sticks do not look at each other than in the next row the number of red (or respectively black) sticks repeats the quantity of black (or respectively red) sticks of the previous row.

In our constructions in Figure 4.1 with the help of computing sticks we used a simple but strict pattern (construction algorithm) so the random walk [12, 13], fluctuations, stochastic and probabilistic processes [12, 13], were not observed. At the same time using new deterministic model of the stepped generalized two-sided form of Pascal's triangle, and non-trajectory computing sticks one can geometrically substantiate the consistency of the probabilistic description of a random walk on a plane [12, 13]. On the contrary by considering a random walk on a plane it is impossible to substantiate the geometric construction of Pascal's triangle.

Figure 4.2 shows a new, stepped generalized two-sided form of Pascal's triangle (the same as in Figure 4.1) built with the help of horizontal and vertical lines (arrows):

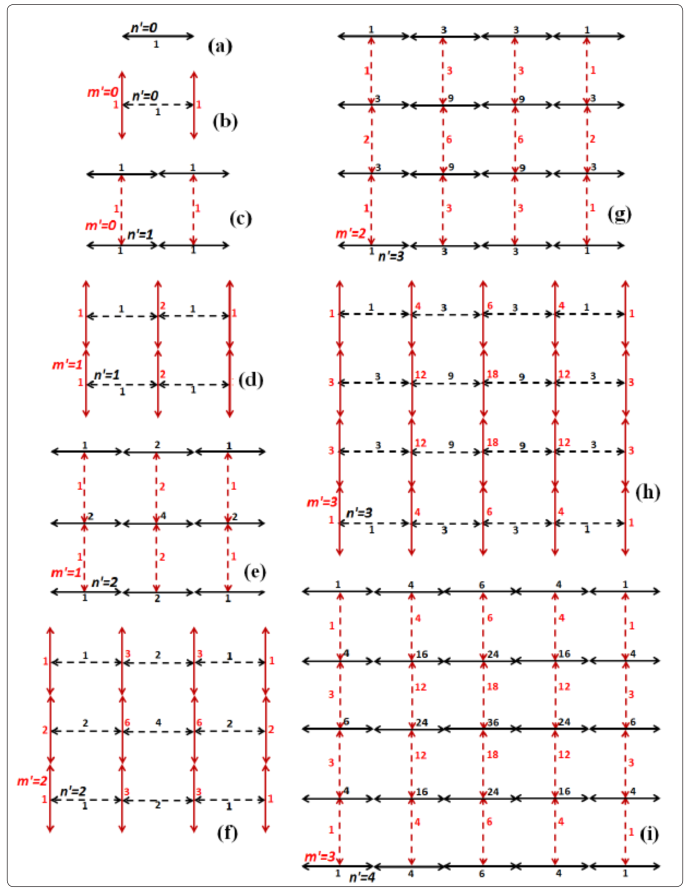

Figure 4.2: The first four whole rows (consisting of five half rows n' and four half rows m') of a stepped generalized two-sided Pascal triangle constructed using horizontal and vertical lines (arrows). For clarity, each fragment of the picture (a - i) contains two rows; repeating rows of black horizontal or red vertical arrows.

In the example shown in Figure 4.2 all arrows have two directions: horizontal black arrows have a direction from left to right and right to left and vertical red arrows have a direction from top to bottom and bottom to top.

The black and red arrows are located one after another and form a step-shaped generalized two-sided Pascal's triangle. The numbers with black arrows are successively summed as in the usual Pascal triangle. The sum of two numbers with two black arrows gives the number with the red arrow; the sum of two numbers with two red arrows gives the number with a black arrow, and so on. The previous rows in Figure 4.2 are shown by dotted arrows.

Let us write down successively the numbers shown in Figure 4.2 (a - i) in the form of the corresponding tables (matrices) shown in Figure 5 (a - i):

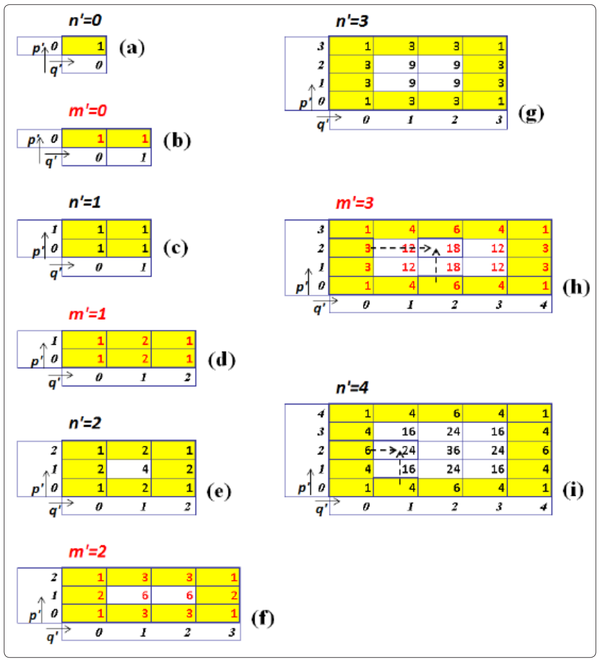

Figure 5: Numbers corresponding to the first four whole rows (consisting of five half rows n' and four half rows m') of a stepped generalized Pascal's double-sided triangle. For clarity the numbers in the fragments of the Figure a, c, e, g, i corresponding to half rows n' black, and the numbers in the fragments of the Figure b, d, f, h corresponding to half rows m' red. The dashed arrows in h and i show the calculation results given below in examples (Equation 9) and (Equation 12). Yellow highlighted areas filled with the usual binomial coefficients of the Pascal triangle.

Our generalized two-sided triangle consists (as one-sided triangle shown in Figure 3.2) of whole rows (matrices) which in turn consist of two half rows. The first whole row consists of two half rows: n'=0, m'=0, the second row of: n'=1, m'=1, the third row of: n'=2, m'=2, etc. Numbers located in half rows (matrices) are indicated by a pair of numbers p' and q'. For the half rows n': p'=0,1,2,…,n'; q'=0,1,2,…,n'. For the half rows m': p'=0,1,2,…,m'; q'=0,1,2,…,m'+1.

All n'- rows and m'- rows can be denoted as k - rows. In this case k=2n' will be even rows and k=2m'+1 odd rows.

Note that n'- rows (n'- matrices) have a square shape, and m'- rows (m'- matrices) have a rectangular shape. The sum of coefficients in the row n'(matrix n') is $2^{2n'}$. The sum of coefficients in the row m' (matrix m') is $2^{2m'+1}$).

Figure 6 shows three-dimensional histograms of two half rows (matrices) shown in Figure 5h, i:

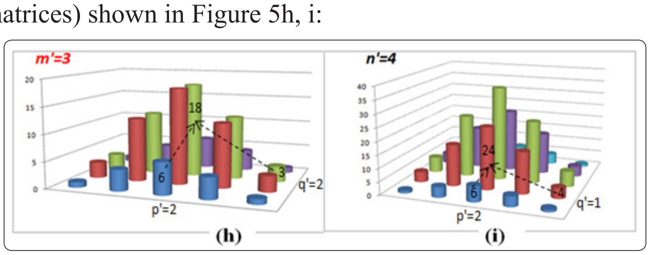

Figure 6: Three-dimensional histograms of half rows (matrices) shown in Figure 5h, i. The dashed arrows show the calculation results given below in examples (Equation 9 and 12).

Calculation formulas and initial conditions

we write out the calculation formulas (similar to the formulas (1, 3, 4, and 5)) and the initial conditions (similar to conditions (2)) for our bilateral generalized stepped Pascal's triangle:

Denote a number located in the n' - half row (in the n' - matrix) as $\begin{pmatrix} n' \\ p' \\ q' \end{pmatrix}$ then we write the recursive expression to calculate it:

$$\begin{pmatrix} n' \\ p' \\ q' \end{pmatrix} = \begin{pmatrix} m'-1 \\ p'-1 \\ q' \end{pmatrix} + \begin{pmatrix} m'-1 \\ p' \\ q' \end{pmatrix}. \qquad (6)$$

It is necessary to specify the numbers of the zero row (n=0) or the initial conditions:

$$\begin{pmatrix} 0 \\ p' \\ q' \end{pmatrix} = 1 \text{ for } p'=0, q'=0 \text{ and } \begin{pmatrix} 0 \\ p' \\ q' \end{pmatrix} = 0 \qquad (7)$$

for other values of p' and q'.

The generalized Newton formula for our generalized Pascal triangle will be:

$$\begin{pmatrix} n' \\ p' \\ q' \end{pmatrix} = \binom{n'}{p'} * \binom{n'}{q'} = \frac{n'!}{p'!(n'-p')!} * \frac{n'!}{q'!(n'-q')!}. \qquad (8)$$

For example for n'=4, p'=2, q'=1 we have:

$$\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = \binom{4}{2} * \binom{4}{1} = \frac{4!}{2!(4-2)!} * \frac{4!}{1!(4-1)!} = \frac{24}{4} * \frac{24}{6} = 24. \qquad (9)$$

(It is shown by dotted arrows in Figure 5i and Figure 6i.)

Denote the number located in the m' - half row (in the m'- matrix) as $\begin{pmatrix} m' \\ p' \\ q' \end{pmatrix}$ then we write the recursive expression to calculate it:

$$\begin{pmatrix} m' \\ p' \\ q' \end{pmatrix} = \begin{pmatrix} n' \\ p' \\ q'-1 \end{pmatrix} + \begin{pmatrix} n' \\ p' \\ q' \end{pmatrix}. \qquad (10)$$

The generalized Newton formula for our generalized Pascal triangle will be:

$$\begin{pmatrix} m' \\ p' \\ q' \end{pmatrix} = \binom{m'}{p'} * \binom{m'+1}{q'} = \frac{m'!}{p'!(m'-p')!} * \frac{(m'+1)!}{q'!(m'+1-q')!}. \qquad (11)$$

For example for m'=3, p'=2, q'=2 we have:

$$\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} = \binom{3}{2} * \binom{4}{2} = \frac{3!}{2!(3-2)!} * \frac{4!}{2!(4-2)!} = \frac{6}{2} * \frac{24}{4} = 18. \qquad (12)$$

(It is shown by dotted arrows in Figure 5h and Figure 6h.)

From the joint consideration of expressions (6) and (10) we can obtain an expression for the successive construction of our generalized triangle using square matrices only for n'- series:

$$\begin{pmatrix} n' \\ p' \\ q' \end{pmatrix} = \begin{pmatrix} n'-1 \\ p'-1 \\ q'-1 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p'-1 \\ q' \end{pmatrix} + \begin{pmatrix} n'-1 \\ p' \\ q'-1 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p' \\ q' \end{pmatrix}. \quad (13)$$

Numerical examples and generalizations

Example 1

Let us give an illustrative numerical example performed in MS Excel using the formula (13). For clarity we repeat the example given in Figures 5, 6 but we assume that in the initial conditions p' and q' are increasing series of numbers starting not necessarily from zero but our initial unit in the row n'=0 (in a square matrix n'=0) is located in the central part of this matrix in Figure 7:

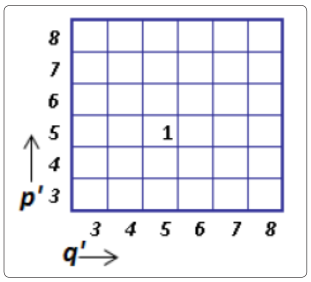

Figure 7: Graphic description of the placement of the initial conditions of the zero row (n'=0) in the central part of the square matrix p'=q'=5.

The terms on the right-hand side of expression (13) can be placed in each of the four small squares (as in MS Excel program) of the square matrix:

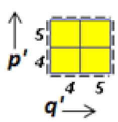

Figure 8: Graphic description of expression (13). Placing the right-hand side of expression in a square-shaped matrix. Each of the four cells contains all four components of the right side of expression (13). Recursive formulas (13) are placed in all rows (matrices) starting with the first: n'=1,2,3,….

The results of the calculations (similar to those shown in Figure 5a, c, e, g, i, and Figure 6i) are shown in histograms in Figure 9:

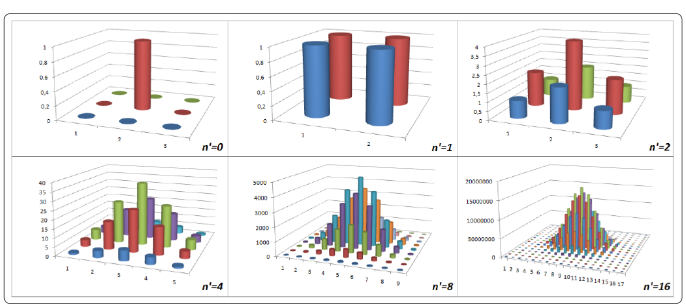

Figure 9: Three-dimensional histograms of n'-rows (n'-matrices) in accordance with the initial conditions (7) and Figure 7 as well as the recursive expression (13) and Figure 8.

In fact in Example 1 using the geometrical construction of the generalized Pascal triangle (deterministic model) we obtained the same results for n'- rows as we can constructed using the probability theory (probability model) for a random walk and Brownian motion on the plane of the square [12, 13]. However in the previous paragraphs 3.1 and 3.2 we considered the second case for m' - rows corresponding to a random walk on the plane of the rectangle.

Example 2

Let us give an illustrative numerical example performed in MS Excel using the more complex (compared to expression (13)) formula (14):

$$\begin{aligned} \begin{pmatrix} n' \\ p' \\ q' \end{pmatrix} = & \begin{pmatrix} n'-1 \\ p'-1 \\ q'-1 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p'-1 \\ q' \end{pmatrix} + \begin{pmatrix} n'-1 \\ p' \\ q'-2 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p' \\ q'-1 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p' \\ q' \end{pmatrix} + \begin{pmatrix} n'-1 \\ p' \\ q'+1 \end{pmatrix} + \\ & \begin{pmatrix} n'-1 \\ p'+1 \\ q'-2 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p'+1 \\ q'-1 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p'+1 \\ q' \end{pmatrix} + \begin{pmatrix} n'-1 \\ p'+1 \\ q'+1 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p'+2 \\ q'-1 \end{pmatrix} + \begin{pmatrix} n'-1 \\ p'+2 \\ q' \end{pmatrix} \end{aligned} \quad (14)$$

The terms on the right-hand side of expression (14) can be placed in each of the twelve small squares of the matrix (as in MS Excel program) which is close in shape to the octagon:

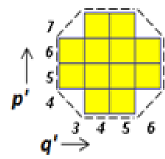

Figure10: Graphic description of expression (14). Placing the right side of expression (14) in a matrix that is close in shape to the octagon (the octagon in turn is closer to the circle than the square in Figure 8): in each of the twelve cells all twelve terms of the right side of expression (14) are placed. Recursive formulas (14) are placed in all rows (matrices) starting with the first: n'=1,2,3,….

We assume the initial conditions are the same as in Example 1 (Figure 7).

The calculation results are shown in histograms in Figure 11:

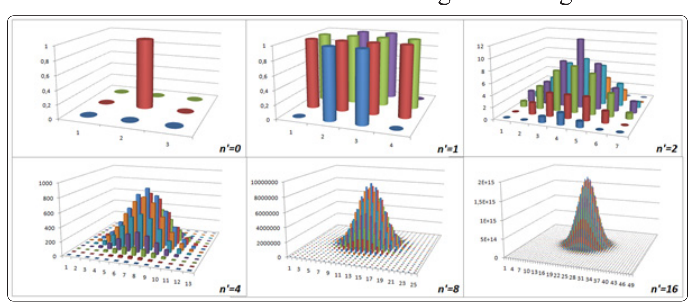

Figure 11: Three-dimensional histograms of n'- rows (n'- matrices) in accordance with the initial conditions (7) and Figure 7 as well as the recursive expression (14) and Figure 10.

From consideration of Examples 1 and 2 it can be seen that in the sequential construction of Example 1 we obtain three-dimensional figures at the base of which lies a square. In Example 2 the base of the figure is an octagon closer in shape to a circle than a square.

Using the method of construction of bilateral generalization stepped Pascal's triangle we can more accurately describe different processes if we generalize the initial conditions (expression (7), Figure 7) and recursive formulas (expressions (13) and (14), Figures 8 and 10).

Figures 12 and 13 shows a graphic representation of examples of generalized initial conditions and generalized formulas respectively:

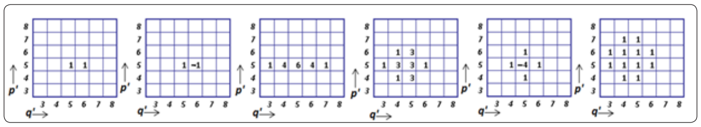

Figure 12: Examples of graphical descriptions of generalized initial conditions

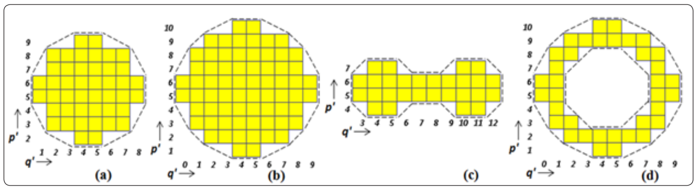

Figure 13: Examples of a graphic description of generalized formulas: a figure close to a dodecagon (a dodecagon is closer in shape to a circle than the octagon in Figure 10) (a); a figure close to a 16-square (a 16-square in turn is closer in shape to a circle than a dodecagon) (b); dumbbell-shaped figure (c); ring (d).

We can continue the generalization of recursive formulas; if before the terms for example in the expression (13) additionally put some coefficients:

$$\begin{pmatrix} n' \\ p' \\ q' \end{pmatrix} = a * \begin{pmatrix} n'-1 \\ p'-1 \\ q'-1 \end{pmatrix} + b * \begin{pmatrix} n'-1 \\ p'-1 \\ q' \end{pmatrix} + c * \begin{pmatrix} n'-1 \\ p' \\ q'-1 \end{pmatrix} + d * \begin{pmatrix} n'-1 \\ p' \\ q' \end{pmatrix}, \quad (15)$$

where for example: a=2,b=3,14,c=-1,d=5, and so on.

It is possible to carry out similar constructions for m'- series of generalized cases, and sequences of alternating series: n'=0, m'=0, n′=1, m'=1, n'=2,…, as shown in Figure 5.

Conclusions

Thus, the formulas for describing the two-sided stepped generalized Pascal triangle turned out to be quite simple because our visual geometric deterministic models are simpler than probabilistic models [12, 13]. On the basis of our new model of bilateral stepped Pascal triangle a substantiation (not associated with trajectories) of the consistency of the probabilistic approach is given to explain the random walk on a plane using geometric constructions and computing sticks for children.

Perhaps our new geometric constructions and recursive formulas will find application to understand the development of processes in biology in optics and acoustics and also in other areas for example in technology [8, 14 and 15].

In our work, we referred [12] only to the great Russian mathematician Andrey Kolmogorov. However, A. Kolmogorov solved only half the problem of random walk and Brownian motion. The great American mathematician Norbert Wiener [16], who perfectly knew Russian and another 20 different languages, went to the Soviet Union to help Kolmogorov. Unfortunately, the historical fact is that A. Kolmogorov did not accept N. Wiener, therefore the problem of random walk and Brownian motion was not fully resolved. The science of Cybernetics was recognized in the Soviet Union as pseudoscience.

Acknowledges

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