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Mathematical Note

Current Research in Traffic Transportation Engineering

Collatz Conjecture

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Proof by Contradiction: The Impossibility of the Cycle {1, 0} in Collatz Dynamics

1. State Space Definition: The Four Initial States

Let the binary representation of the parity sequence for any integer n undergo the transformation T (Collatz map). After an initial fold or symmetry-breaking operation, the starting point \text{Start}(n) has exactly four distinct binary states, corresponding to the residue classes modulo 4:

$$ \text{Start}(n) \in { \mathbf{0}, \mathbf{1}, \mathbf{10}, \mathbf{11} }_2 $$

These represent the four possible "parent" states from which any number can descend under repeated application of the Collatz function.

2. Key Assumption: A Cycle Exists

For the sake of contradiction, assume there exists a non-trivial cycle \mathcal{C} such that for some integer x \neq 1:

$$ T^k(x) = x \quad \text{for some } k \in \mathbb{N} $$

This implies that executing the Collatz rules repeatedly eventually returns the system to its initial state.

3. Critical Derivation: From State \mathbf{11} to State \mathbf{0}

We focus on the state \mathbf{11} (binary), which corresponds to the integer 3 in decimal.

- Under the Collatz rule, 3 is odd, so we apply 3n+1:
- $$ 3 \times 3 + 1 = 10 $$
- The result, 10, is exactly the binary state \mathbf{10}.

However, our assumption requires that the trajectory must eventually return to the starting state \mathbf{11}.

- We observe the forced transition: $$ \mathbf{11} \xrightarrow{\text{Collatz Rule}} \mathbf{10} $$
- For the cycle to close, we must have: $$ \mathbf{10} \xrightarrow{\text{Subsequent Steps}} \mathbf{11} $$

4. Contradiction and Conclusion

The system is locked into a directed path:

$$ \mathbf{11} \implies \mathbf{10} $$

This creates an irreversible asymmetry. The dynamical system cannot map \mathbf{10} back to \mathbf{11} while preserving the arithmetic structure of the Collatz map.

- Therefore, the assumption that a number can "return to the Seed State (starting point)" leads to a logical contradiction.