

## Coexistence of Relativity between Observers and the Absoluteness of Inertial Systems

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**Abstract**

Einstein derived the Lorentz and its inverse transformations based on the principles of the relativity of the laws of physics and the constancy of light's speed. However, only the principle of the constancy of light's speed can induce the Lorentz and its inverse transformations, indicating that the relativity of the laws of physics arises from the constancy of light's speed. Einstein also assumed that the relativity of inertial systems further establishes the relativity of the laws of physics. However, using light and rigid rulers together enables distinguishing between rest and constant-velocity systems (called the absoluteness of inertial systems): if the lengths measured by the rigid and light rulers are the same, it is a rest system; otherwise, it is a constant-velocity system. This study presents new interpretations of the twin paradox and Michelson–Morley experiment to explain the coexistence of the relativity between observers and the absoluteness of inertial systems.

**Keywords:** Special Relativity, Relativity between Observers, Absoluteness of Inertial Systems, Rest System, Constant-Velocity System, Twin Paradox, Michelson–Morley Experiment

**Introduction**

Einstein formulated the principle of special relativity and derived the Lorentz transformation and its inverse using the following two axioms: relativity of the laws of physics and the principle of the constancy of the velocity of light [1]. However, he did not physically prove the mutual independence of these axioms; he merely postulated that the relativity of the laws of physics is established from the relativity of inertial systems [2]. The relativity of an inertial system signifies that the physical methods applied in inertial, reference, and motion systems are indistinguishable. Since Einstein did not physically prove the relationship between the relativities of the laws of physics and inertial systems, it is important to investigate the physical ideas behind the relativity of the laws of physics.

The concept of relativity was introduced by Galileo Galilei, who laid the foundations of classical mechanics. He mentioned the principle of relativity in *Dialogue Concerning the Two Chief World Systems*, published in 1632. Using “a thought experiment in a ship,” he argued that the motion of an object in a ship is independent of the ship's velocity [3]. His principle of relativity, which holds for the inertial system defined later by Newton, describes characteristics similar to those defined by laws of mechanics for observers moving at constant speed with respect to each other. The law of inertia states that an object at rest remains at rest, and an object in motion always moves in the same direction at a constant speed unless an external force acts on it. In his book, Newton divided time-space into absolute time and absolute space and their relative counterparts. He argued that

absolute time and space are immeasurable and that only relative time and space are measurable quantities [4].

The Galilean transformation and its inverse are described using the following framework from classical mechanics. With respect to a reference system  $O$  at rest, a dynamic motion system  $O_0'$  moves at a constant velocity  $v$  in the  $x$ -direction. For an observer at  $P(x,y,z,t)$  in the reference system, the coordinates  $P''(x',y',z',t')$  of an observer in the motion system are expressed as

$$x'=x-vt, y'=y, z'=z, \text{ and } t'=t, \quad (1)$$

Which is referred to as the Galilean transformation.

Conversely, in the system  $P''(x',y',z',t')$ , the coordinates  $P(x,y,z,t)$  are expressed as

$$x=x'+vt, y=y', z=z', \text{ and } t=t', \quad (2)$$

Which is referred to as the inverse Galilean transformation.

In measuring the Galilean transformation and its inverse, observers in the reference and motion systems use a rigid ruler and the mechanical clock of the reference system.

Advancements in electromagnetism led to a reanalysis of the concept of relativity from a new perspective. In 1864, the British physicist J. C. Maxwell discovered that light is a type of electro-

magnetic wave that can be described using four basic equations, providing a complete description of electromagnetism. According to Maxwell's theory, the speed of light is  $c = 1/\sqrt{\varepsilon_0\mu_0}$  where  $\varepsilon_0$  and  $\mu_0$  denote the permittivity and permeability of vacuum (free space), respectively [5]. The establishment of light as a wave led physicists of the time to hypothesize that it was transmitted through a medium called the ether that was stationary in space. To verify this hypothesis and detect the presence of this stationary ether, an experiment based on the laws of classical mechanics using interference between light waves was performed in 1886 by Michelson and Morley. However, the experiment could detect none of the interference phenomena predicted by the laws of classical mechanics [6].

In 1895, Fitzgerald and Lorentz formulated the Fitz Gerald–Lorentz contraction to explain the results of the Michelson–Morley experiment. However, their report did not gain the attention of physicists at that time [7]. In 1899, Lorentz suggested that expressing the laws of electromagnetism in the same form in different inertial systems would require the use of different temporal frames and derived the Lorentz transformation by adding equations for time and length transformations based on the fact that the speed of light is invariant under the transformations of Maxwell's equations [8].

Lorentz discovered Lorentz's and its inverse transformation during his research related to electromagnetism, and those are described as follows. With respect to a reference system  $O$  at rest, a moving system  $O'$  moves with a constant velocity  $v$  in the  $x$ -direction. For an observer at  $P(x,y,z,t)$  in the reference system, the coordinates of an observer in the moving system  $P'(\xi,\eta,\zeta,\tau)$  are expressed as

$$\xi=k(x-vt), \eta=y, \zeta=z, \text{ and } \tau=k(t - vx/c^2), \quad (3)$$

Which is referred to as the Lorentz transformation.

Conversely, in the coordinate system  $P'(\xi,\eta,\zeta,\tau)$ , the coordinates  $P(x,y,z,t)$  are expressed as

$$x=k(\xi+v\tau), y = \eta, z = \zeta, \text{ and } t=k(\tau+v\xi/c^2), \quad (4)$$

Which is referred to as the inverse Lorentz transformation.

Under the Lorentz transformation and its inverse, observers in the reference and motion systems use light as a ruler, whereas the observer in the reference system uses a rigid ruler.

In 1905, Einstein postulated the relativity of the laws of physics and the principle of the constancy of the speed of light as axioms, from which the Lorentz transformation and its inverse were derived. He suggested that the Galilean transformation could be regarded as an approximation of the Lorentz transformation because the speed of light,  $c$ , is very large [9]. However, the Galilean transformation is not strictly an approximation of the Lorentz transformation, as transformations and their inverses differ if the observers use different tools. The Galilean transformation and its inverse hold because the observers in both systems use

a rigid ruler. Thus, the principle of the constancy of a rigid rod can be applied to the Galilean transformation. By contrast, the Lorentz transformation and its inverse are established because the observers in different systems use light and atomic clocks (the atomic clock had not been invented when the theory of special relativity was presented). In the Lorentz transformation, the principle of the constancy of the speed of light is applied. Herein, the principles of constancy of rigid rod (or the principles of constancy of a rigid ruler) and the speed of light are used as axioms to derive the relativity of physical laws premised on special relativity from the principle of the constancy of the speed of light. Using this framework, we further attempt to reveal the absoluteness, as opposed to the relativity of inertial systems.

In the Methodology section, we define two axioms related to rigid rods, and light and atomic clocks are used to construct reference and motion systems. The principle of the constancy of a rigid rod is necessary to measure the distance between two points using a rigid ruler, and the principle of the constancy of the speed of light is necessary to synchronize the clocks. For the reference system, two axioms are defined: the length of a rigid rod is invariant regardless of the position of the rod, and the speed of light measured through experiments in which light travels back and forth on a rigid rod is invariant. For the motion system, two axioms are defined: if a rigid rod in the motion system overlaps a rigid rod in the reference system, the lengths of the two rods are the same, and the speed of light in the motion system is calculated using the speed  $c$  of light in the reference system and the concept of the vicinity. As the light in the reference system and that in the vicinity within the motion system arrive at the reference system observer and the motion system observer, the two observers measure the speed of light as  $c$  regardless of the light source [10].

In the Results and Discussion section, it is shown that, through the application of the two axioms, relativity between observers and the absoluteness of inertial systems coexist. As the reference and motion system observers both use light and atomic clocks, the Lorentz transformation and its inverse can be induced by applying only the principle of the constancy of the speed of light. This is referred to as relativity between observers in the Lorentz transformation and its inverse; here, relativity between observers implies that the resting and constant-velocity systems are distinguishable, with observers saying that one is stationary and the other appears to move. This establishment of relativity between observers naturally leads to the relativity of the laws of physics. It is further proved that the relativities of length and time in the Lorentz transformation and its inverse can be derived using only light.

It is shown that the relativity of inertial systems in special relativity does not hold but that inertial system absoluteness does. On the premise of the relativity of inertial systems, Einstein assumed that a motion system observer could use a rigid ruler, light, and a clock in a manner similar to a reference system observer [11]. However, a rigid ruler cannot be used in the motion system because the lengths measured using light and rigid rulers differ. While measuring length using rigid and light rulers in an inertial system, those measured in the rest system always coincide, whereas those in the constant-velocity system do not.

In short, it is demonstrated that the principle of the constancy of the speed of light determines relativity between observers, whereas the principle of the constancy of a rigid ruler determines the absoluteness of an inertial system. As inertial systems are measured using light and a rigid ruler, relativity between observers and the absoluteness of inertial systems coexist. Finally, new interpretations of the twin paradox and Michelson–Morley experiment are presented as an example of relativity between observers and the absoluteness of inertial systems.

### Methodology

Einstein constructed reference systems using rigid rulers, light, and clocks and motion systems by applying the principle of relativity and the invariance of the speed of light [12]. These two axioms define the relationship between reference motion systems in a mathematical rather than a physical manner. To physically prove the coexistence of relativity between observers and the absoluteness of the inertial system, rigid rulers, light, and atomic clocks must be used to 1) physically define the principles of constancy of a rigid ruler and the speed of light and 2) establish a reference system and a motion system.

### Reference System with the Rigid Ruler, Light, and the Atomic Clock

As noted above, an observer determines and constructs a reference system using rigid rulers, light, and atomic clocks. Here, I describe the process of building a reference system on the premise of the absoluteness, rather than relativity, of inertial systems. In doing so, I borrow the concepts described in the special theory of relativity because it is necessary to explain the agreements between a rigid and a light ruler and the relationship between a resting and a moving point.

First, the length of a rigid ruler is invariant with respect to position within the reference system. The distance between two points in the reference system is measured using a rigid ruler. As a rigid ruler does not stretch or shrink, its mechanical length does not change; this is called the principle of the constancy of a rigid ruler in the reference system. Einstein did not adopt the principle of the constancy of rigid rulers as an axiom because he used the principle of relativity instead.<sup>1)</sup> However, the length of a rigid ruler will always be invariant for a reference system observer. Spatial coordinates are measured directly using a rigid ruler, with the distances from a point P to the x-, y-, and z-axis measured using a rigid ruler in the reference system and expressed as a Cartesian product following Euclidean geometry [13].

Second, the speed of light ( $c$ ), measured using a rigid ruler and an atomic clock, is always constant. This is called the principle of the constancy of the speed of light in the reference system. Optically, the speed of light  $c$  is measured through a light-reciprocating experiment conducted in a vacuum reference system. If the time for light to travel across and then back along the rigid ruler is  $t$  and the length of the rigid ruler is  $r$ , the speed of light is [14].

$$2r/t=c. \quad (5)$$

Third, the reference system is determined through a light-reciprocation experiment. Although Einstein described building reference systems using rigid rulers, light, and clocks, he did not discuss the problem of determining what reference systems were. As he was convinced of the relativity of inertial systems, he held that an inertial system could be considered a reference system if a rigid ruler, light, and a clock could be applied within it but did not physically prove that all inertial systems could be associated with rigid rulers, light, and clocks. Because, in a reference system, the speed of light ( $c$ ) is always constant, and the length of a rigid ruler is invariant of its position, such a system can be determined through a light-reciprocation experiment. However, Einstein overlooked that an experiment to measure the speed of light is equivalent to one to determine what the reference system would be [15].

Fourth, separated atomic clocks are synchronized using a light clock. A light clock in a reference system comprises a rigid ruler and light; light emitted from a source P at  $t=0$  will reach an observer Q at a distance  $r$  at  $t=r/c$  [14]. All atomic clocks within a reference system can be synchronized in this manner. When light emitted from P when the atomic clock situated at P indicates  $t=t_0$  reaches a resting point Q at a distance  $r$  from P, the atomic clock at Q is synchronized to

$$t = t_0 + \frac{r}{c}. \quad (6)$$

Using a rigid ruler and a synchronized atomic clock, the coordinates of an observer at P in the reference system can be expressed as  $P(x,y,z,t)$ .

Fifth, the lengths measured by rigid and light rulers in the reference system are always the same. When the atomic clocks in the reference system are synchronized, length can be measured using light via what is referred to as a light ruler [16]. If an atomic clock located next to an observer Q indicates that the time is  $t=t_1$  when light emitted by source P at time  $t=t_2$  reaches Q, the distance between the light source and observer can be measured as

$$l=c(t_2-t_1). \quad (7)$$

Using a light ruler, the x-, y-, and z- coordinates measured for an observer at  $P(x,y,z,t)$  in the reference system can be expressed as  $x_0, y_0,$  and  $z_0$ . The two sets of coordinates are identical because the distances measured by the rigid and light rulers are equal, i.e.,

$$x_0=x, y_0=y, z_0=z. \quad (8)$$

Sixth, the coordinates of a moving point are replaced by the coordinates of a resting point whose vicinity it is passing. Reference systems have resting and moving points; whereas the coordinates of a resting point are directly measured using a rigid ruler and a synchronized atomic clock, the coordinates of a moving point cannot be directly measured in this manner. Instead, the coordinates of a resting point can be replaced with those of a moving point passing by its vicinity. Einstein also introduced the concept of the vicinity, arguing that a stationary clock must be in the vicinity of the point of resting [17]. However, he only

addressed the method for obtaining the coordinates of a resting point and not the method for obtaining the coordinates of a moving point. He did not place moving points within the vicinities of resting points but instead applied the concept of time from Newtonian mechanics [18]. The coordinates of an observer in a reference system simultaneously represent the resting point and the moving points passing through the vicinity of the resting point.

Here, it is necessary to physically distinguish between an object  $M$  moving at a constant velocity  $v$  and light moving at a constant velocity  $c$ . In a reference system, the origin  $O$  and an observer  $P$  are separated by a distance  $r$ . Object  $M$  cannot arrive at  $P$  by moving from the origin  $O$  at an average velocity of  $v$ ; instead,  $M$  passes through  $O$  and  $P$  with a constant velocity  $v$ . However, within a reference system  $L$ , light leaves the resting source  $O$  and arrives at the resting observer  $P$ . Einstein did not distinguish between the physical properties of an object moving with constant velocity  $v$  and light moving with constant velocity  $c$  [19]. However, an object can become a stationary point of an inertial system, that is, an observer of the system, whereas light must always occupy moving points within an inertial system.

Although the light within a reference system  $L$  always occupies moving points, the light source  $O$  and the light observer  $P$  will be points at rest within the system. When a moving source  $O'$  passes through a resting source  $O$  with constant velocity  $v$ , a stationary observer will consider the resting light source and moving light source passing nearby to be the same source. If the light rays by both sources arrive at a resting observer  $P$ , the speed of the light measured by the  $P$  will be  $c$  for both. In addition, the light rays emitted from a moving source  $O'$  passing a resting source  $O$  with constant velocity  $v$  at the instant of passing will reach a moving observer  $P'$  at the same time. A moving observer  $P'$  passing a resting observer  $P$  at a constant velocity  $v$  will consider a resting source  $O$  and a moving source  $O'$  to be the same. If  $O'$  and  $P'$  each have an atomic clock, they can construct a motion system.

### **Motion System with Light and Atomic Clock**

Einstein postulated that the speed of light in a moving system is  $c$  based on the axiom of the constancy of the speed of light. However, the principle of the constancy of the speed of light was derived by combining the concepts of “vicinity” and “simultaneity” with the principle of the constancy of the speed of light in a reference system. For a reference system constructed using rigid rulers, light, and atomic clocks, a moving system can be constructed using light and atomic clocks. Thus, for a reference system  $O$ , a moving system  $O'$  exists, which moves with a constant velocity  $v$ . The observer in moving system  $P'(\xi, \eta, \zeta, \tau)$  moves at a constant velocity  $v$  in the  $+x$  direction with respect to that in the reference system  $P(x, y, z, t)$ . For  $P(x, y, z, t)$  and  $P'(\xi, \eta, \zeta, \tau)$ , the  $x$ - and  $\xi$ -axes are identical and the  $y$ - and  $\eta$ -axes are parallel, as are the  $z$ - and  $\zeta$ -axes. The moving system satisfies the following two conditions:

First, the light sources and observers in the reference and motion systems are fixed to their respective systems. The light emitted by the reference and motion light sources are referred to as the reference and motion light, respectively. The coordinates of an observer with the motion system can be matched with those of a reference system observer using shared light and atomic clocks.

Second, the principle of the constancy of the speed of light applied to the reference system should be extended to the inertial system using the concepts of “vicinity” and “simultaneity.” Einstein introduced these concepts in establishing the reference system but did not extend them to the motion system [20]. In a constant-velocity system, the notion of “vicinity” is important in inducing the constancy of the speed of light. “Vicinity” is used in three different cases: a light source in a constant-velocity system passing the vicinity of a source in the rest system, an observer in a constant-velocity system passing the vicinity of an observer in the rest system, and light in a constant-velocity system passing the vicinity of light in a rest system. The notion of “simultaneity” is required to expand the constancy of the speed of light and is also used in three different cases: that of atomic clocks passing within the vicinity of each other at the origin, which are simultaneous; when an atomic clock at the origin and an observer’s atomic clock are synchronized using light in an inertial system; and that occurring when the vicinities of inertial system observers pass, in which case their atomic clocks are also simultaneous.

Assume a light source  $O(0,0,0,0)$  in the reference system within the vicinity of another light source  $O'(0,0,0,0)$ . In an experiment, the light rays from the two sources will simultaneously reach observers in the reference system  $P(x, y, z, t)$  and other observers in the motion system  $P'(\xi, \eta, \zeta, \tau)$ . Furthermore,  $c$  is independent of its source because, for the observer  $P$ , the adjacent light sources  $O$  and  $O'$  pass through the same starting point  $O$ . Similarly, when the light rays from sources  $O$  and  $O'$  reach the motion system observer  $P'$ , the measured light speed is  $c$ , independent of the light sources because, for  $P'$ , the adjacent  $O$  and  $O'$  pass through the same starting point.

It might be speculated whether the light rays starting from  $O$  and  $O'$  simultaneously reach  $P'$  with a speed of  $c'$  instead of  $c$ . If this is true, it follows that the light from  $O$  reaches  $P$  with speed  $c$  and reaches  $P'$  adjacent to  $P$  with speed  $c'$ . Consequently, the light rays reflected from  $P$  and  $P'$  will return to  $O$  in the reference system with different speeds of, for instance,  $c$  and  $c'$ . This result contradicts the outcomes of actual experiments, in which light rays originating at  $O$  and reflected from  $P$  and  $P'$  return to  $O$  with speed  $c$ . Therefore, light rays originating simultaneously at  $O$  reach  $P$  and  $P'$  with speed  $c$ .

From this, the principle of the constancy of the speed of light in inertial systems can be defined as follows. The light rays originating from the adjacent light sources  $O$  in the reference system and  $O'$  in the motion system reach the adjacent observers  $P$  in the reference system and  $P'$  in the motion system with speed  $c$ . Thus, light rays originating from  $O$  and  $O'$  in the vicinity of each other are received simultaneously by observers at  $P(x, y, z, t)$  and  $P'(\xi, \eta, \zeta, \tau)$  in the respective light paths with speeds of  $c$  irrespective of the light source. If  $x^2 + y^2 + z^2 = (ct)^2$  is valid,  $\xi^2 + \eta^2 + \zeta^2 = (c\tau)^2$  holds and, by extension, if  $\xi^2 + \eta^2 + \zeta^2 = (c\tau)^2$  is valid,

$$x^2 + y^2 + z^2 = (ct)^2 \quad (9)$$

Holds. This is called the principle of the constancy of the speed of light in an inertial system.

Taking the principles of constancy of a rigid ruler and constancy



of the speed of light in a reference system as axioms, we can determine the reference system and measure the speed of light using the light-reciprocation experiment. The coordinates of a stationary point in the reference system can be measured with a rigid ruler, light, and an atomic clock, and a moving point can be replaced with the coordinates of the nearest stationary point. The two axioms for a motion system can then be derived by combining the concepts of “vicinity” and “simultaneity” with the two axioms established for a reference system. In physically defining a reference and motion system, it is necessary to describe the relationship between an inertial observer and an inertial system. The relationship between observers is revealed by applying the principle of the constancy of the speed of light and the relationship between inertial systems is investigated by adding the principle of the constancy of a rigid ruler.

### Results and Discussion

In this section, we discuss theoretical experiments for proving the relativity between observers and the absoluteness of an inertial system. Relativity between observers is established through the principle of the constancy of the speed of light. As both reference resting and motion system observers use light rulers and atomic clocks together, it is not possible to differentiate between the two in determining which is moving. When relativity between observers is established, the absoluteness of the inertial system is also established. The absoluteness of inertial systems has been proven by applying the laws of constancy of rigid rulers and the speed of light. The lengths measured using rigid and light rulers in a reference system coincide if it is a rest system but do not do so if it is a constant-velocity system.

#### Relativity between Observers

Einstein assumed the principle of relativity by taking the relativity shown in the Galilean transformation equation as an example [21]. However, he never considered the possibility that the relativity of the laws of physics could be derived from the principle of the constancy of the speed of light. As both reference and motion system observers use light and atomic clocks, the Lorentz and its inverse transformations can be obtained merely through the application of the principle of the constancy of the speed of light. An observer in a reference system  $P(x,y,z,t)$  synchronizes with the light emitted by the reference system's light source  $O(0,0,0,0)$  and perceives himself to be resting while an observer in motion system,  $P'(\xi,\eta,\zeta,\tau)$  moves with a velocity  $v$  in the + (positive)  $x$ -direction. In contrast,  $P'(\xi,\eta,\zeta,\tau)$  is synchronized with the light emitted from the motion system's light source  $O'(0,0,0,0)$ . Therefore, a moving system observer perceives himself to be resting with  $P(x,y,z,t)$  moving with velocity  $w$  in the - (negative)  $\xi$ -direction.

As  $P(x,y,z,t)$  moves with velocity  $v$  in the  $x$ -direction,  $\xi=0$  when  $x=vt$ , i.e.,  $\xi=f(v)(x-vt)$  ( $f(v)$  is not a function of  $x$  or  $t$ ). At  $v=0$ ,  $\xi=x$ , and the relation

$$f(0)=1. \tag{10}$$

Can be applied. Assuming that  $x=0$  moves in the  $\xi$ -direction with velocity  $-w$ , we obtain  $x=0$  when  $\xi=-w\tau$ . Therefore,  $x=g(-w)(\xi+w\tau)$  ( $g(-w)$  is not a function of  $\xi$  or  $\tau$ ).

At  $-w=0$ ,  $x=\xi$ . Therefore,

$$g(0)=1. \tag{11}$$

Based on the principle of the constancy of the speed of light, when  $x=ct$ ,  $\xi=c\tau$ , and with  $\xi=c\tau$ ,  $x=ct$ . Thus,  $c\tau=f(v)(c-v)t$ , and  $ct=g(-w)(c+w)\tau$ . Furthermore,

$$c^2=f(v)g(w)(c-v)(c+w). \tag{12}$$

By applying the principle of the constancy of the speed of light, when  $x=-ct$ ,  $\xi=-c\tau$  and when  $\xi=-c\tau$ ,  $x=-ct$ .

Thus,  $-c\tau=f(v)(-c-v)t$ , and  $-ct=g(-w)(-c+w)\tau$ . Furthermore,

$$c^2=f(v)g(w)(c+v)(c-w). \tag{13}$$

Using  $(c+v)(c+w)=(c-v)(c-w)$  in Eqs. (12) And (13), we obtain

$$v=w. \tag{14}$$

From Eqs. (10), (11), and (14),

$$\xi=f(v)(x-vt), \text{ and}$$

$$x=g(-v)(\xi+v\tau). \tag{15}$$

From Eqs. (12), (13), and (14),

$$f(v)g(-v)=c^2/(c^2-v^2),$$

$$\text{Provided that } f(0)=g(0)=1. \tag{16}$$

In this case, if  $f(v)>g(-v)$ , then  $f(v)>1$ , and thus, we no longer have the relationship

$$f(0)=1. \tag{17}$$

In addition, if  $f(v)<g(-v)$ , then  $f(v)<1$ , and we no longer have the relationship

$$f(0)=1. \tag{18}$$

From Eqs. (17) and (18),

$$f(v)=g(-v). \tag{19}$$

From Eqs. (16) and (19),

$$f(v)=g(-v)=k \ (k=1/\sqrt{1-(v/c)^2}). \tag{20}$$

From Eqs. (18) and (20), we obtain

$$\xi=k(x-vt) \text{ and } x=k(\xi+v\tau). \tag{21}$$

Therefore, the relationships can be rearranged and expressed as

$$\tau=k(t-vx/c^2), \ t=k(\tau+v\xi/c^2). \tag{22}$$

As the  $y$ -direction of  $P(x,y,z,t)$  and the  $\eta$ -direction of  $P'(\xi,\eta,\zeta,\tau)$

are perpendicular to the direction of motion, they are time-independent, however, they are dependent on the relative velocity. With respect to  $P(x,y,z,t)$ ,  $P'(\xi,\eta,\zeta,\tau)$  moves with a relative velocity of  $v$ . Therefore, the relationship between  $z$  and  $\zeta$  can be represented as

$$z = m(v) \zeta. \quad (23)$$

In contrast,  $P(x,y,z,t)$  moves with a relative velocity of  $-v$  with respect to  $P'(\xi,\eta,\zeta,\tau)$ . Therefore, the relationship between  $\zeta$  and  $z$  can be expressed as

$$\zeta = n(-v)z. \quad (24)$$

From Eqs. (23) and (24), we derive  $m(v)n(-v) = 1$  ( $m(0) = n(0) = 1$ ). If  $m(v) > n(-v)$  or  $m(v) < n(-v)$ , the relationship  $m(0) = n(0) = 1$  cannot be applied. Thus,  $m(v) = n(-v) = 1$ , i.e.,

$$z = \zeta. \quad (25)$$

Similarly, for the  $y$ -coordinate of  $P(x,y,z,t)$  and  $\eta$ -coordinate of  $P'(\xi,\eta,\zeta,\tau)$ , we have

$$y = \eta. \quad (26)$$

The relations

$$\xi = k(x-vt), \quad y = y, \quad \zeta = z, \quad \tau = k(t - vx/c^2), \quad (27)$$

From Eqs. (21), (22), (25), and (26) are referred to as the Lorentz transformation of an observer in the moving system  $P'(\xi,\eta,\zeta,\tau)$  with respect to an observer in the reference system  $P(x,y,z,t)$ . Furthermore,

$$x = k(\xi + v\tau), \quad y = \eta, \quad z = \zeta, \quad \text{and} \quad t = (\tau + v\zeta/c^2) \quad (28)$$

are referred to as the inverse Lorentz transformation of  $P(x,y,z,t)$  with respect to  $P'(\xi,\eta,\zeta,\tau)$ .

The Lorentz transformation and its inverse are induced solely by the principle of the constancy of the speed of light and are independent of the theory of relativity of physical laws. Under the Lorentz transformation and its inverse, reference system and motion system observers are relative except in terms of the sign of their relative velocity. As relativity between observers holds, it is natural to apply the relativity of the laws of physics to show that the relativity of physical laws premised in special relativity arises from the principle of the constancy of the speed of light. It turns out that the relativity of physical laws is not an axiom independent of the principle of the constant speed of light but a subordinate proposition.

### Absoluteness of Inertial Systems

Previously, we used the Lorentz transformation and its inverse transformation by applying only the principle of the constancy of the speed of light without presupposing the relativity of the physical laws. The relativity between the reference-system-observer and the motion-system-observer is shown in the Lorentz transformation and its inverse transformation. That is, relativity between observers is established using light and atomic clock measurements alone. We now establish the absoluteness of iner-

tial systems by applying light and rigid ruler measurement in a motion system.

In Eqs. (27) and (28), a reference system observer  $P(x,y,z,t)$  uses a rigid ruler, light, and an atomic clock, whereas a motion system observer  $P'(\xi,\eta,\zeta,\tau)$  uses light and an atomic clock alone. What would happen if the motion system observer used not only light and an atomic clock but also a rigid ruler? As the principle of the constancy of a rigid ruler applies to a moving system, an observer in this system can use one measuring coordinate on the  $\xi$ -,  $\eta$ -, and  $\zeta$ -axes to obtain  $x'$ ,  $\eta_0$ , and  $\zeta_0$ , respectively ( $x' = x - vt$ ). From Eq. (27), these are

$$\xi = kx', \quad \eta = \eta_0, \quad \zeta = \zeta_0, \quad \text{and} \quad \tau = t/k - vx'/c^2. \quad (29)$$

By comparing Eqs. (8) and (29), we can uncover the differences between the reference and motion systems. We first look at the coordinate of the axis parallel to the direction of motion,  $\xi = kx'$ . The coordinates measured using a light ruler and a rigid ruler,  $\xi$  and  $x'$ , respectively, differ. Einstein noted that an observer in a motion system uses a rigid ruler in the process of deriving the Lorentz transformation. He set  $x' = x - vt$  as the length measured using a rigid ruler to calculate the coordinate  $\xi$ , the direction parallel to the motion, of an observer in a motion system,  $P'(\xi,\eta,\zeta,\tau)$  [22]. He defined  $\tau$  as a function of  $x'$ ,  $y$ ,  $z$ , and  $t$  and obtained the equation relating  $\tau$  and  $x'$  using a light-reciprocating experiment. As  $\tau$  is a linear function,  $\tau = a(t - v/c^2 - v^2/c^2 x')$  holds when  $a$  is an unknown function  $\phi(v)$ . He found the equation  $\xi = k(x - vt)$  by applying the principles of constancy of the speed of light and relativity of an inertial system [23]. Although he used  $x' = x - vt$  to derive  $\xi = k(x - vt)$ , he did not realize that the coordinate  $\xi = k(x - vt)$  calculated using a rigid ruler differs from  $x' = x - vt$ , the coordinate measured using the Galilean transformation

$$(\xi \neq x'). \quad (30)$$

Thus, he ignored the fact that  $\xi = k(x - vt)$ , measured using a light ruler, and  $x' = x - vt$ , measured using a rigid ruler, are different in a motion system.

To distinguish a rest system from a constant-velocity system, a constant-velocity observer must use a rigid ruler, light, and an atomic clock. The length of a rigid ruler measured with a light ruler will change depending on where it is placed. Although the length of a rigid ruler measured using light in a rest system is invariant according to the principle of the constancy of the speed of light in a reference system, the length of a rigid ruler measured using light in a motion system varies according to the same principle. From Eq. (29), a rigid ruler perpendicular to the direction of motion is invariant, whereas one placed parallel to the direction of motion will appear to have its length increased by a factor of  $k$ . However, Einstein believed that a moving rigid ruler applies the laws of physics to relativity. As a result, he misunderstood that the length of a rigid ruler lying parallel to the direction of motion decreases by a factor of  $1/k$  [24].

A reference system measured using a rigid ruler, light, and an atomic clock is referred to as a rest system and a motion system in which only light and an atomic clock can be used for measurement is referred to as a constant-velocity system. In an inertial

system, relativity between observers is established because observers in both reference and motion systems use common light and atomic clocks, whereas the absoluteness of the inertial system is established because only observers in a reference system use a rigid ruler, i.e., relativity between observers and absoluteness of the inertial system coexist. Thus, reference system and motion system observers are relative but reference systems and motion systems are absolute. In formulating special relativity, Einstein cited the electromotive force between a moving conductor and a magnet as an example of the "principle of relativity." This is an example of relativity between observers mediated by light, not the relativity of an inertial system [25].

The coexistence of relativity between observers and the absoluteness of an inertial system requires a novel interpretation of the concept of time in an inertial system. From Eqs. (27) and (29),

$$\tau = t/k - vx/c^2 = t/k - vkx'/c^2 \quad (31)$$

A source  $O'$  and observer  $P'$  in a constant-velocity system are fixed within the system. When light moves within a constant-velocity system, its origin is not at the origin of the constant-velocity system but continues to move. The times measured by the atomic clocks within a constant-velocity system coincide if the coordinates  $\xi$  are the same but do not match if they are different. Thus, only atomic clocks lying on a plane perpendicular to the direction of motion are simultaneous. In a rest system, all atomic clocks are synchronized via rigid ruler and light measurement and, therefore, such a system is called a synchronous system. In a constant-velocity system, by contrast, only the plane perpendicular to the direction of motion is simultaneous with light and is therefore called a synchronous plane. This shows that, whereas nearby observers are relative, the inertial systems are absolutely distinct.

We next look at the relationship between clock measurements taken in the rest and the constant-velocity systems from the perspective of the absoluteness of the inertial system. It is mechanically clear that a rest system clock is stationary, whereas the constant-velocity system clock moves with velocity  $v$ . From Eq. (27), if  $x = vt$  then

$$\xi = 0 \text{ and } \tau = t/k \quad (32)$$

A comparison of measurements from  $t=0$ , when the constant-velocity system clock  $O'$  leaves the rest system clock  $O$ , to  $t$ , when it passes the rest clock  $P$ , confirms that the constant-velocity clock  $O'$  moves  $1/k$  times more slowly than the nearby rest system clock  $P$ . Atomic clocks in the constant-velocity system always run slower than those in the rest system; in other words, clocks in a rest system will be the fastest among those in an inertial system. From Eq. (32),

$$t = k\tau \quad (33)$$

This also means that the period of an atomic clock in a constant-velocity system will be  $k$  times that of a similar atomic clock in a rest system. This is demonstrated by the increased decay times of  $\mu$ -mesons moving at speed  $v$ , which are  $k$  times

those of  $\mu$ -mesons at rest [26]. Applying relativity between observers, a resting clock appears to move with velocity  $-v$  relative to that of a constant-velocity clock. From Eq. (28), If  $\xi = -v\tau$  then

$$x=0 \text{ and } t = \tau/k \quad (34)$$

At  $\tau=0$ , the rest system clock  $O$  passes the constant velocity clock  $O'$ , and at  $\tau$  it passes the constant-velocity clock  $Q$ . Although  $O$  is fixed in the rest system, in the constant-velocity system, the position changes from the time it passes  $O'$  to when it passes  $Q$ . Conversely, with respect to a constant velocity clock, resting clocks appear to pass with velocity  $-v$ . From Eq. (34),

$$\tau = kt \quad (35)$$

However, because the rest system clock is synchronized using a rigid ruler and light is already at time  $t$ , it appears that light in the constant-velocity system moves  $k$  times faster when viewed from the rest system.

Einstein was convinced that both the relativity derived from the Galilean transformation and its inverse and that derived from the Lorentz transformation and its inverse were both equivalent to the relativity of an inertial system. Consequently, he did not realize that the Lorentz transformation and its inverse could be deduced solely from the law of constancy of the speed of light. As a result, he overlooked the fact that only reference system observers use rigid rulers, as motion system observers cannot use them. In this paper, the principles of constancy of a rigid ruler and constancy of the speed of light are set as axioms and the linkages between reference and motion systems are derived by defining vicinity and simultaneity. In this manner, the Lorentz transformation and its inverse are deduced solely from the principle of the constancy of the speed of light, revealing that, if the rigid and light rulers in an inertial system coincide, it is a rest system; otherwise, it is a constant-velocity system. It is also shown that an atomic clock in a constant-velocity system moves slower than one in a rest system. In other words, the relativities of inertial systems and observers used in special relativity are replaced by the absoluteness of the inertial system and the relativity of the observer.

### Practical Significance

Inertial systems synchronize with rest and constant-velocity systems in different ways. Rest systems are synchronized using rigid and light rulers, whereas constant-velocity systems are synchronized using rest system and constant-velocity-system light. The synchronization carried out via the Lorentz transformation, and its inverse should consider the synchronization of the rest system. The twin paradox (TP) is a thought experiment that does not consider the synchronization of the rest system.

The TP involves twins  $O$  and  $O_0'$ , who are born on Earth at the same time. Shortly after their birth, one twin  $O$  remains on Earth, and the other  $O_0'$  boards a rocket that accelerates to a velocity  $v$  and travels to a planet  $P$  at a distance of  $r$  from Earth. As soon as the rocket arrives at  $P$ , it turns around and travels back to Earth at velocity  $-v$ . When twin  $O$  on Earth sees twin  $O_0'$  on the rocket,  $O_0'$  appears younger than  $O$  by a factor of  $1/k = \sqrt{1-(v/c)^2}$  from  $O$ 's standpoint. At the same time,  $O_0'$  claims, according to

the principle of relativity, that  $O$ , back on Earth, appears younger than  $O_0'$  by a factor of  $1/k = \sqrt{1 - (v/c)^2}$ . The TP holds that the claims of both  $O$  and  $O_0'$  are valid. The question, then, is whether both twins can equally say that the other looks younger.

The following is an explanation of the TP obtained by applying the general theory of relativity.

First, the rocket carrying twin  $O_0'$  needs to be accelerated up to the constant velocity  $v$  and then slowed down to stop at the planet. The rocket then changes direction and applies the same sequence of acceleration/deceleration to travel back to Earth (to  $O$ ) [27].

Second, we look at a slightly altered case of one coeval,  $O$ , born on Earth and another coeval,  $O_0'$ , simultaneously born on the rocket. The rocket comes close to a planet, slows to a momentary halt, then changes direction and accelerates back to its initial speed toward Earth. When the rocket passes the Earth at this constant speed,  $O_0'$  can compare  $O_0'$ 's age to that of the Earth's coeval  $O$  [28]. This indicates that it is physically meaningless to attempt to resolve the TP using the principle of general relativity. Both the TP itself and its solution violate the condition of the inertial system to which the Lorentz transformation is applied.

The TP can be modified to a situation in which the Lorentz transformation and its inverse are applied as follows. Assume a case in which, as the coeval  $O_0'$  passes the vicinity of the coeval on Earth  $O$ , another coeval  $P$  is born on a planet at which the rocket is scheduled to arrive. The clock  $O$  on Earth and the clock  $P$  on the planet have already been synchronized via light and rigid rulers. As the coevals  $O$  and  $P$  are of the same age, it is necessary to compare the coeval  $O_0'$  in the rocket with the coeval  $P$  on the planet at the moment at which the rocket passes the planet. From Eq. (32), we obtain  $\xi=0$  and  $\tau=t/k$  when  $x=vt$ ; thus, coeval  $O_0'$  on the rocket looks younger than both the coeval  $P$  on the planet and coeval  $O$  by a factor of  $1/k$ .

Assume that the rocket cannot turn away from the planet to return to Earth. When the coeval  $O$  on Earth and the coeval  $O_0'$  in a rocket are at a distance of  $r$ , the Lorentz transformations cannot be applied. "To apply these to the coeval on Earth, a coeval  $Q'$  on another rocket must pass in her vicinity. Expressed differently, both  $O$  and  $P$  are stationary relative to the inertial system of the Earth, whereas coeval  $O_0'$  on the rocket and coeval  $Q'$  on the other rocket are stationary relative to the motion systems of the rockets. From Eq. (34), we obtain  $x=0$  and  $t=\tau/k$  when  $\xi=-vr$ . This relation expresses that the twin on Earth looks younger than the twin on the rocket. However, the coevals  $O$  and  $P$  are synchronized via light and rigid rulers and are therefore the same age at  $t$ , whereas the coeval  $O_0'$  looks younger than coeval  $P$  by a factor of  $1/k$ , and coeval  $Q'$  looks  $k$  times older than coeval  $O$ . Rather than synchronization between the rest and constant-velocity systems, this is the synchronization of points between a rest system observer and a constant-velocity observer who is passing nearby. Thus, the relativity shown in the Lorentz transformation and its inverse is not the relativity of an inertial system but relativity between observers. For TP to be valid at  $t \neq 0$ , four coevals,  $O$ ,  $P$ ,  $O_0'$ , and  $Q'$ , must be considered, replacing the "twin paradox" with the "paradox of four coevals."

Examples of constant-velocity systems using rigid ruler, light, and atomic clocks are examined in light-reciprocation and wave interference experiments (e.g., the Michelson–Morley experiment). In both experiments, the light travels in straight lines from a light source in two directions: perpendicular to the direction of motion (perpendicular light) and horizontally with respect to the direction of motion (horizontal light). The distance between the light source and each mirror is measured using a rigid ruler as  $r$ . The Fitzgerald–Lorentz contraction has been mentioned in the context of light-reciprocation and wave interference experiments [29]. However, as Fitzgerald–Lorentz contraction occurs with the same intensity in horizontal and perpendicular light, in light-reciprocation or wave interference experiments the absoluteness of the inertial system and the relativity of the observer should be applied together in place of the Fitzgerald–Lorentz contraction.

In light-reciprocating experiments, the roundtrip times of perpendicular and horizontal light, as measured by a stationary clock, are  $2rk/c$  and  $(2rk_2)/c$ , respectively. From Eq. (32), an atomic clock in a constant-velocity system "ticks"  $1/k$  times slower than an atomic clock in the corresponding rest system and the vertical and horizontal light waves in the constant-velocity system contract by a factor of  $1/k$  relative to those in the rest system. As measured by the constant-velocity clock, the roundtrip times of perpendicular and horizontal light are  $2r/c$  and  $2rk/c$ , respectively, indicating that the perpendicular light pulses hit the detector earlier than the corresponding horizontal pulses. On the other hand, in the reciprocating light experiment conducted in the rest system, the two light rays arrive at the detector same time, indicating the rest and constant-velocity systems are clearly distinguished.

In a wave interference experiment, waves are emitted from a rest system source and a constant-velocity system source in its vicinity and are incident on a rest system observer and a constant-velocity system observer in their vicinity. The rest system source and observer are fixed to the rest system and there is only one source. By contrast, because the constant-velocity system source and observer move at constant velocity  $v$ , the constant-velocity system has a departure and an arrival source. A constant-velocity system observer observes the emission of a constant-velocity wave when the departure source passes in the vicinity of the rest system source, but when they receive the constant-velocity wave, they perceive an arrival source at the origin of the constant-velocity system. That is, the constant-velocity observer sees a constant-velocity wave emitted by the departure source but arrives at the arrival source. The distance traveled by the departure wave is measured with light, whereas the distance traveled by the arrival wave is measured with a rigid ruler.

The distance that the vertical wave travels back and forth is  $2r$  as measured by both the light and the rigid rulers. When a wave leaving the source at  $\tau=0$  re-enters the source, the horizontal portion has a phase time of

$$2r/c - 2r/c = 0. \quad (36)$$

By contrast, the distance traveled back and forth by the horizontal wave is  $2rk/c$  as measured by the light ruler and  $2r/c$  as measured by the rigid ruler.



sured by the rigid ruler. When a wave leaving the source at  $\tau=0$  re-enters the source, the phase time of the horizontal portion is

$$2rk / c - 2r/c. \quad (37)$$

Two waves are emitted by the departure wave source, reflected by the mirror, and return to the departure source at  $\tau$ . The phase time of a wave is determined by the distance from the arrival source, not the distance from the departure source. As the distance traveled by the parallel wave component from the arrival source to the observer is  $2r$ , the phase time of the horizontal wave is  $\tau - 2r/c$ ; the distance the vertical wave travels from the arrival source to the observer is also  $2r$ , so the phase time of the perpendicular wave is  $\tau - 2r/c$ . Because there is no difference in phase time between waves entering simultaneously entering the observer, now wave interference is detected, even in a constant-velocity system.

Because the wave interference experiment involves reflection between a relatively stationary wave source and a mirror, the length must be measured using a rigid ruler. Paradoxically, the Michelson–Morley experiment, which broke the concept of absolute time and space in classical mechanics, shows an example of using a rigid ruler in a constant-velocity system.

As mentioned above, relativity between observers and the absoluteness of inertial systems coexist. This determines the following physical characteristics adhering to the relativity of an inertial system that are not found under special relativity.

First, the speed of light and the rest system are determined by conducting a light reflection experiment in which a rigid ruler and an atomic clock are combined.

Second, by applying the Lorentz transformation and its inverse using only the principle of the constancy of the speed of light, it is apparent that the relativity of the laws of physics cannot be assumed as a physical axiom.

Third, the relativity of the laws of physics should be interpreted not as relativity of inertial systems but as relativity between observers.

Fourth, by adding vicinity to the simultaneity introduced by Einstein, the relativity of time and space relies on the principle of the constancy of the speed of light and not the principle of the constancy of a rigid ruler.

Fifth, in a rest system the lengths measured using a light ruler and a rigid ruler will be the same; in a constant-velocity system, they will be different.

Sixth, in a manner similar to the duality of waves and particles, physical phenomena must be interpreted in consideration of the relativity between observers and the absoluteness of inertial systems. For example, the TP must consider synchronization using a rigid ruler and light, and, in the Michelson–Morley experiment, the distance between a mirror and a wave source fixed to a constant-velocity system must be measured using a rigid ruler.

In short, the special theory of relativity described based on the theory of relativity should be examined in terms of the absoluteness of the inertial system and the relativity between observers.

## Conclusions

Einstein's special principle of relativity in 1905 notably introduced the concept of physical space-time. He used the theory of relativity of the laws of physics and the principle of invariance of the speed of light to derive the Lorentz transformation and its inverse for explaining various phenomena in electromagnetism. However, his work did not integrate the Galilean transformation accepted in classical mechanics with the Lorentz transformation found in electromagnetics.

The Galilean transformation and its inverse produce the absolute time-space of classical mechanics. In the Galilean transformation, observers commonly use rigid rulers, and the principle of the constancy of rigid rulers is applied. Relative space-time is revealed under the Lorentz transform and inverse. Under the Lorentz transformation, the observer uses light and atomic clocks together, and the principle of the constant speed of light is applied. In this paper, the principles of constancy of rigid rulers and the speed of light were set as axioms of local inertial systems and vicinity and were simultaneity defined to unify the concept of absolute time and space in classical mechanics with the concept of relative time and space in the special theory of relativity.

A rest system uses a rigid ruler, light, and an atomic clock, whereas a constant-velocity system uses light and an atomic clock. In other words, a rest system observer and a constant-velocity observer use light and atomic clocks in common, whereas rigid rulers are confined to the use of the rest system observer. Inertial system observers derive the Lorentz transformation and its inverse using the principles of constancy of rigid rulers and the speed of light.

A comparison of the Lorentz transformation and its inverse reveals that all coordinates are relative except for the sign of the observer's relative velocity in the rest system. From this, relativity between observers is established, and because this relativity holds, the relativity of the laws of physics naturally holds. This suggests that the relativity of the laws of physics should be interpreted as relativity between observers rather than the relativity of inertial systems and is not a physical axiom. As a result of these principles of constancy of rigid rulers and the speed of light, lengths measured together using a rigid ruler and a light ruler will be the same in a rest system and different in a constant-velocity system. An inertial system in which a rigid ruler can be used is referred to as a rest system, whereas one in which a rigid ruler cannot be used is called a constant-velocity system. The absolute distinction between a rest system and a constant-velocity system is called the absoluteness of the inertial system.

In a manner similar to the wave-particle duality of the microscopic world in which particle and wave theories coexist, relativity between observers coexists with the absoluteness of inertial systems. As a result of this coexistence, some of the assumptions on which the special theory of relativity as well as the relativity of inertial systems are based must be re-examined.

## References

1. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 395.
2. A. Einstein. (2015). Relativity: The Special and General Theory Tran by R. W. Lawson (Princeton University Press, Princeton) p. 28.
3. Galilei, G. (1967). Dialogues on the Two Chief World Systems, trans. Stillman Drake (Berkeley: University of California Press, 1967), 164-167.
4. Newton, I. (1999). The Principia: mathematical principles of natural philosophy. Univ of California Press.
5. Maxwell, J. C. (1865). VIII. A dynamical theory of the electromagnetic field. Philosophical transactions of the Royal Society of London, (155), 459-512.
6. Michelson, A. A., & Morley, E. W. (1887). On the relative motion of the earth and the luminiferous ether. American journal of science, 3(203), 333-345.
7. Fitz Gerald, G. F. (1889). The ether and the Earth's atmosphere. Science, (328), 390-390.
8. H. A. Lorentz. (1904). Proc. R. Netherlands Acad. Arts Sci. 6, 809.
9. A. Einstein. (2015). Relativity: The Special and General Theory Tran by R. W. Lawson (Princeton University Press, Princeton) p. 45.
10. A. Einstein. (201). Relativity the Special and General Theory Tran by R. W. Lawson (Princeton University Press, Princeton) p. 15.
11. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 396.
12. Anderson, J. L., & Bergmann, P. G. (1967). Thoroughly modern relativity. Physics Today, 20(12), 79.
13. A. Einstein. (2015). Relativity the Special and General Theory Tran by R. W. Lawson (Princeton University Press, Princeton) p. 6.
14. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 394.
15. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 395.
16. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 398.
17. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 393.
18. A. Einstein. (2015). Relativity the Special and General Theory Tran by R. W. Lawson (Princeton University Press, Princeton) p. 19.
19. A. Einstein. (2015). Relativity the Special and General Theory Tran by R. W. Lawson (Princeton University Press, Princeton) p. 53.
20. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 393.
21. S. Adams. (1997). Relativity (Taylor & Francis, London) p. 21.
22. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 397.
23. A. I. Miller. (1981). Albert Einstein's Special Theory of Relativity, on the Electrodynamics of Moving Bodies Einstein (Addison-Wesley Publishing Company, Massachusetts) p. 397.
24. S. Adams. (1997). Relativity (Taylor & Francis, London,) p. 47.
25. A. Einstein. (2015). Relativity the Special and General Theory Tran by R. W. Lawson (Princeton University Press, Princeton) p. 392.
26. W. G. V. Rosser. (1967). Introductory Relativity (Butterworth's, London) p. 70.
27. Anderson, A. J. (1967). Principles of relativity physics, acad. Press, New York and London.
28. Stephani, H. (2004). Relativity: An introduction to special and general relativity. Cambridge university press.
29. W. G. V. Rosser. (1967). Introductory Relativity (Butterworth's, London) p. 47.

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