

## And Where are Fluctuations In Quantum-Mechanical Wave Function?

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## Abstract

The generalized complex numbers containing weight coefficients for a paraxial case are described. An evident analogy between the wavy systems of trajectories, the module and an argument of complex numbers and the module and an argument of wave functions is drawn. Their geometrical interpretation is given on the plane and on the sphere. The symmetric and asymmetrical systems of wavy trajectories are considered. The examination of different types of wavy and zigzag trajectories of our systems of rays is presented. An evident analogy of these systems and wave functions describing finding of an electron in a potential well and near an atomic nucleus is drawn. The possible arrangement of a wave of de Broglie of an electron on the sphere of Bohr radius is shown. In the offered work great mathematicians whose works are connected with researches of complex numbers are marked out in {braces}: {Cardano, Hamilton, Gauss, Cauchy and Euler}.

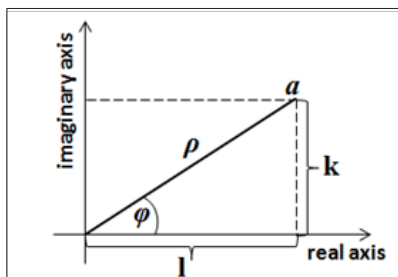
**Keywords:** Complex numbers; Wave function; Electron; Nucleus; Zigzag; Trajectories; Geometrization of physics.

## Introduction

A. Sommerfeld, one of founders of quantum mechanics in the work has noted an opportunity existence of communication between discrete variety of Riemann and quantum mechanics [1]. A. Sommerfeld has asked a question in [1]: “And where are fluctuations in wave function?” In our works we considered discrete variety of systems of the rays inclined under small angles to an axis (paraxial approach) of which we made different types of wavy and acyclic trajectories [2-6]. Then we investigated geometrical properties of these trajectories which revealed directly from consideration of drawings or by simple calculations by means of MS Excel of program.

In work Complex Geometry of Nuclei and Atoms was investigated [7]. In work the potential energy termin Schrödinger equation for Hydrogen Atom was discussed [8]. In present work we will consider communication of our geometrical system of wavy trajectories and quantum-mechanical wave function of Schrödinger.

Geometrical interpretation of complex numbers is presented in Fig.1 [9]:



**Figure 1:** Geometrical interpretation of complex numbers

The form of record of complex numbers comes in three types. Rectangular form {Gauss}:

$$a = l + ki, \quad (1)$$

Where  $i$  is imaginary unit.

Trigonometrical form {Cauchy}:

$$a = \rho(\cos\varphi + i\sin\varphi), \quad (2)$$

Where length (distance)  $\rho$  is the module of complex number

$$|a| = \rho = \sqrt{l^2 + k^2}, \quad (3)$$

And corner  $\varphi$

$$\varphi = \arctg \frac{k}{l} \quad (4)$$

It is an argument of a complex number.

Exponent form {Euler}:

$$a = \rho e^{i\varphi}. \quad (5)$$

Two complex numbers  $a$  and  $a^*$  are called mutually conjugate if their real parts are equal and imaginary differ only in the sign:

$$a = 1 + ki = \rho(\cos\varphi + i\sin\varphi) = \rho e^{i\varphi}, \quad (6)$$

$$a^* = 1 - ki = \rho(\cos\varphi - i\sin\varphi) = \rho e^{-i\varphi}. \quad (7)$$

The multiplication of two mutually conjugate numbers will be:

$$a^* a = |a|^2 = \rho^2. \quad (8)$$

By analogy with complex numbers wave function  $\psi$  in quantum mechanics can be also presented in the form of “module” and “argument”. We give this formula from [1]:

$$\psi = |\psi| e^{i\frac{S}{\hbar}}, \quad (9)$$

Where  $|\psi|$  is “module” and  $S/\hbar$  is “argument” (or phase [1]) of complex function and  $\hbar$  it is Planck constant.

To similarly complex numbers “square of the module” of wave function [1, 10] is:

$$\psi^* \psi = |\psi|^2. \quad (10)$$

Schrödinger at first assumed that  $\psi^* \psi$  there is continuously smeared charge density [1]. Born has gone further and interpreted  $\psi^* \psi$  as the probability density (the probability referred to unit of volume [1,10]).

Comparing expressions (5) and (9) it is possible to see that value  $\frac{S}{\hbar}$  is proportional to a corner  $\varphi$  in Fig.1:

$$\frac{S}{\hbar} \sim \varphi. \quad (11)$$

Value  $|\psi|$  is proportional (expressions (3), (5) and (9)) to distance  $\rho$ :

$$|\psi| \sim \rho. \quad (12)$$

Comparing expression (8) and (10) it is possible to see proportional values:

$$|\psi|^2 \sim \rho^2. \quad (13)$$

The power of levels that in Bohr’s theory depends on the term  $n$  [3, 4, 10]:

$$E_n = \sim \frac{1}{n^2}, \quad (14)$$

Where main quantum number  $n=1,2,3,\dots$ . In the simplest case of finding of an electron in a potential hole the Schrödinger equation as parameter enters the total energy of a particle of  $E$  [10]:

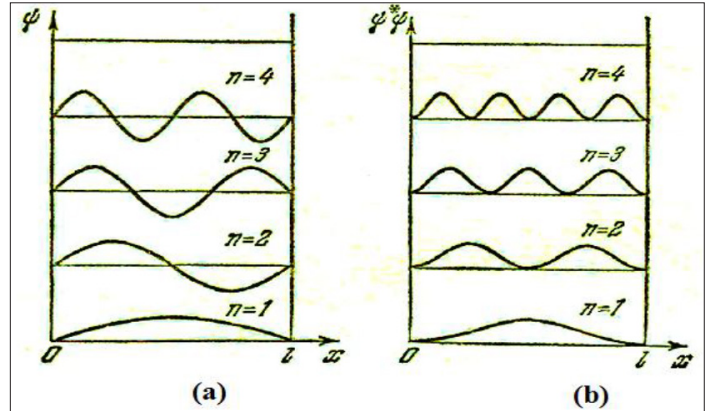
$$\frac{d^2\psi}{dx^2} + \frac{2m^2}{\hbar^2} E\psi = 0, \quad (15)$$

Where  $m$  is the mass of a particle (electron) and  $x$  is an axis along which moves a particle in a potential hole. In case of a discrete spectrum own values of energy can be numbered and compared with own values of function [10]:

$$\left. \begin{array}{l} E_1, E_2, \dots, E_n, \dots \\ \psi_1, \psi_2, \dots, \psi_n, \dots \end{array} \right\} \quad (16)$$

The value  $E_1$  has the certain value in a state of  $\psi_1$ , the value  $E_2$  has the certain value in a state of  $\psi_2$ , etc.

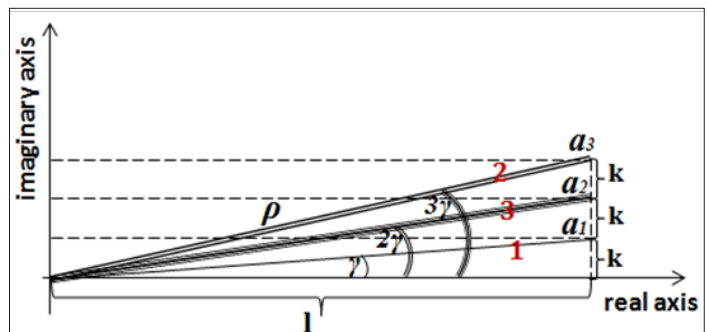
In Figure 2: The scheme of wave function of finding of an electron in a potential well is represented:



**Figure 2.1:** The scheme of the wave function  $\psi$  in one in one-dimensional infinitely deep potential well (a); and the scheme of density of probability of  $\psi^* \psi$  of detection of an electron at various distances from walls of this well (b).  $l$  is the width of well. (Fig.23.2 is from the monograph) [10].

### The Generalized Complex Numbers, Weight Coefficients and Small Angles

The geometrical interpretation of several complex numbers with small angles and various weight coefficients is presented in Figure 3:



**Figure 3:** Geometrical interpretation of generalized complex numbers with small angles  $\gamma$ ,  $2\gamma$ ,  $3\gamma$  and various weight coefficients 1, 3, 2 (are highlighted in red color).

{It is known from the history of mathematics that the first generalization of complex number in a form of quaternion was presented by Hamilton.} Here we describe another type of generalization of complex numbers.

We will enter weight coefficients for the numbers represented in Figure 3. The number of  $a_1$  consists of one line inclined at an angle  $\gamma$  therefore  $a_1$  has weight coefficient of  $b_1=1$ . The number of  $a_2$  consists of three lines at an angle  $2\gamma$  therefore  $a_2$  has weight coefficient

of  $b_2 = 3$ . The number of  $a_3$  consists of two lines at an angle  $3\gamma$  therefore  $a_3$  has weight coefficient of  $b_3 = 2$ , etc. Thus the numbers a represented in Fig.3 are characterized by one more parameter, it is weight coefficient of b therefore we will call them the generalized complex numbers.

The expressions for distances in this case it is possible to write down approximately similar to expressions (2) and (3) (paraxial approach taking into account weight coefficients):

$$\left. \begin{aligned} a_1 &\approx \rho_{b_1}(1 + \gamma i) \approx \rho_{b_1} \approx l_{b_1} \sim b_1, \\ a_2 &\approx \rho_{b_2}(1 + 2\gamma i) \approx \rho_{b_2} \approx l_{b_2} \sim b_2, \\ a_3 &\approx \rho_{b_3}(1 + 3\gamma i) \approx \rho_{b_3} \approx l_{b_3} \sim b_3, \\ &\dots\dots\dots \\ a_n &\approx \rho_{b_n}(1 + 3\gamma i) \approx \rho_{b_n} \approx l_{b_n} \sim b_n. \end{aligned} \right\} \quad (17)$$

The expression for small angles in this case it is possible to write down approximately similar to expressions (4):

$$\left. \begin{aligned} \gamma &\approx \frac{k}{1} \\ 2\gamma &\approx \frac{2k}{1} \\ 3\gamma &\approx \frac{3k}{1} \\ &\dots\dots\dots \end{aligned} \right\} \quad (18)$$

For odd values of angles:

$$(2n - 1)\gamma \approx \frac{(2n-1)k}{1}, \quad (19)$$

And by analogy with expression (5) for a case of small angles and weight coefficients for odd values of angles we will write down:

$$a_{(2n-1)} \sim b_{(2n-1)} e^{(2n-1)\gamma i}. \quad (20)$$

Comparing expression (9) to expression (20) and from Fig.3 it is possible to find the following compliances:

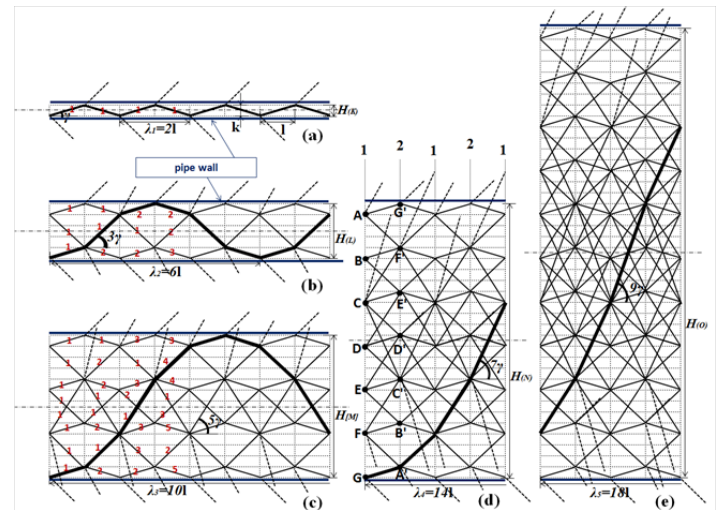
$$\left. \begin{aligned} \psi &\sim a_{(2n-1)}, \\ |\psi| &\sim b_{(2n-1)}, \\ \frac{S}{h} &\sim (2n - 1)\gamma. \end{aligned} \right\} \quad (21)$$

**Symmetric System of Trajectories X and Wave Function**

Our model consists of wavy trajectories. In our paraxial model we assume that wavy trajectories going in a long pipe or a well.

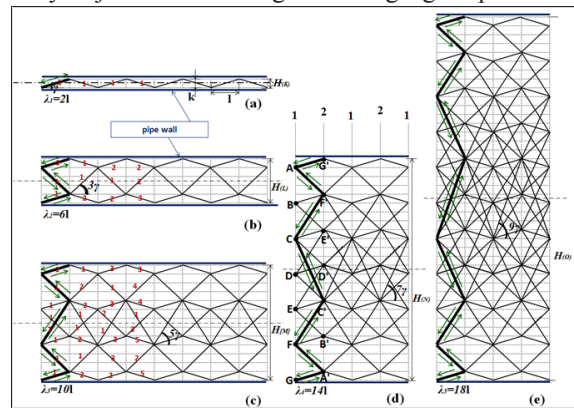
In Figure 4.1 and Figure 4.2 a pattern is provided. In a reduced scale projections to the drawing plane the symmetric system of wavy

trajectories of different height  $H$ :  $H_{[K]}$  (a),  $H_{[L]}$  (b),  $H_{[M]}$  (c),  $H_{[N]}$  (d), and  $H_{[O]}$  (e) are provided. Wavy trajectories consist of the links similar to pieces (the generalized complex numbers) represented in Fig.3.



**Figure 4.1:** The symmetric system of wavy trajectories of different height of  $H$ . The thick lines depict wavy trajectories with a length of “waves” of  $\lambda_1$  (a),  $\lambda_2$  (b),  $\lambda_3$  (c),  $\lambda_4$  (d),  $\lambda_5$  (e) of  $\lambda_{max}$ . The dotted line has designated the acyclic trajectories which are continuation of periodic (wavy) trajectories.  $H$  is the height of system of trajectories or diameter of a pipe the analog of a distance  $l$  between walls of a potential well in Fig.2.1. In (d) numbers 1 and 2 of lines of section of ray system at consecutive passes (iterations) in the direction from left to right are shown. Distribution of rays on angles: points A-G leave seven links (with various weight coefficients) of periodic trajectories under angle  $\gamma$  to an axis, points A-F leave six links of periodic trajectories under angle  $3\gamma$ , points of C – F leave four links of periodic trajectories under angle  $5\gamma$ , the point of C leaves one link of periodic trajectories under angle  $7\gamma$ ; it is similar for points  $A' - G'$ , etc. Distribution of rays on section: a point of G there is one link, points of A and B on two links, points of D – F on three links (with various weight coefficients) of periodic trajectories under various angles to an axis, etc. Wavy trajectories of various length are located between pipe walls on all width of system. On (a – c) by red numbers have designated weight coefficients.

For better understanding of the characteristics of our ray system and wave function module and argument we can present in Fig. 4.2 the same wavy trajectories as in Fig. 4.1 in zigzag shape:



**Figure 4.2:** Another view of the symmetric system of wavy

trajectories of different height of  $H$ . Zigzag shape of wavy trajectories. The direction of zigzag wavy trajectories construction is shown by green arrows. The distribution of weight coefficients for zigzag trajectories coincides with distribution of weight coefficients for wavy trajectories (in normalized form for stationary case) in Fig. 4.1 because these trajectories consist of the same links.

From Figure 4.1 and Figure 4.2 some conclusions can be drawn:

- The wavy trajectory is located in a rectangular coordinate grid with cells length of  $l$  and cells height of  $k$  where  $l \gg k$ .
- The simplest wavy system of trajectories  $H_{[K]}$  height contains only one wavy trajectory with a length of "wave"  $\lambda_1 = \lambda_{max} = 2l$ , height of  $v_1 = v_{max} = k = H_{[K]}$ , and an angle  $p$  of inclination  $p_1 = p_{max} = \gamma$ . This wavy trajectory consists only of two phases  $s_1$  and  $s_2$  that is of the links inclined on angles of  $+\gamma$  and  $-\gamma$  respectively.
- Wavy system of trajectories of  $H_{[L]}$  height contains two types of wavy trajectories with lengths of "waves"  $\lambda_1 = 2l$  and  $\lambda_2 = \lambda_{max} = 6l$ , height  $v_1 = k$  and  $v_2 = v_{max} = 5k = H_{[L]}$  and angles  $p_1 = \gamma$ ,  $p_2 = p_{max} = 3\gamma$ . This system contains two wavy trajectories  $\lambda_1$  and three wavy trajectories  $\lambda_2$ . Each trajectory  $\lambda_1$  consists of two phases  $s_1$  and  $s_2$  that is of the links inclined on angles of  $+\gamma$  and  $-\gamma$  respectively. Each trajectory  $\lambda_2$  consists of four phases  $s_1$  and  $s_2$  that is of the links inclined on angles of  $+\gamma$  and  $-\gamma$  and two phases  $s_3$  and  $s_4$  that is of the links inclined on angles of  $+3\gamma$  and  $-3\gamma$  respectively, etc.

The length  $\lambda$  of wavy trajectories depends on the number  $n$  and grows linearly [6]:

$$\lambda_n = 2(2n - 1)l \sim n, \quad (22)$$

When  $n = 1, 2, 3, \dots$

The height  $v$  of system of wavy trajectories depend on the number  $n$  and grows quadratically:

$$v_n = (2n^2 - 2n + 1)k \approx H(n) \sim n^2 \sim \lambda_n^2. \quad (23)$$

The tilt angle  $p$  of the links that gives rise to trajectories depends on the number  $n$  and grows linearly:

$$p_n = (2n - 1)\gamma \sim n \sim \lambda_n. \quad (24)$$

Points of branching of links in our system will depend from each other at  $q$  distance:

$$q_m = 4km, \quad (25)$$

where  $m = 0, 1, 2, \dots$

The links inclined on identical angles can be imposed at each other therefore their weight coefficients  $b$  can be various (as well as in Figure 3). At each consecutive pass (iteration) of rays weight coefficients are summarized as it is shown in Fig. 4 by red numbers. In detail the rule of calculation of weight coefficients (including by means of a special arithmetic parallelepiped) of links at consecutive creation of system of wavy trajectories is provided in [2-4].

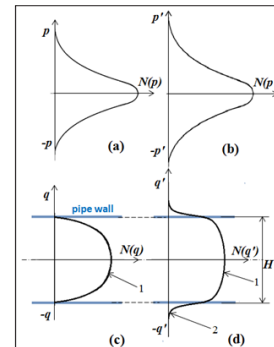
Our numerical calculations show that our system reaches steady state after redistribution of weight coefficients for links of wavy

trajectories less than after hundred iterations [2,4].

For the simplest case represented in Figure 4.1a weight coefficients it isn't redistributed between links and for the case represented in Figure 4.1b redistribution of weight coefficients between links for a stationary case it can be counted not only in numbers but also analytically by a solution of the cubic equation on Cardano formula [11]. {It is known from the history of mathematics that after Cardano formula appearance complex numbers began to use in mathematics.} In other cases distribution of weight coefficients between links of system was carried out by the numerical account.

In works we have counted the normalized distribution of  $N$  weight coefficients of  $b$  (the energy extending along links proportional values of weight coefficients) for our system of rays [2-4]. We considered distribution of number of rays on angle of  $N_p$  and on section of  $N_q$  of our system.

In Fig.5 the results of our calculations as envelopes of distributions are shown:



**Figure 5:** Distribution of weight coefficients (energy) of links (the energy extending along links) of trajectories of system of the rays represented in Fig.4.1 for great values of height of  $H$ . (a) is angular distribution of energy for links of periodic (wavy) trajectories in a pipe (bell-shaped form). (b) is angular distribution of energy for links of periodic (wavy) and acyclic trajectories in a pipe and outside of a pipe (bell-shaped form). (c) is distribution of energy on pipe section for links of periodic (wavy) trajectories in a pipe; the parabolic form of a curve 1 (close to a parabola of the fourth degree) is close to a form of distribution of laminar streaming of volume of liquid in pipes and to a form of a curve in Fig.2.1a. (d) is distribution of energy on pipe section for links of periodic (wavy) and acyclic trajectories in a pipe and outside of a pipe [2]; the parabolic form of a curve 1 (close to a parabola of the seventh degree) is close to a form of distribution of turbulent streaming of volume of liquid in pipes and to a form of a curve in Figure 2.1a; curves 2 are close to curves of exponents describing tunnel effect [2,10].

Proceeding from the above text drawings and formulas it is possible to see that the phases describing an inclination of links of our system of rays correspond to a phase (expression) of wave function [11]:

$$s_1, s_2, s_3, s_4, \dots \sim S. \quad (26)$$

That is tilt angles of links of our wavy system of trajectories correspond to a phase and an argument of wave function.

Angular distribution of weight coefficients of links (the energy extending along links) of trajectories of system of rays is:



$$N_{p_n} = N_{(2n-1)\gamma} \sim E_n \quad (27)$$

The parabolic form of a curve 1 in Figs.5c, d is close to a form of distribution of streaming of volume of liquid in pipes and to a form of a curve in Fig.2.1a therefore it is possible to assume [2]:

$$N_{q_m} = N_{Akm} \sim |\psi|. \quad (28)$$

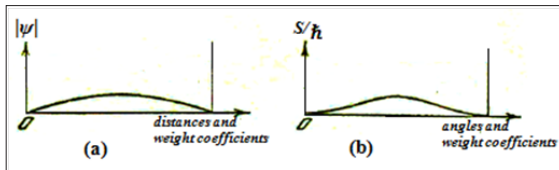
We will give the following reasoning's concerning Eq. (28): to receive distribution of  $N_{q_m}$  of weight coefficients on section in our system, we at first summarize weight coefficients in each of points of branching (Figure 4.1d) of all links leaving these points and then we find the general distribution of weight coefficients for these points on section. This distribution is formally similar to binomial distribution and to probability of detection of an electron in a potential well [2-4]. We will note that from this example it is visible that distribution not always has the habitual bell-shaped form as in Fig. 2.1b as in this case distribution is clamped in a pipe or a well.

The bell-shaped form of a curve in Figs.5a, b is close to a curve form in Fig.2.1.b therefore it is possible to assume:

$$N_{p_n} = N_{(2n-1)\gamma} \sim \frac{S}{h}, \quad (29)$$

We will give the following reasonings concerning Eq.(29): to receive distribution of  $N_{p_n}$  of weight coefficients on an angle in our system, we at first summarize weight coefficients of all links leaving all points of branching (Figure 4.1d) under identical angles and then we find the general distribution of weight coefficients for these points on an angle. This distribution is formally similar to wave function of the movement of an electron as all wavy trajectories of our system consist of these links. We will note that from this example it is visible that distribution of a wave (Figure 2.1b) in this case distribution has the bell-shaped form.

According to our reasonings we offer the changed version of Fig. 2.1. That is Fig. 2.2:



**Figure 2.2:** The changed version of Fig. 2.1 it is received from analogy between complex numbers and wave function which consists of two parts: module (a) and argument (b).

Our symmetric system of trajectories  $X$  can be characterized by the longest wavy trajectory  $\lambda_{max}$ . In turn  $\lambda_{max}$  can be characterized (according to expressions (21)) by one characteristic link  $\chi$  inclined on the greatest angle to a system axis. The link is characterized by two components: an angle and weight coefficient. The tilt angle of this link can be designated as  $p\lambda_{max}$  from expression (24) and Fig.4.1 and weight coefficient of this link as  $b\lambda_{max}$  it is calculated for stationary distribution in numbers. Knowing values of  $p\lambda_{max}$  and  $b\lambda_{max}$  we can define all other elements of symmetric (Fig.4.1) system of trajectories. By analogy with expressions (5), (9) and (20) we will write expression for a characteristic link  $\chi$  of the symmetric system of trajectories in a form:

$$\chi \sim b\lambda_{max} e^{p\lambda_{max}i} \quad (30)$$

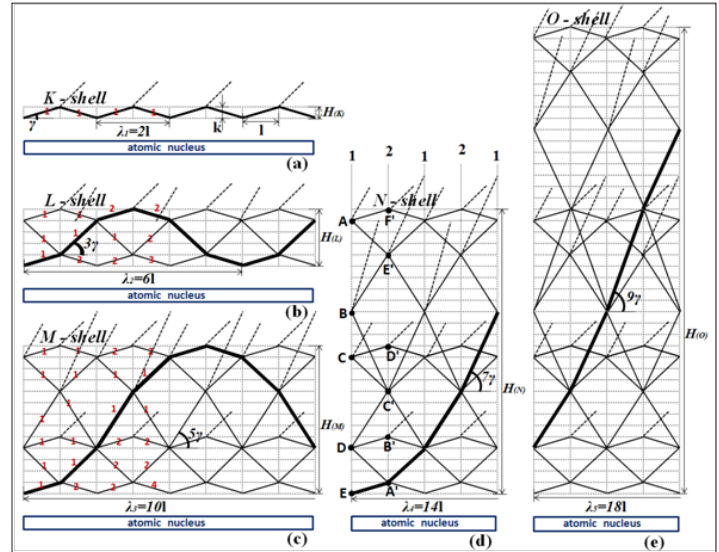
Because of symmetry of this wave function according to us it is optional to enter in addition a concept of a square of the module of function  $|\psi|^2$  here.

### Asymmetrical System of Trajectories X' And Wave Function

Our model consists of wavy trajectories. In our paraxial model we assume that wavy trajectories going around an atomic nucleus. We assume that wavy trajectories going around a nucleus come back to a given starting point so that the orbits are stationary in touch with the de Broglie's assumption given in [6].

In Figure 6: A patterns is provided. In a reduced scale projections to the drawing plane for shells of atom and main quantum number  $n$ :  $K$ -shell,  $n = 1$  (a),  $L$ -shell,  $n = 2$ (b),  $M$ -shell,  $n = 3$  (c), and  $N$ -shell,  $n = 4$  (d) are provided.

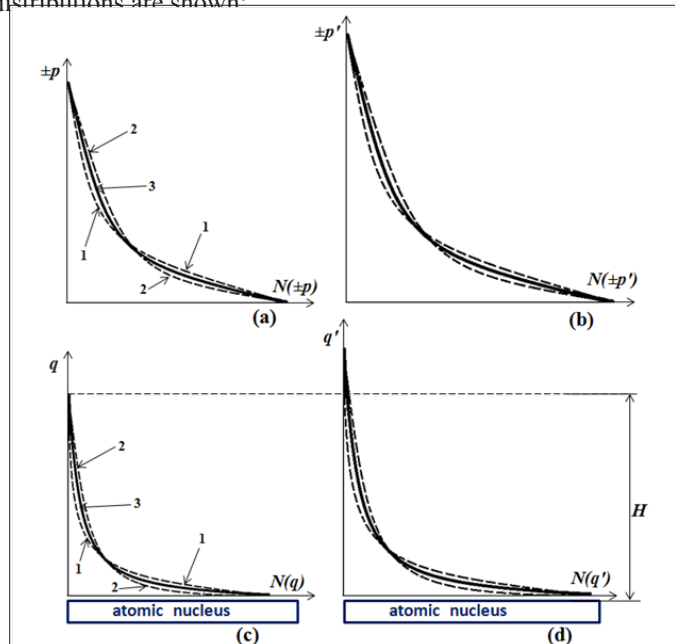
By consideration of geometry on the sphere of atom [6] the plane of the drawing lies on an arch of a big circle of the sphere. Wavy trajectories consist of the links similar to pieces represented in Figs 3, 4.



**Figure 6:** First four shells of the atom. The thick lines depict wavy trajectories with a length of "waves" of  $\lambda_1$  (a),  $\lambda_2$  (b),  $\lambda_3$  (c),  $\lambda_4$  (d),  $\lambda_5$  (e) of  $\lambda_{max}$ . The wavy trajectory is located in a rectangular coordinate grid with cells length of  $l$  and cells height of  $k$  where  $l \gg k$ . The dotted line has designated the acyclic trajectories which are continuation of periodic (wavy) trajectories.  $H$  is height of system of trajectories near an atomic nucleus. In (d) numbers 1 and 2 of lines of section of ray system at consecutive passes (iterations) in the direction are shown from left to right. Quantity of points of branching in (d) for various numbers of passes of 1 and 2 lines of section of wavy trajectories is various generally for asymmetrical system (c – e) but quantity of links is identical. Wavy trajectories of various lengths except the shortest are tied by the wave trough to atomic nucleus. It is possible to construct here zigzag wavy trajectories the same as in Fig. 4.2.

In works we have counted the normalized distribution  $N$  of weight coefficients  $b$  (the energy extending along links) for this system of rays [2-4]. We considered distribution of number of rays on angle  $\pm N_p$  and on section  $N_q$  of our system.

In Figure 7: The results of our calculations as envelopes of distributions are shown:



**Figure 7:** Distribution of weight coefficients of links (the energy extending along links near an atomic nucleus) of trajectories of system of the rays represented in Fig.6 for great values of height of  $H$ . (a) is angular distribution of energy for links of periodic (wavy) trajectories. (b) is angular distribution of energy for links of periodic (wavy) and acyclic trajectories being continuation of periodic trajectories. (c) is distribution of energy on the section of system of rays for links of periodic (wavy) trajectories. (d) is distribution of energy on the section of system of rays for links of periodic (wavy) and acyclic trajectories being continuation of periodic trajectories. The direction of axes of diagrams in Fig.7 corresponds with arrangement of elements in Fig.6. In works the direction of ordinate axis of similar diagrams was opposite i.e. top-down; it is standard practice in quantum mechanics to show in such a way the direction of electron energy increasing [3,4,10].

Because of asymmetry of system of the wavy trajectories represented in Figs.6c-e the distribution of rays on angle  $\pm N$  and on section  $N$  within consecutive passes (iterations) of 1 and 2 lines of section of wavy trajectories (Fig.6d) differ among themselves. Therefore in Fig.7. Curves distributions for line 1 and line 2 are described by various curves (are shown by dashed lines). The average value of distribution is shown by continuous curves 3 in Fig.7 [3,4]. Thus it is possible to explain geometrically observed effect of splitting of spectral lines of atom [4].

The asymmetrical system of trajectories  $X'$  it is possible as well as symmetric to characterize by the longest wavy trajectory  $\lambda'^{max}$ . In turn  $\lambda'^{max}$  can be characterized (according to expression (21)) by one characteristic link  $\chi'$  inclined on the greatest angle to a system axis. The link is characterized by two components: an angle and weight coefficient. The tilt angle of this link can be designated as  $p\lambda'_{max}$  (from expression (24) and Fig.6) and weight coefficient of this link as  $b\lambda'_{max}$  it is calculated for stationary distribution in numbers). Knowing value of  $p\lambda'_{max}$  and  $b\lambda'_{max}$  we can define all other elements of asymmetrical (Fig.6) system of trajectories.

By analogy with expressions (5), (9), (20) and (30) we will write expression for a characteristic link  $\chi'$  of the asymmetric system of trajectories in a form:

$$\chi' = b\lambda'_{max} e^{p\lambda'_{max} i}. \quad (31)$$

We will note that values of characteristic links for symmetric ( $p\lambda'_{max}$  and  $b\lambda'_{max}$ ) and asymmetrical ( $p\lambda'_{max}$  and  $b\lambda'_{max}$ ) the systems of trajectories generally (except the cases represented in Figs.4.1a, b and Figs.6a, b) are different from each other as it is different systems.

Because of asymmetry of this wave function, according to us, it is possible to enter in addition a concept of a square of the module of function  $|\psi|^2$  for an explanation of splitting of electronic levels because of asymmetry of passes of rays to various time points in volume here.

### Waves of De Broglie on Sphere

In work we have offered new, Belt model of Atom [6]. In work we have assumed that length of the shortest wavy trajectory of our system coincides with de Broglie wavelength in the first Bohr orbit of atom [6]:

$$\lambda_1 = \lambda_e, \quad (32)$$

Where the wavelength of this electron in agreement with de Broglie's formula is [12]:

$$\lambda_e = \frac{2\pi\hbar}{m_e v} \approx 3,1 * 10^{-10} \approx [\text{m}], \quad (33)$$

Where Planck constant is  $\hbar \approx 1,05 * 10^{-34} [\text{J} * \text{s}]$  and the mass of an electron is  $m_e \approx 9,1 * 10^{-31} [\text{kg}]$ . The speed  $v$  of the movement of an electron on such first circular orbit is  $v \approx 0.0073c \approx 2,2 * 10^6 [\text{m/s}]$  and  $c \approx 3 * 10^8 [\text{m/s}]$  [12].

Concerning the geometrical size of the first atomic shell the Bohr radius  $R_B$  is approximately [10]:

$$R_B \approx 5,3 * 10^{-11} [\text{m}]. \quad (34)$$

It is also possible to write down:

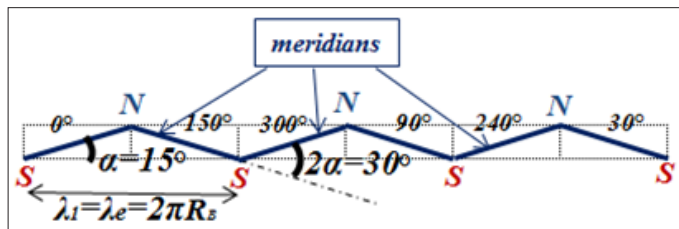
$$\lambda_e \approx 2\pi R_B. \quad (35)$$

In the previous paragraph we assumed that one of projections of our system of trajectories lies in the plane of the drawing and in the plane of an arch of a big circle of the sphere (on the plane, the perpendicular surface of the sphere on a meridian) and all angles multiple to an angle  $\gamma$  lie in this plane. However, it is possible to offer another projection of our system of trajectories; this projection lies on the surface of the sphere. In this case links of trajectories are located on sphere meridians at an angle  $2\alpha$  to each other. It means that two our projections are perpendicular each other. By consideration of geometry on the sphere of atom the plane of the drawing lies on an arch of a big circle of the sphere. Wavy trajectories consist of the links similar to pieces represented in Figures 3, 4, 6 [6].

In work the simple example has been described. Some traveler travels around the planet Earth on meridians from South Pole of S to the North Pole of N, then back to the South Pole of S, etc [13].

Every time when crossing points of the South Pole of  $S$  he turned left on a corner  $2\alpha$  and every time when crossing points of the North Pole of  $N$  he turned right on a corner  $2\alpha$ . It turned out that such traveler at repetition of many iteration will pass on all meridians of Planet Earth.

In Fig.8 Trajectories for a corner  $\alpha=15^\circ$  are shown:



**Figure 8:** Trajectories of the movement of traveler or electron on meridians on the globe and on the sphere with a Bohr radius of  $R_B$ .

The correction of calculations can be checked by means of the globe. If the globe sizes are very small and its radius is equal to  $R_B$  then it is possible to assume that the electron is capable to move along all big circles passing along all meridians after a while. As a result all surface of the sphere of atom will be covered with electron trajectories at various values of a corner  $\alpha$ .

It should be noted that above we considered trajectories of particles with half-integer spin for example electrons [14]. However trajectories of particles with integer spin for example photons on the sphere won't settle down on all meridians of the globe and will settle down only on two meridians [14]: on one meridian in the West and on one in the East hemisphere. Too it is easy to check it on the globe. Particles with whole spin can move lengthways to meridians on the globe as follows: when crossing  $S$  Pole they turn to the left on a corner  $2\alpha$ , further when crossing  $N$  Pole they turn to the left on a corner  $2\alpha$  again, further when crossing  $S$  Pole they turn to the right on a corner  $2\alpha$ , further when crossing  $N$  Pole they turn to the right on a corner  $2\alpha$  again, further when crossing  $S$  Pole they turn to the left on a corner  $2\alpha$ , etc. That is two turns to the right, then two turns to the left, then two to the right, etc.

## Conclusion

Our geometrical models presented in this work are approximate. The application of the generalized complex numbers containing weight coefficients for the paraxial case allow us to draw evident analogy between module and argument of complex numbers, our system of wavy trajectories and quantum-mechanical wave function:  $N_{qm} = N_{4km} \sim |\psi|$ , Eq.(28) and  $N_{pm} = N_{(2n-1)\gamma} \sim S/\hbar$ , Eq.(29). The examination of different types of wavy and zigzag trajectories of our system of rays perhaps helps us to answer A. Sommerfeld question: "And where are fluctuations in wave function?" It is possible that more complicated models consisting both of symmetric and of asymmetrical the systems of rays and also models more precisely the considering features of geometry on the sphere for the systems of wavy trajectories will be closer to reality.

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