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Adaptability of Power Law Exponential Decline Model for Hydraulically Fractured Unconventional Reservoirs during and After Linear Flow

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Abstract
Reserve estimation of unconventional formations is a new challenge to reservoir engineers due to the geological uncertainty and complex flow patterns evolving in the multi-fractured horizontal wells (MFHWs). Some predicting models have been presented and widely used in MFHWs exhibiting a long-term linear flow, such as stretched-exponential production-decline (SEPD), power law exponential decline (PLE) and Duong's model. Plenty of successful field applications of these models seem to have demonstrated their availability and correctness especially in the transient linear flow period.

Due to the limitation of reservoir boundaries or size of stimulated volume, any fractured tight reservoir will eventually exhibit a boundary-dominated-flow (BDF). The models above which show "goodness of fit" in linear flow may not be used or will cause great error when used to predict production in BDF period.

This paper compared the newly developed PLE model with the traditional Arps' hyperbolic decline model in terms of production historic match, decline rate and decline exponent during and after linear flow. The analysis result demonstrated that PLE model actually cannot match production decline characteristics as previously thought when only linear flow appears and it is a model which should be used in the transition period rather than linear flow period as applied in the past few years. The wrong usage of the model will cause great error to reserve estimation. The modified steps to predict production in different flow pattern are given in this work. The outcome of this work should help the industry to forecast production and ultimate reserve more accurately in tight oil and shale gas reservoirs.

Keywords: Unconventional reservoirs, Multi-fractured horizontal wells, Production decline, tight oil, shale gas reservoirs

Introduction

The world has seen a substantial growth in unconventional oil and gas over the last two decades and the wide application of multi-fractured horizontal wells (MFHWs) have allowed producing at economical rate from these low permeable oil and gas resources. Because of the very low matrix permeability, unconventional reservoirs producing with MFHWs usually exhibit long transient linear flow. Recently, significant advances have been made in the development of analytical method for analyzing production data from MFHWs producing from unconventional reservoirs. Many researchers have shift from traditional decline curve to newly developed empirical equations represented by ILK, Valko (2009) and Doung (2010) [1]. A point of view has been commonly accepted by reservoir engineers that traditional decline methods will overestimate reserves when applied on transient linear flow and only the newly developed models can correctly predict the ultimate recovery.

Three examples are analyzed in this work with different models. Two of which are the synthetic data from simulation with different producing time, the other comes from real field production. For the purpose of flow regime diagnostic, β-derivative function is also introduced as in the appendix. The work in this paper proves that even some models can match the historical data very well it can still give the wrongful predicting results without considering the flow regime and economical rate limit. Finally, we discuss a workflow for the production prediction based on decline exponent, β-derivative and the economic rate limit.

Analysis of Empirical Methods

Finding the appropriate forecasting methodologies for unconventional plays such as tight/shale gas has always been the target of reservoir engineers in the past two decades. Since ILK et al. presented a rate-time relation, power law exponential decline (PLE) model, a sin Equation (1); the discussion about this model and other empirical methods never stops [1].

$$q = q_0 \exp\left(-D_\infty t - \frac{D_1}{n} t^n\right) \quad (1)$$

On one hand the PLE model has been widely used for the production analysis and prediction in unconventional reservoirs. On the other hand, doubts still exist for its inconvenience of the four unknown parameters in the equation which cannot easily be solved by common linear regression. The key issue is whether this model can be really applicable to the whole production period or just suitable to the long-term linear flow period in unconventional reservoirs. Most of researchers compare it with Arp's hyperbolic model or modified hyperbolic model. Here we demonstrated its deficiency in matching the production history especially the decline parameters such as decline rate and decline components (the definition of D and b are list in Appendix A).

In the three Arps' decline models the hyperbolic model can be used to match the production data in transient linear flow, the well-known hyperbolic model has the following form:

$$q(t) = \frac{q}{\left[1 + bD_i t\right]^{1/b}} \qquad (2)$$

It is pointed out the analysis of production data from tight/shale gas wells using (2) typically results in a value of decline exponent b parameter greater than unity and a declining decline rate *D*, which will cause an overestimation of ultimate recovery. Maley (1985) suggested that at some point in time the hyperbolic decline should switch to an exponential decline. Robertson and Ilk introduced two decline limit value D_{limit} and $D\infty$, but the determination of these two values have no theory foundations and are mainly based on experience [1,2]. In this study, we examined the flow regime change which is the nature and reason of the model transition and proposed a new workflow for reserve estimation of unconventional reservoirs based on the diagnostic and modeling of *b* plot and *β*- derivative.

A simulation model is built to simulate the production of MFHW in tight reservoir. The physical model is described in Figure 1. The size of reservoir and fracture parameters is list in Table 1. Two cases are derived from the reservoir simulation with different producing time, one of which just shows a linear flow and the other case is a combination of line flow and boundary dominated flow (BDF) with a long producing history. The calculation of decline parameters such as *D* and *b* are list in Appendix A. The numerical simulation model is used to illustrate the inappropriate usage of PLE model in two different cases.

Table 1: Reservoir model parameters to generate synthetic production data

Parameters	Values	Units
Porosity,φ	0.10	
Viscosity,μ	0.30	cp
Compressibility, c_t	3×10^{-6}	psi^{-1}
Thickness,h	100	ft
Initial pressure, p_i	5000	psi
Bottom-hole-pressure, p_{wf}	500	psi
Permeability,k	0.001	md
Half-fracture-length, x_f	250	ft
Fracture spacing, y_e	150	ft
Well-bore radius, r_w	0.35	ft

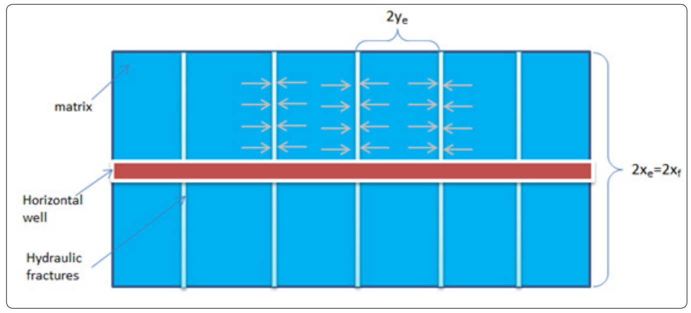

Figure 1: the Multi-fractured horizontal well in a tight reservoir

Case Analysis 1: simulated case- pure linear flow

We start with calculating the decline exponent *b* and decline rate *D* from the flow rate data of case 1, which is shown in Fig 2. From Fig 2and Fig 4 we can see that for case 1 both the decline component *b* and *β*–derivative function show a horizontal straight line with a value of 2 and 0.58, respectively, which indicate a linear flow. Meanwhile the hyperbolic model can perfectly match b, D parameters and *β*–derivative. But the PLE model cannot perfectly match the b trend line due to its decline nature which can be seen from the function of *b* in Equation (A-5).

The production rate matches are shown in Fig 6. From Fig 6 we can see there is an early deviation from real production for PLE, while the HYP model shows a good agreement during the linear flow period. Both the Power Law and Hyperbolic model can match the production data reasonably. The PLE model yields a more pessimistic prediction even in the period of linear flow. So we can say the PLE model implies a higher decline trend even during pure linear period. It can be concluded that if the production rate limit is reached before the end of linear flow, the hyperbolic model should give a more reliable prediction than PLE model.

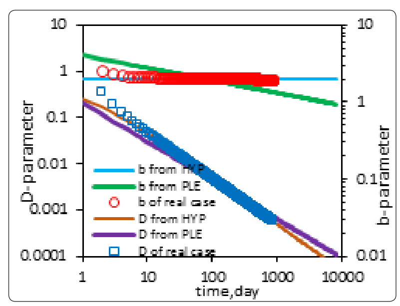

Figure 2: *b* and *D* parameters of case 1

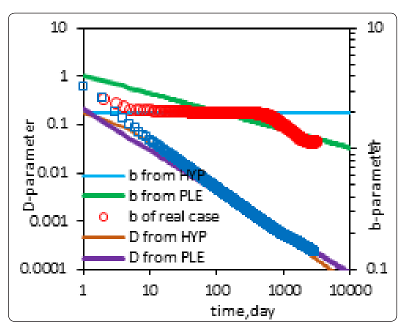

Figure 3: *b* and *D* parameters of case 2

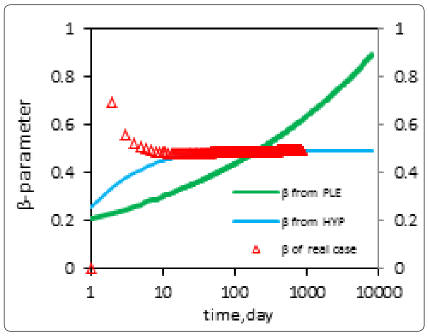

Figure 4: β–derivative function of case 1

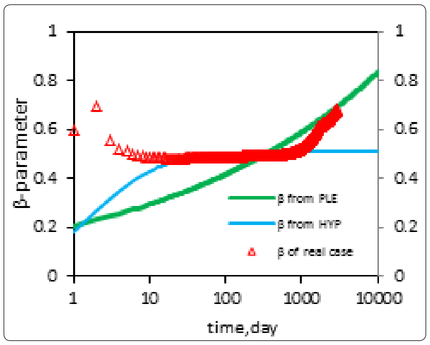

Figure 5: β–derivative function of case 2

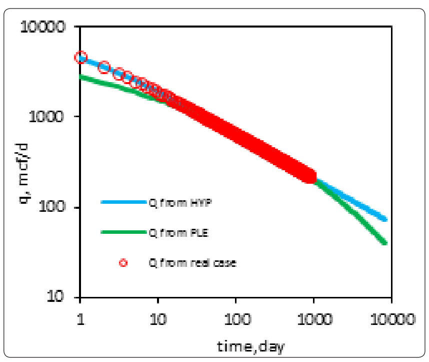

Figure 6: production match of case 1

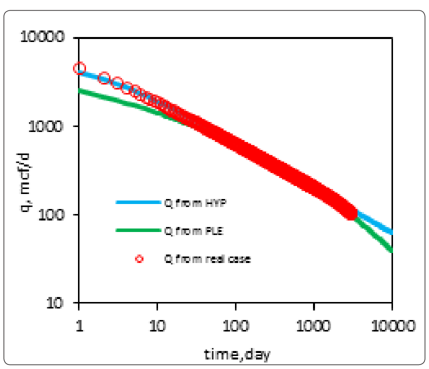

Figure 7: production match of case 2

Case Analysis 2: simulated case-linear flow + BDF

We extended the producing time in case 1 to 3000 days in the simulation model, leading to a final boundary dominated flow. The decline rate, decline component and β-derivative derived from production data are shown in Fig3 and Fig5. Both b and β deviate from the linearflow straight line, which are indications of transition to BDF. PLE model seem to have a better match of decline component b and β-derivative, butit cannot match the whole flow without identification of flow regimes.Especially it does not match the early data very well. This can also be seen from the production plot in Fig 7.

Importance of Flow Regime Diagnosis

It has been realized that we shoulduse decline modelsselectively according to the diagnosis of flow regimes. Matter (2009) proposed to treat the transient flow and the BDF differently. He suggested a modified PLE model in transient flow while anArps exponential decline model when the BDF is reached. In the two-segment decline approaches, Clarkson (2014) suggestedthat the PLE model is unable to practically forecast both transient and boundary-dominated flow in many cases and gave a modified hyperbolic equation representing boundary-dominated flow.

$$q = \begin{cases} q_i \exp\left[-D_i t^n - D_\infty t\right] & t \le t_{elf} \\ \dfrac{q_{elf}}{\left(1 + b_{bdf} D_{bdf} t_{bdf}\right)^{1/b_{bdf}}} & t > t_{elf} \end{cases} \quad (3)$$

P.Liang(2012) pointed out that in some shale gas reservoirs transition period between the primary and compound linear flow can last for 2-3 log cycles. For example, if the primary linear flow ends after 1 year of production, the transition period can last for 100-1000 years. So before the BDF is reached, the production rate may reach an economical value. In some cases transition period may become the dominant flow regime. Because of decline nature of decline rate in the previous analysis, we suggest that PLE model should be used after the end of linear flow.While during the linear flow, hyperbolic decline or Duong model should be used. This conclusion is just on the opposite side of Clarkson's suggestion in 2014.

So flow regime diagnosis is the crucial step in the selection of appropriate models for production forecasting of unconventional reservoirs. The key point is to know the end of linear flow or beginning of transition flow period.

Distance of Investigation for Linear Flow

When used to a horizontal well completed with multi-stage hydraulic fractures as shown in Fig1, most of the decline models will shows some limitations due to the multiple flow regimes, including boundary-dominated flow (BDF). The transient linear flow period is typically the most common transient flow regime observed for the model in Fig 1 and the end of it is caused primarily by fracture interference. The distance of investigation (DOI) of linear flow will help to determine the end of linear flow when the boundary or half fractures inter-distance is reached by pressure propagation. Wattenbarger et al. (1998) proposed a DOI for constant bottom-hole pressure as given in Equation4,

$$y_{inv} = 0.159\sqrt{\left(k/\phi\mu c_t\right)_i t} \quad (4)$$

H. Behmanesh et al. (2015) restudy the calculation of DOI for linear flow and suggest a constant value of C is 0.194 and 0.180 respectively for constant pressures based on unit impulse method and method of intersection.

For multi-fracture formations, when $y_{inv}=y_e$, marks the end of linear flow, so

$$t_{elf}=\left[\frac{y_e}{2\times 0.159}\sqrt{\left(\frac{\phi\mu c_t}{k}\right)_i}\right]^2=\frac{39.56\phi\mu c_t y_e^2}{k} \qquad (5)$$

MortezaNobakht et al. (2012) gave another method of t_{elf} if the slope m of $1/q$ vs. square root of t plot is given.

$$t_{elf}=\left[\frac{Ah\left(\phi\mu c_t\right)_i m\left(p_{pi}-p_{pwf}\right)}{200.6T}\right]^2 \qquad (6)$$

The time value from the above equations may be smaller than the real time of linear flow in practical reservoirs due to the real complex fracture network of micro-nature fracture and hydraulic fractures. So it is better to determine the end of linear flow with the analysis of production data combining theoretical calculations.

The New Procedure for Production Prediction

As we demonstratedin the first two cases,the aforementioned models cannot give a reliable prediction without considering the flow regime and economic rate limit. The new procedures we propose are list as following:

1. Compute the decline component b and β–derivative based on real production data;
2. Observe the b and β–derivative plots with time, if both of them appear to be a horizontal line, hyperbolic model or Duong model can be selected to match the production and determine the model parameters;
3. Estimate the end of linear flow t_{elf} based on Equation (5-6) and predict the flow rate at the end of line flow, Q_{elf};
4. Obtain the limit production rate Q_{limit} through economic analysis. If Qlimit>Qelf, hyperbolic model or Duong model can be used to predict the future flow rate and the EUR when flow rate reach Q_{limit}.
5. Otherwise in step 4, PLE model can be used to make prediction between Q_{elf} and Q_{limit}, the production data after linear flow is used to determine the PLE model parameters;
6. Otherwise in step 2, if end of linear flow is seen from the relation plots of b and β–derivative with time, to determine the t_{elf} from these two plots. Use hyperbolic model to match the data for $t<t_{elf}$, and use PLE model to match the data for $t>t_{elf}$ and make predictions.

Case 3: Field Example Application

This example comes from a fractured shale gas reservoir. From Fig 8, it can be seen both the PLE and Hyperbolic model can match the production data very well. In Fig 9 the hyperbolic model gives *a b* value of 1.5 and PLE model shows a declining b, whilehyperbolic model fits the D parameter better than PLE model. In Fig 10 both hyperbolic model and PLE model can match the β-parameter of real data, but they develop along different directions as for the predictions. The hyperbolic model gives a gradually stable β with anapproximate value of 0.65, indicating a linear flow, but the PLE model shows ansteadily increasing β parameter, which wrongfully indicateevolution of a boundarydominated flow. But the real time of linear flow is about 1000 days from Equation 5, which means the current flow regime is still linear flow. The straight line relationship of 1/q and t in Fig 11 also demonstrates the existence of this flow regime.As pointed out by J. Seshadri, the power law exponential model does not provide any significant tangible advantage over the modified hyperbolic.

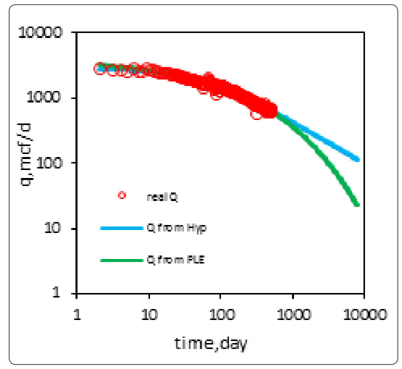

Figure 8: Match of Production Rate in Case 3

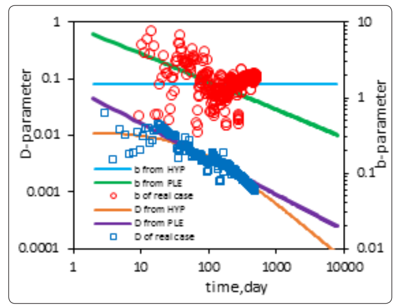

Figure 9: Match of Decline D and b in Case 3

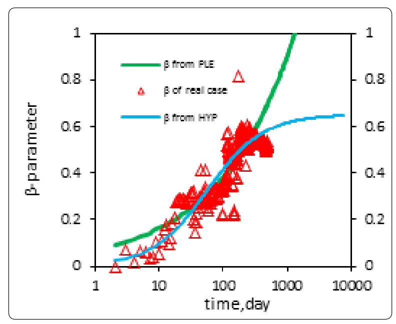

Figure 10: Match of β-parameter in Case 3

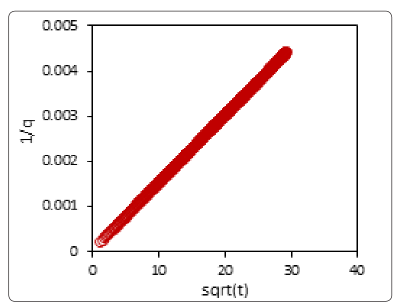

Figure 11: 1/q and t Relationship in Case 3

The 20-year production rate q_{20} and 20-year cumulative production G_{p20} are estimated with these two models respectively. The results are list in Table 2, in which we can see the hyperbolic tends to overestimate the production because it assume a life-time linear flow, while the PLE model underestimate the production due to an early-time transition to boundary dominated flow.

According the procedure suggested before, a production economic limit value of is 20MSCF/D and the end of linear flow is about at 3550 days. First we use the hyperbolic model to match the real production data which showing a linear flow and PLE model is then used to make prediction of transition period based the predicted data of hyperbolic model before the end of linear flow. The result of this combined model is also list in Table 2 showing a reasonable production trend.

Table 2: Production Prediction Based on Hyperbolic and PLE Model

Model	q_i Mscf/D	D_i	b or *n*	q_{20} Mscf/D	G_{p20} MMscf
Hyperbolic	2688	0.011	1.463	104.9	2.01
PLE	3400	0.121	0.428	12.7	1.25
Combined-Model	1000	0.077	0.358	86.5	1.87

Conclusions

The inappropriateness of the commonly accepted PLE model for forecasting MFHWs exhibiting purely linear flow has been analyzed through simulated and field cases. An integration of analytical methods with empirical models has been provided as an effective approach to history-match and production forecasting for unconventional reservoirs. In reservoirs exhibiting a transition to boundary dominated flow after linear flow, the hyperbolic should first be used to match the pure linear flow period and then the PLE model can be selected to model the transition flow. For the prediction of reserves in unconventional reservoirs the economical limit value of flow rate cannot be neglected.

Nomenclature

A	cross sectional area, ft^2
b	decline exponent
b_{bdf}	decline exponent for boundary dominated flow
C	constant factor
c_t	compressibility, psi^{-1}
D	decline rate, 1/day
D_i	initial decline rate, 1/day
D_1	a constant decline rate, 1/day
D_{bdf}	decline rate in boundary-dominated flow, 1/day
D_∞	decline rate when t reaches infinite, 1/day
h	net-pay thickness, ft
k	permeability, md
m	constant factor
n	constant factor
p	reservoir pressure, psi
p_{pi}	pseudo initial reservoir pressure, psi^2/cp
p_{pwf}	pseudo well flowing pressure, psi^2/cp
q_i	initial production rate, stb/d
q_{elf}	flow rate at the end of linear flow, std/d
t	time, day
t_{elf}	time at end of linear flow, day
t_{bdf}	time during boundary-dominated flow, day
T	temperature0, R
x_f	halffracturelength, ft
y_e	fracture spacing, ft
Ø	porosity, dimensionless
µ	viscosity, cp
EUR	Estimated Ultimate Recovery, BSCF

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