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Accelerated Motion towards Relativistic Velocities Described by Newtonian Mechanics

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Abstract
This paper presents a simple geometric derivation of the equations of motion for an object that accelerates from zero speed towards a relativistic velocity. This is not done by using the Theory of Special Relativity, but with Newtonian Mechanics only. The results are identical to those derived by the Theory of Special Relativity, with the exception of the expression for time dilation, in case the clock postulate is not applied. The clock postulate is used by Special Relativity and assumes that the time dilation of an accelerating object at any moment in time depends on its contemporary velocity only. It is shown that the clock postulate is violated in case of continuous longitudinal acceleration, and an alternative expression for accelerated time dilation is presented without the need for the clock postulate.

1. Introduction

In the year 1887 the American physicists Albert Michelson and Edward Morley obtained the first experimental results indicating that the perceived speed of light is independent of the motion of the observer with respect to the light source [1]. This observation could not be explained by the then prevalent physical models for waves travelling through a medium of propagation. This initiated a line of research that eventually led to the theory known as 'Special Relativity' [2]. Special Relativity (SR from now on) is still used today to describe the motion of objects with velocities approaching the speed of light, as opposed to Newtonian Mechanics, which so far have only been used for velocities much smaller than the speed of light. One of the fundamentals of SR is that no solid object can ever reach the speed of light, regardless of how fast and how long it accelerates. As a result, the classical expressions for traveled distance $d(t)$ and perceived velocity $v(t)$ after a time t for an object with initial position 0, initial velocity 0 and acceleration a,

$$d(t) = \tfrac{1}{2} at^2 \tag{1a}$$

$$v(t) = at \tag{1b}$$

are replaced by alternative expressions [3]:

$$d(t) = \frac{c^2}{a}\left(\sqrt{1 + a^2t^2/c^2} - 1\right) \tag{2a}$$

$$v(t) = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}} \tag{2b}$$

where c is the speed of light *in vacuo*. In addition, the perceived acceleration $a'(t)$ of the object diminishes over time from its initial value a to

$$a'(t) = a\left(\frac{1}{\sqrt{1 + \frac{a^2t^2}{c^2}}}\right)^3 . \tag{2c}$$

It can be easily verified that Equation (2b) assures that the velocity $v(t)$ is limited to values smaller than c, regardless the value of at, whereas Equation (2c) assures that the perceived acceleration $a'(t)$ eventually becomes 0. For $at \ll c$ (for low velocities compared to the speed of light) Equations (2) reduce to the Newtonian expressions represented by Equations (1).

SR is based on a number of postulates, one of which is an undeclared assumption made by Albert Einstein in his landmark paper [2], later referred to as the *clock postulate*. The clock postulate states that the Lorentz factor

$$\gamma(t) = \frac{1}{\sqrt{1 - \frac{v^2(t)}{c^2}}} \quad (3)$$

depends only on the instantaneous value of the velocity *v(t)* and not on any of its derivatives. In other words, the clock postulate assumes that the value of *γ(t)* in an accelerated frame of reference is identical to its value in a momentarily co-moving inertial frame. In this paper it will be shown that this assumption is incorrect, which leads to an inaccuracy in the familiar expression for time dilation of a continuously accelerated object.

2. Special Relativity described by Newtonian Mechanics

We will use Newtonian mechanics to describe the motion of a 'relativistic rocket', with the inclusion of time dilation effects. So far these equations have only been derived by SR with the use of Lorentz transformations, but it will be shown that Newtonian Mechanics suffice by using 3 alternative postulates that lay the foundation for a 'simplified theory of relativity':

1. All objects and all observers move through a 4-dimensional Euclidean space, with uniform velocity equal to the speed of light *c*, in a direction perpendicular to the three orthogonal dimensions *x*, *y* and *z* that set up the 3-dimensional space that we see around us. We will call this the *w* direction. Despite being an ordinary spatial dimension, this *w* dimension is 'hidden' for any observer, in the sense that it cannot be directly perceived. All that an observer perceives are the 3 dimensions perpendicular to his own velocity vector.
2. At any point in time, objects manifest themselves only in the 3-dimensional space that contains their current position and that is perpendicular to their direction of motion through the 4-dimensional space. We call this their 'space of existence'.
3. Observers can perceive an object only if they are located inside the object's space of existence.

Note that the postulates of SR have been abandoned, so that any known result derived by SR (such as that no solid mass can ever reach the speed of light) does not necessarily apply to our new theory - at least not yet. Nevertheless, it will be shown that, with above alternative postulates, 17th century Newtonian Mechanics can reproduce the relativistic equations of motion with a surprisingly simple geometrical method, and without the need for a clock postulate.

3. Travelled distance during continuous acceleration

Figure 1 represents the projection of a 4-dimensional Euclidian space, with a hidden *w*-dimension along the vertical axis and an arbitrary direction of our 3-dimensional world along the horizontal axis, for which we have chosen the *z* direction. At time $t = 0$, an observer (1) and a rocket (2) are both located at point A. According to postulate 1, both move with the speed of light *c* in the *w* direction, which is perpendicular to the *x, y* and *z* directions.

At $t = 0$, the engine of the rocket starts to run at constant power, accelerating the rocket into the *z* direction. We will not consider mass loss due to fuel consumption, so that the rocket will maintain a constant acceleration *a* throughout its entire journey. As soon as the rocket starts to move, its trajectory in 4D space will start to deviate from the *w* axis. As a result, according to postulate 2, its space of existence (which is by definition perpendicular to the rocket's trajectory) will start to tilt in a clockwise direction in the *z-w* plane, so that the acceleration of the rocket always remains perpendicular to its 4-dimensional velocity vector, as a result of which the rocket will perform a circular orbit in the *z-w* plane. This orbit is indicated in Figure 1 by the dotted circle segment running from point A to point C. From Newtonian Mechanics we know the relationship between the centripetal acceleration *a*, the orbital velocity *v* of an object and the radius of curvature *R* of the resulting circular motion:

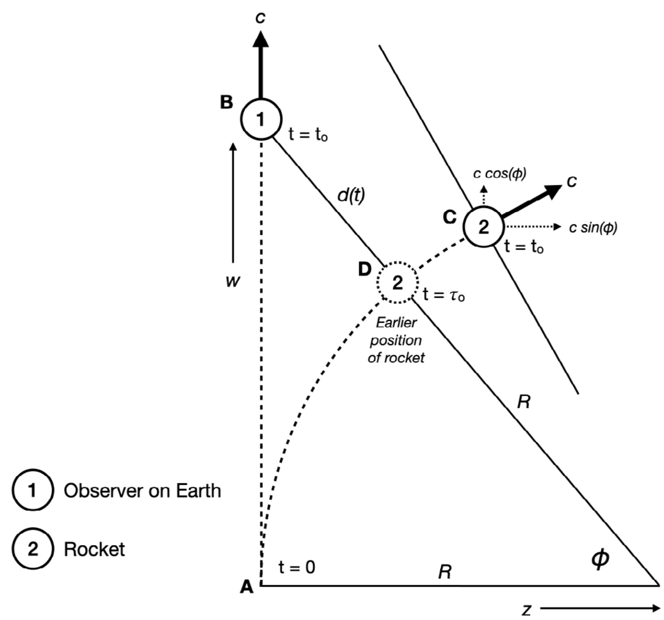

Figure 1: *Relativistic rocket (2) leaving Earth (1) at point A with constant acceleration a. The situation is shown in the z-w plane at a time t_o after the launch. According to the first postulate, at launch the rocket (2) already moves in the w direction with the speed of light, and because its acceleration is perpendicular to its velocity it will perform a circular motion in the z-w plane. The Earth (1) will continue to move along a straight line into the w direction.*

$$a = \frac{v^2}{R}\,. \quad (4)$$

Because in the 4D space the acceleration of the rocket remains perpendicular to its direction of motion, its orbital velocity remains unaltered and equal to the speed of light *c*, so that

$$R = \frac{c^2}{a}\,. \quad (5)$$

Figure 1 shows the situation at a time $t_o > 0$. The Earth has moved in a straight line from point A to point B, whereas the rocket has moved the same distance along a circular orbit from point A to point C. However, the observer on Earth will not be able to see the rocket in point C. This is the result of the 2nd postulate, which states that matter manifests itself only in the 3-dimensional space perpendicular to its velocity vector. In Figure 1, a part of this 'space of existence' of the rocket has been indicated by a solid line piece through point C (the total space of existence would have to be represented by an infinitely long line). This space of existence does not contain the position of the observer (point B). As a result, according to the 3rd postulate, the observer will not be able to see the rocket. However, Figure 1 also shows that the observer is located inside the space of existence of the rocket at an earlier time τ_o, when it was at point D. Therefore, at time t_o the observer will see the rocket at position D, at an earlier time τ_o, at a distance *d(t)* as indicated in Figure 1. One might argue that, at $t = t_o$, the observer cannot see the rocket at an earlier time τ_o because it will no longer be there. Keep in mind, however, that the concept of time in this study is not the same as how we experience time in our daily life. In this study, an observer can see an object when he is located inside the space of existence of that object, regardless the corresponding time stamps. At non-relativistic velocities, the spaces of existence of the observer and the rocket, or of two observers, will (nearly) coincide and the values of t_o and τ_o will be (nearly) identical. This describes the experience in our daily life: two observers always seem to see each other at the same time t and seem to share the same past at an earlier time $t - \Delta t$. However, in this paper each observer manifests himself at various times, whereas it is perfectly possible for both observers to see the other observer at an earlier time τ_o, provided that the space of existence of one observer intersects with the current position of the other. The exact value of τ_o will be derived later in this paper. We will first derive the expression for *d(t)*, which can be done with simple trigonometry, see Figure 1. At time t the distance AB amounts to *ct*, so that, using the Pythagorean theorem, we immediately find

$$d(t) = \sqrt{R^2 + c^2t^2} - R = R(\sqrt{1 + c^2t^2/R^2} - 1). \quad (6)$$

Substitution of Equation (5) leads to

$$d(t) = \frac{c^2}{a}(\sqrt{1 + a^2t^2/c^2} - 1). \quad (7)$$

This expression is identical to the one derived by SR [3].

Note that the derivation of this result is much simpler than the traditional route followed by SR (a Lorentz transformation for acceleration, followed by twofold integration over time). In essence, *d(t)* is simply the distance between a point on a straight line and a point on a circle.

4. Perceived velocity and acceleration

The velocity v_z of the rocket as observed from Earth is obtained simply by taking the time derivative of Equation (7):

$$v_z(t) = \frac{1}{2}\frac{c^2}{a}\frac{1}{(\sqrt{1 + a^2t^2/c^2}}\cdot\frac{2a^2t}{c^2} = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2t^2} + \frac{1}{c^2}}} \quad (8)$$

whereas differentiating once more reveals the expression for the perceived acceleration a_z:

$$a_z(t) = -\frac{1}{2}(\frac{1}{a^2t^2} + \frac{1}{c^2})^{-3/2}\cdot\frac{-2}{a^2t^3} = (\frac{1}{\sqrt{\frac{1}{a^2t^2} + \frac{1}{c^2}}})^3\cdot\frac{1}{a^2t^3} \quad (9)$$

$$= (\frac{1}{\frac{1}{at}\sqrt{1 + \frac{a^2t^2}{c^2}}})^3\cdot\frac{1}{a^2t^3} = a(\frac{1}{\sqrt{1 + \frac{a^2t^2}{c^2}}})^3.$$

Expressions (8) and (9) can be further simplified. In Figure 1, it can be seen that the velocity of the rocket can be decomposed into a z-component and a w-component, $v_z = c\sin(\phi)$ and $v_w = c\cos(\phi)$, so that

$$\cos(\phi) = \sqrt{1 - \sin^2(\phi)} = \sqrt{1 - \frac{v_z^2}{c^2}} = \frac{1}{\gamma} \quad (10)$$

where γ is the Lorentz factor from SR. The angle ϕ changes as the rocket accelerates, and can be written as (see Figure 1)

$$\phi(t) = \tan^{-1}(ct/R), \quad (11)$$

so that after substitution of Equation (5)

$$\phi(t) = \tan^{-1}(at/c). \quad (12)$$

Combining Equations (10) and (12) gives

$$\frac{1}{\gamma} = \cos(\tan^{-1}(at/c)). \quad (13)$$

By using the relationship

$$\cos(\tan^{-1}(x) = \frac{1}{\sqrt{1 + x^2}} \quad (14)$$

this can be written as

$$\frac{1}{\gamma} = \frac{1}{\sqrt{1 + \frac{a^2t^2}{c^2}}}. \quad (15)$$

After substitution of (15) into Equations (8) and (9), we find:

$$v_z(t) = \frac{at}{\gamma} \tag{16}$$

$$a_z(t) = \frac{a}{\gamma^3}\,. \tag{17}$$

Equations (16) and (17) are identical to the expressions for the rocket's velocity and acceleration as derived by SR [3].

5. Time dilation

Now we will compare the distances travelled through the 4-dimensional space by the observer on Earth and by the rocket at the time of its observation. Figure 2 shows the situation at time t_o when the Earth (1) has moved a distance $T = ct_o$ in the 'hidden' w direction, and the rocket (2) has performed a section of a circular orbit with curved length ct_o and radius R = c^2/a. As explained before, the observer on Earth will not be able to see the rocket at its position at $t = t_o$. Instead, it will see the rocket at an earlier position at time $t = \tau_o$ (indicated by the dotted circle), when the rocket had performed a section of a circular orbit with total length S. By definition:

$$\phi = \frac{S}{R} \tag{18}$$

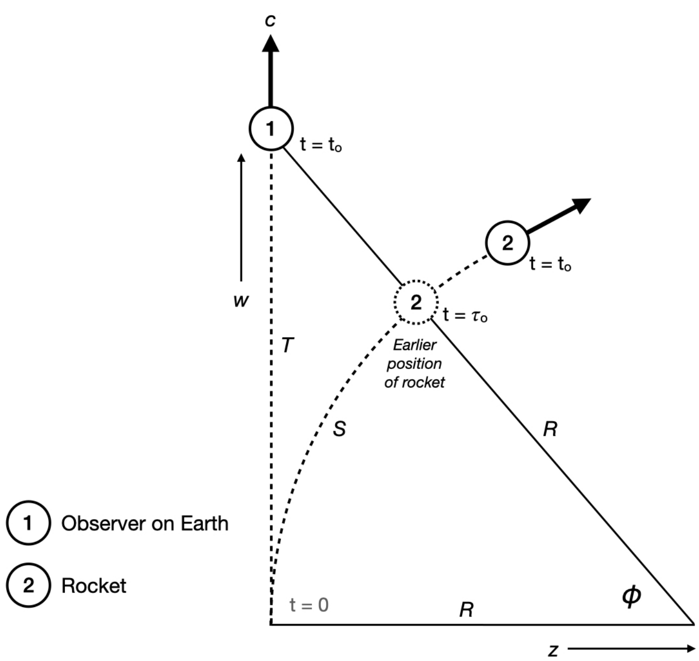

Figure 2: *Time dilation as experienced by observer on Earth (1) when seeing the rocket at its past position (2, dotted circle).*

and also

$$\phi = \tan^{-1}\left(\frac{T}{R}\right) = \tan^{-1}\left(\frac{ct_o}{R}\right) \tag{19}$$

so that, with Equations (18) and (5)

$$S = R\tan^{-1}\left(\frac{ct_o}{R}\right) = \frac{c^2}{a}\tan^{-1}\left(\frac{at_o}{c}\right). \tag{20}$$

The rocket has a constant orbital velocity equal to c, so that the time it took for the rocket to perform the circular segment with length S amounts to $\tau_o = S/c$ and, therefore, after substitution of Equation (20)

$$\tau_o = \frac{c}{a}\tan^{-1}\left(\frac{at_o}{c}\right). \tag{21}$$

The third postulate states that, at time $t = t_o$, the observer on Earth will see the rocket at time $t = \tau_o$. Since $\tau_o < t_o$, this will be perceived by the observer as a 'time dilation' because it will appear as if the time aboard the rocket has been running slower. Note that expression (21) is different compared with the familiar expression from SR, which has the following form [3]:

$$\tau_o = \frac{c}{a}\sinh^{-1}\left(\frac{at_o}{c}\right). \tag{22}$$

The cause of this discrepancy will be discussed in the next paragraph. Here we will demonstrate that expression (21) makes, at least intuitively, perfect sense. Taking the inverse of Equation (21) yields

$$t_o = \frac{c}{a}\tan\left(\frac{a\tau_o}{c}\right). \tag{23}$$

Equation (23) contains a singularity when $a\tau_o/c$ approaches the value of $\pi/2$, for which to will become infinitely large. This happens when

$$\tau_o = \frac{\pi c}{2a}\,, \tag{24}$$

at which point the Equations of motion (7), (8) and (17) reduce to:

$$d(t) = ct \tag{25}$$

$$v_z(t) = c \tag{26}$$

$$a_z(t) = 0 \tag{27}$$

so that the observer on Earth will eventually see the rocket move away with (almost) the speed of light, and with (almost) zero acceleration. This makes perfect sense. The angle ϕ in Figure 2 will then be equal to

$$\phi(\tau_o) = \frac{S(\tau_o)}{R} = c\frac{\pi R}{2cR} = \frac{\pi}{2}\,. \tag{28}$$

Indeed, it can be seen in Figure 2 that t_o will become infinitely large when ϕ approaches the value $\pi/2$. This means that the observer will have to wait an infinitely long time before he can see the rocket reach the speed of light. In other words, *in any finite amount of time the observer will never see the rocket reach the speed of light, regardless its acceleration a.* This finding is in perfect agreement with SR and all experimental observations so far, but is just a little more carefully formulated than "no solid object can ever reach the speed of light" as often stated stated by those who refer to conclusions reached by SR.

6. Violation of the Clock Postulate

The clock postulate states that an accelerating clock, at any point in time, compared to a clock that is stationary with respect to an observer, always has its timing slowed down by a factor

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} . \qquad (29)$$

In other words, the time dilation factor depends only on the instantaneous velocity v of the clock on a given moment, and not on any of its derivatives, such as acceleration. SR makes use of the clock postulate, however, for many years it was not known if the clock postulate were valid or not. Only in the year 1977 Bailey et al. [4] tested the clock postulate for muons subject to a transverse acceleration of up to ~10^{18} g, showing no impact on time dilation. Three years later, Roos et al. [5] also measured the decay of Sigma baryons subject to a longitudinal acceleration up to 5.0×10^{15} g. Again, no deviation from ordinary time dilation was measured.

In the remainder of this paragraph it will be shown that the combination of particle lifetime and the amount of longitudinal acceleration in the Roos experiment [5] was not large enough to demonstrate the discrepancy between Equations (21) and (22). As such, one could argue that the clock postulate was indeed not violated, at least in practical terms. However, it will now be demonstrated that in essence any accelerating clock violates the clock postulate, which explains the discrepancy between Equations (21) and (22).

We will start with the traditional derivation of time dilation as experienced by a stationary observer watching an accelerating clock. Making use of the clock postulate and Equation (15) we get

$$\tau_o = \int_0^{t_0} \frac{1}{\gamma} dt = \int_0^{t_0} \frac{1}{\sqrt{1+\frac{a^2t^2}{c^2}}} dt \qquad (30)$$

Using

$$\int \frac{1}{\sqrt{1+t^2}} dt = \ln(|t+\sqrt{1+t^2}|)+C, \qquad (31)$$

we find

$$\tau_o = \frac{c}{a}\ln\left(\frac{a}{c}t_o + \sqrt{1+\frac{a^2t_o^2}{c^2}}\right) = \frac{c}{a}\sinh^{-1}\left(\frac{at_o}{c}\right). \qquad (32)$$

As shown earlier, a simple geometric derivation for time delation, with the use of Figure 2, resulted in a different expression, given by Equation (21):

$$\tau_o = \frac{c}{a}\tan^{-1}\left(\frac{at_o}{c}\right) .$$

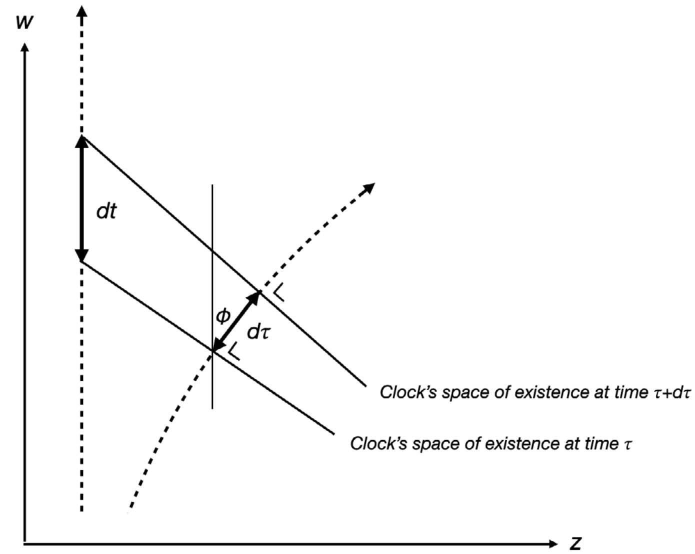

Figure 3: *Violation of the clock postulate shown in the z-w plane in the case of an accelerating clock (curved trajectory) observed by a stationary observer (vertical trajectory). Because the clock's spaces of existence at successive moments in time are never parallel to each other, the relationship* $d\tau = \cos(\phi)\, dt = 1/\gamma\, dt$ *is not valid. The error gets worse for larger values of* ϕ*, that is, for acceleration towards higher velocities.*

The discrepancy can be explained by the violation of the clock postulate. See Figure 3. The observer's trajectory in the *z-w* plane is represented by a dotted vertical line, and the trajectory of the accelerating clock by a circular segment. Two successive spaces of existence for the clock are shown, each perpendicular to its curved trajectory, intersecting the trajectory of the stationary observer. It is immediately seen that the time intervals dt and $d\tau$ are not related by $dt = d\tau/cos(\phi) = \gamma d\tau$. As long as the clock accelerates, its trajectory in the *zw*-plane will be curved, and as a result its successive spaces of existence are never parallel, not even for infinitely small values of dt and $d\tau$. This fact violates the clock postulate, which explicitly assumes that the clock's successive spaces of existence would (at least pairwise) be parallel, i.e., identical to a momentarily co-moving inertial frame.This is never the case.

The discrepancy between Equations (21) and (22) has been quantified in Figure 4, which compares the functions $\sinh^{-1}(x)$ and $\tan^{-1}(x)$. In case of the Roos experiment [5], with maximum acceleration $a = 5.0 \times 10^{15}$ g and a lifetime of the Sigma baryons in the order of a nanosecond, we find

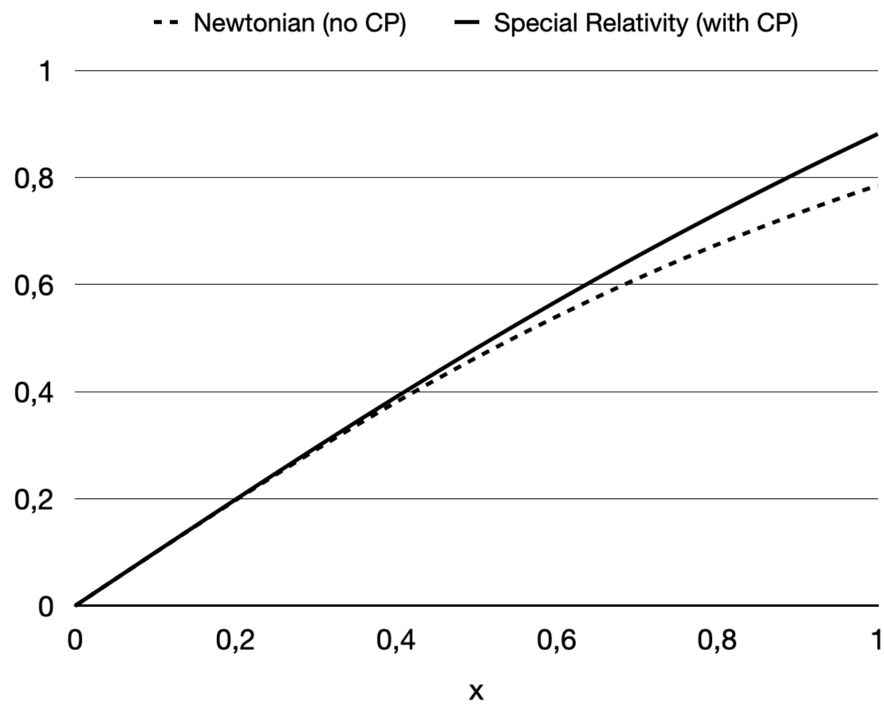

Figure 4: *The functions* $tan^{-1}(x)$ *(dashed curve) and* $sinh^{-1}(x)$ *(solid curve) corresponding to the expressions for time dilation of a continuously accelerated object according to Newtonian Mechanics, as proposed in this paper (Equation 21, dashed curve), and Special Relativity with assumption of the clock postulate (Equation 22, solid curve).*

$$x = at_o / c = 5.0 \times 10^{15} \times 9.81 \times 10^{-9} / 3.0 \times 10^{8} \approx 0.16.$$

For such a low value of x, the difference between $\sinh^{-1}(x)$ and $\tan^{-1}(x)$ is only 0.4%, which is well beyond the measurement accuracy of the Roos experiment which was reported to be several per cents [4]. We predict that a repeated experiment, with sufficiently higher particle energy or longer time measurements, will eventually show a time dilation that cannot be explained by the current formalism of SR.

7. Lorentz contraction, mass increase and the invariability of the speed of light

The Newtonian model presented in this paper also explains the phenomena known as time dilation at constant velocity, Lorentz length contraction, as well as relativistic mass increase and the invariability of the speed of light. We will show this by starting with the classical example of distance travelled by muons in the Earth's atmosphere. The probability that muons can reach the Earth is affected by relativistic corrections of their mean lifetime. If no time dilation would exist, muons would decay in the upper regions of the atmosphere, however, as a consequence of time dilation they are present in considerable amounts also at much lower heights. This phenomenon will now be explained by using Newtonian Mechanics only.

Consider a muon with life time $\Delta\tau$, traveling through the atmosphere with a relativistic velocity, while it is being observed by an observer on Earth. We assume that the muon travels with constant velocity during its entire life time. In the 4-dimensional space both the muon and the observer on Earth travel with the speed of light c, with an angle ϕ between their directions of motion (see Figure 5).

At time $t = t_o$, the muon starts its journey at position A. At the same time the observer is in position C, inside the space of existence of the muon, and therefore the observer perceives the muon at a distance $d(t_o)$. At time $t = t_o + \Delta t$ the observer has moved by a distance $c\Delta t$ and finds himself at position D, whereas the muon has moved by the same distance to position E. Because the observer is not in the muon's space of existence, he will not be able to perceive the muon at position E. Instead, he will perceive the muon at an earlier time $t = t_o + \Delta\tau$ at position B, at a distance $d(t_o + \Delta t)$. From Figure 1 it can be seen that during the time Δt, the observer sees the muon move over a distance

$$s = d(t_o + \Delta t) - d(t_o) = c\,\Delta t \sin(\phi) = v_z\,\Delta t. \quad (33)$$

where

$$v_z = c\sin(\phi)\ . \quad (34)$$

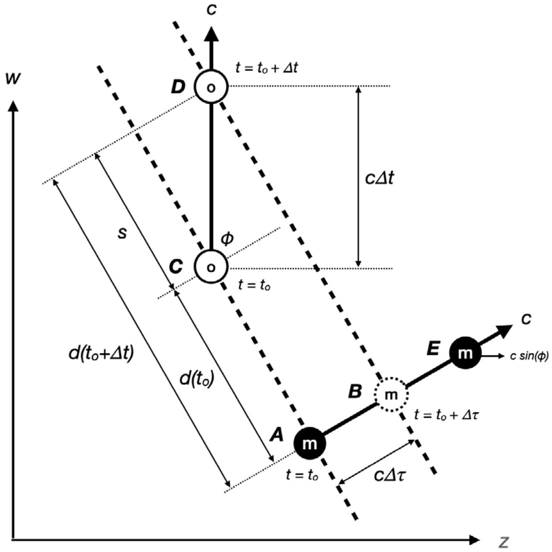

Figure 5: *A muon, moving from A to B during a time $\Delta\tau$, is observed by an observer on Earth, moving from C to D in a time Δt. The observer on Earth at $t = t_o$ is located inside the space of existence of the muon at $t = t_o$, whereas at $t = t_o + \Delta t$ he is located inside the space of existence of the muon at $t = t_o + \Delta\tau$. Therefore at $t = t_o + \Delta t$ the observer sees the muon at an earlier, 'time dilated' moment. According to the observer on Earth, the muon has travelled a distance $s = v_z \Delta t$ during a time Δt.*

We see that the muon's velocity v_z is perceived by the observer as its 4-dimensional velocity vector projected on the observer's space of existence. From Equation (33) we see that the muon, with lifetime τ, travelling a distance $c\Delta\tau$ in the 4-dimensional space, is perceived by the stationary observer as a particle that travels a distance $v_z \Delta t$ during a time Δt. In Figure (5) we also see that

$$\frac{\Delta\tau}{\Delta t} = \cos(\phi) = \sqrt{1 - \sin^2(\phi)} = \sqrt{1 - (\frac{v_z}{c})^2} = \frac{1}{\gamma} \tag{35}$$

where γ is the Lorentz factor known from SR. As a result, to the observer it will seem as if the lifetime of the muon is longer, or its time is running slower, by a factor of γ. This is the effect of time dilation as known from SR.

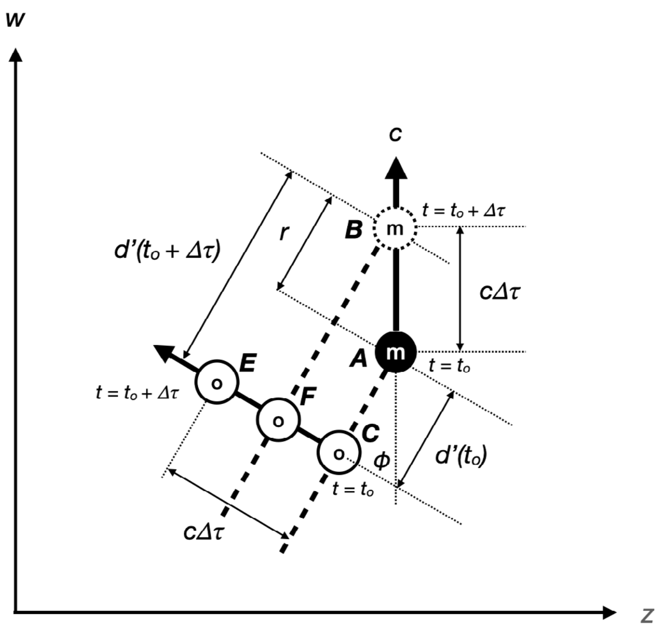

Figure 6: *The muon, moving from A to B during a time $\Delta\tau$, is accompanied by a second observer, watching the observer on Earth moving from C to F. During the life time $\Delta\tau$ of the muon the second observer sees the observer on Earth move over a distance $r = v_z \Delta\tau$.*

Now imagine a second observer moving along with the muon, see Figure 6. In the time $\Delta\tau$ that the muon and the second observer move from position A to position B, the observer on Earth moves from position C to position E. However, at time $t = t_o + \Delta\tau$ the second observer will not see the observer on Earth at position E, but at position F instead. During the life time $\Delta\tau$ of the muon the second observer sees the observer on Earth move over a distance

$$r = d'(t_o + \Delta\tau) - d'(t_o) = c\,\Delta\tau\,\sin(\phi) = v_z\,\Delta\tau. \tag{36}$$

Consequently, also the second observer perceives the velocity of the observer on Earth as the latter's 4-dimensional velocity vector projected on the second observer's space of existence. In addition, the second observer will experience to have traveled a distance $v_z\,\Delta\tau$ across the Earth during the life time $\Delta\tau$ of the muon. Because the observer on Earth sees the muon travel a distance $v_z\,\Delta t$ during his life time, as shown before, the second observer perceives distances on Earth contracted by a factor

$$\frac{v_z \Delta\tau}{v_z \Delta t} = \gamma\,, \tag{37}$$

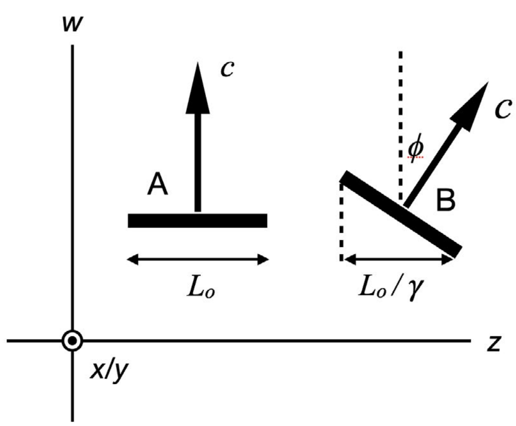

Figure 7: *Two observers with relative velocity will see each other's length as the orthogonal projection of the actual length on their own space of existence.*

where we made use of Equation (35). This is the familiar Lorentz factor for length contraction known from SR. It is straightforward to see that this effect will be symmetrical: both observers will see the length of the other contracted in the direction of motion by a factor of γ. As a result, similar to the observed velocity, both observers perceive each other's length as the projection of the actual length orthogonally projected on their own space of existence, see Figure 7.

A similar reasoning explains why the mass of a particle traveling at relativistic velocity seems to increase by a factor of γ. When the particle moves into the z-direction and the observer applies a force F in the z-direction on the particle (the way it happens, for example, in a linear accelerator), the effective force as experienced by the particle - that is, the force that will contribute to its centripetal acceleration - will be equal to $F\cos(\phi) = F/\gamma$. This reduced effective force will be interpreted by the observer as a mass increase by a factor of γ. This can be easily seen from Newton's second law $a = F/m$, which in this particular case takes the form

$$a = F/\gamma m_o \tag{38}$$

where m_o is the rest mass of the particle. The reduced effective force F/γ (in stead of F) will be interpreted by the observer as an increased effective mass γm_o (in stead of m_o).

Even the invariability of the speed of light can be easily explained by the model presented in this paper. Imagine a light source emitting a light wave with a hyper-spherical wave front in a 4-dimensional space, that is, a wave front described as

$$x^2 + y^2 + z^2 + w^2 = (ct)^2\,, \tag{39}$$

which would correspond to the solution of a 4-dimensional version of Maxwell's Equations. According to postulate 1, an observer will perceive this wave front as a 3-dimensional projection on its own space of existence, regardless its own velocity with respect to the light source. Consequently, this projection will always take the shape

$$(x_o - p)^2 + (y_o - q)^2 + (z_o - r)^2 = (ct)^2 \tag{40}$$

where x_o, y_o and z_o represent three orthogonal dimensions perpendicular to the observer's velocity vector, whereas the location of the light source perceived by the observer is (p,q,r). Equation (40) shows that the observer will always see the wave front approach with velocity c, regardless his own velocity with respect to the light source. It is simply the result of a 3- dimensional projection of a 4-dimensional wave front. Interestingly, the same observation would be made in case the light wave would use a medium of propagation, such as a luminiferous aether, which would provide an alternative explanation for the observations made by Michelson and Morley [1].

8. The Expanding 4D Universe

So far it has been shown that, with a set of alternative postulates, Newtonian Mechanics can reproduce the equations of motion for objects accelerating towards relativistic velocities, including Lorentz contraction, time dilation, mass increase and the invariability of the speed of light. One might argue that one of the alternative postulates, postulate 1, violates the scientific consensus that no solid object can ever reach the speed of light. However, this is a conclusion derived from SR, and since we explicitly abandoned SR for the model presented in this paper (albeit temporarily), postulate 1 is perfectly acceptable. This is confirmed by the conclusion derived from our alternative model, which states that no observer can ever see an object reach the speed of light in any finite amount of time. This conclusion is in perfect agreement with SR as well as with countless experimental results. However, it seems appropriate to put postulate 1 into a context, and hypothesise why the Earth would be moving through a 4-dimensional space with the speed of light. In essence this can be explained by a 4-dimensional version of the classical Big Bang theory (see Figure 8). Imagine all matter of the universe concentrated in a single point in a 4-dimensional Euclidian space. Again, this is an Euclidian space with 4 orthogonal spatial dimensions, *w, x, y*, and *z*, not space-time as known from SR. At the birth of the Universe, a Big Bang initiated the symmetrical expansion of matter into all 4 orthogonal dimensions with the speed of light *c* (for the sake of simplicity we assume a constant expansion velocity). As a result, all matter in the Universe got distributed over the hypersurface of a 4-dimensional hypersphere, described by:

$$w^2 + x^2 + y^2 + z^2 = c^2 t_b^2 \qquad (41)$$

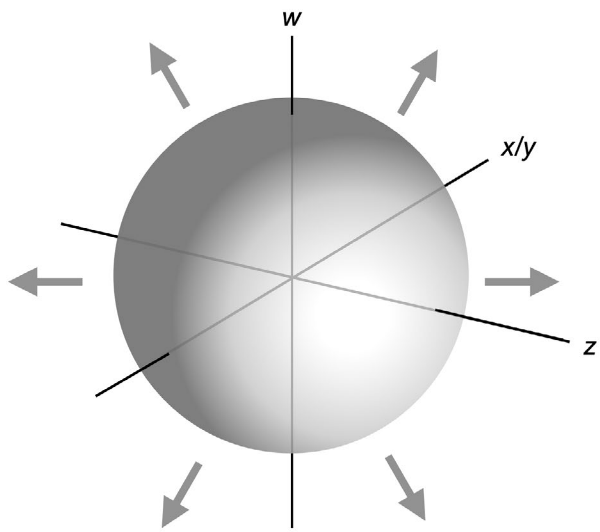

Figure 8: *Schematic of the 4-dimensional expanding Universe, showing a hypersphere in a 4-dimensional Euclidian space, expanding into 4 orthogonal spatial dimensions. All matter in the Universe is concentrated over the hypersurface of the 4-dimensional hypersphere that started to expand with the speed of light c at the moment of the Big Bang. The radius of the sphere is therefore equal to* $R = ct_b$*, where* t_b *is the time in seconds since the occurrence of the Big Bang.*

where c is the speed of light and t_b the time since the Big Bang.

Figure 9 illustrates what happens when an observer on Earth (in point A) looks at a far away celestial object (in point B), shown in the *z-w* plane. Both the observer and the celestial object are moving at velocity c through the 4-dimensional universe, however, because of the large distance between them there is is a significant angle φ between their velocity vectors. According to postulate 1, the observer in point A can only see the 3-dimensional space extending over x, y and z, indicated by the horizontal dotted line. The observer in point A is also located inside the space of existence of the celestial object when it was at position C, and, according to postulate 2, will therefore perceive the celestial object at a distance

$$d = R \sin(\phi) = ct_b \sin(\phi)\ , \qquad (42)$$

where t_b is the time since passed since the Big Bang. Since the observer in point A is not able to see the *w* dimension, the velocity of the celestial object is perceived as the projection of its actual velocity *c* projected on the *xyz*-space (as shown in the previous section), so that the observer on Earth sees the celestial object recede with an apparent velocity

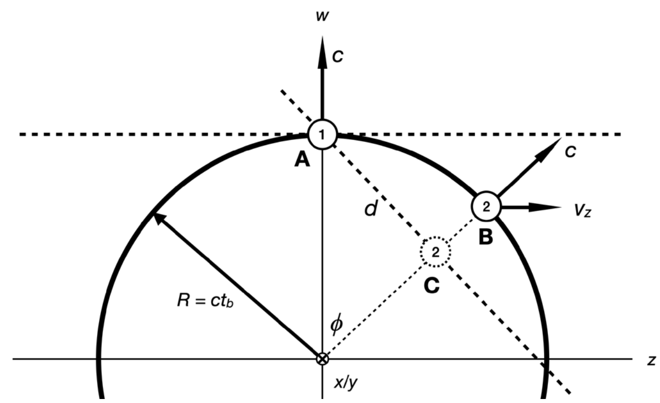

Figure 9: *An observer (1) on Earth in point A watching a far away celestial object (2) in point B. Both the observer and the celestial object are located at the outer shell of an expanding 4- dimensional hypersphere. The observer is located inside the space of existence of the celestial object at an earlier time (dotted circle), when it was in point C. The observer will therefore measure a distance* $d = ct_b \sin(\varphi)$*. At the same time, since the observer in A cannot discern the w-dimension, the perceived velocity of the celestial object is the projection of its actual velocity on the z-axis,* $v_z = c \sin(\varphi)$*. In fact, the ratio* H_o *of the perceived velocity and distance of a far away celestial object is always the same for any observer anywhere in the Universe.*

$$v_z = c \sin(\phi) \qquad (43)$$

From equations (42) and (43) it directly follows that the ratio of velocity and distance of celestial objects amounts to:

$$H_o = v_z / d = 1/t_b \qquad (44)$$

which is the reciprocal of the age of the Universe, also known as Hubble's constant. We see that a simple '4D Big Bang' model not only elegantly describes the relationship between distance and receding velocity of all celestial objects, but also provides a rationale why the Earth would move through a 4-dimensional

Euclidian space with the speed of light, as stated by postulate 1. As a matter of fact, the Earth is located at the edge of the 4-dimensional expanding Universe, however, because of the collapsed dimension in its direction of motion (the *w*-direction in Figure 9), an observer on Earth perceives to be in the center of a 3-dimensional spherically expanding Universe with dimensions *x, y* and *z*.

9. Discussion and Conclusions

Using an alternative set of postulates than those used by SR, we applied Newtonian Mechanics and a very simple geometrical method to derive the relativistic equations of motion for an accelerating object, up to velocities approaching the speed of light. The resulting expressions for travelled distance, perceived velocity and perceived acceleration were identical to those traditionally obtained by SR. Also the mechanisms of Lorentz length contraction, mass increase and the invariability of the speed of light were explained with simple geometrical methods. Interestingly, the mechanism for the invariability of the speed of light would even be valid in case electromagnetic radiation would be propagating through a luminiferous ether. This provides an alternative interpretation of the observations by Michelson and Morley in 1887 [1], and suggests that a luminiferous ether might still exist, contrary to current scientific consensus.

The expression derived for time dilation during longitudinal acceleration, from zero speed towards a velocity close to the speed of light, was different from the expression known from SR. This finding was attributed to a violation of the clock postulate during longitudinal acceleration, which has not yet been verified nor falsified experimentally. The experiment by Bailey et al. [4] only considered centripetal acceleration and not longitudinal acceleration as discussed in the this paper, whereas the combination of particle lifetime and acceleration in the Roos experiment [5] was too small to demonstrate the discrepancy. We predict that a repeated experiment, with sufficiently higher particle energy or longer time measurements, will eventually show a time dilation that cannot be explained by the current formalism provided by SR. We emphasise, however, that the model presented in this paper does not violate any experimental verifications of SR performed so far.

Obviously, much more work would be required to have the proposed model provide a mathematically consistent formulation that completely covers SR and all of its implementations and adjacencies in contemporary physics, including consistency with the flat space time limit of general relativity (which is currently a topic of the author's ongoing research). However, the elegant simplicity of the presented model, the suggestion that Relativity is not complementary to Newtonian Mechanics but that both formalisms are in essence one and the same, the alternative postulates, which might provide new insights into the actual nature of our Universe, the suspected violation of the clock postulate during longitudinal acceleration, and last but not least the possibility that a luminiferous aether might exist after all, are considered interesting enough for a communication within the scientific community, and hopefully inspires further research.

		Special Relativity	Newtonian Mechanics
Travelled distance during acceleration		$d(t) = \frac{c^2}{a}(\sqrt{1+a^2t^2/c^2}-1)$	Identical
Perceived velocity during acceleration		$v_z(t) = \frac{at}{\gamma}$	Identical
Perceived acceleration		$a_z(t) = \frac{a}{\gamma^3}$	Identical
Lorentz contraction at constant velocity		$L = L_o/\gamma$	Identical
Mass increase at constant velocity		$m = \gamma m_o$	Identical
Time dilation at constant velocity		$\tau = 1/\gamma\ t$	Identical
Time dilation during longitudinal acceleration	*With Clock Postulate*	$\tau = \frac{c}{a}\sinh^{-1}\left(\frac{at}{c}\right)$	Identical
	Without Clock Postulate	*N/A*	$\tau_o = \frac{c}{a}\tan^{-1}\left(\frac{at_o}{c}\right)$

Table 1: *Overview of the expressions for various physical quantities derived by the traditional Theory of Relativity, and the expressions derived in this paper by using Newtonian Mechanics and an alternative set of postulates. All results are identical, but Newtonian Mechanics also provide an expression for time dilation during longitudinal acceleration without the use of the Clock Postulate.*

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