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Review Article

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About Nature of Nuclear Forthes

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Annotation

After the discovery of the proton-neutron composition of nuclei, the problem of nuclear forces nature became especially urgent. Nowadays, nuclear forces are explained by the action of a special strong interaction that occurs when nuclear nucleons exchange special particles - gluons. This article proves that the attraction between protons and neutrons can be explained by the well-known quantum mechanical effect, which was first described about a hundred years ago. This is the attraction between two protons that occurs when they are exchanged by electron (in this case relativistic). This makes it possible, abandoning the gluon model, to obtain quantitative estimates of the magnitude of the mass defect of both light and heavy nuclei.

Keywords: Proton, Neutron, Nuclear Forth, Electron, Light Nuclei, Heavy Nuclei, Defect of Mass

1. Introduction

British scientist William Gilbert formulated about 400 years ago a postulate that can be considered the main postulate of the natural sciences [1]. According to Gilbert, all theoretical constructions claiming to be scientific should be tested and confirmed experimentally.

Despite the fact that nowadays it is apparently impossible to find a researcher who would disagree with this statement, a number of modern physical theories do not satisfy this principle [2].

In the physics of the microcosm, this refers to those theories from which it is impossible to calculate the main characteristic parameters of the objects under study. These include existing models of nuclear physics, which do not make it possible to calculate the masses of atomic nuclei.

An alternative approach to solving this problem is discussed below.

This new approach to the problem of the nature of strong interaction is based on the effect of attraction described by the classics of quantum mechanics almost a hundred years ago. This attraction occurs between protons during the exchange of electron [3]. To describe the attraction of nuclear objects, it is necessary to take into account that electrons in this case must be relativistic [4, 5].

In this case, neutron is considered as a composite particle consisting of proton and relativistic electron, which makes it possible to fairly accurately estimate the mass of neutron, its magnetic moment and decay energy.

What are the features of the forces acting between nucleons inside nuclei?

The enormous binding energy of nucleons indicates that these inter-nucleon forces are created by a very intense interaction. This interaction has the character of attraction if the nucleons are at short distances from each other, despite the strong electrostatic repulsion between the protons.

Nuclear forces are short-acting - at distances between nucleons exceeding about $2 \cdot 10^{-13}$ cm, their action is no longer detectable. At distances less than 10^{-13} cm, the attraction of nucleons is replaced by repulsion.

The intra-nuclear interaction is not Coulomb, it does not depend on the charge of the nucleons.

The nuclear forces depend on the mutual orientation of spins of interacting nucleons. For example, neutron and proton are held together to form deuteron only when their spins are parallel to each other.

Nuclear forces have a saturation property (which means that each nucleon in the nucleus interacts with a limited number of nucleons). This property follows from the fact that the binding energy per nucleon is approximately the same for all nuclei, starting from ^4_2He. In addition, the saturation of nuclear forces is also indicated by the proportionality of the volume of the nucleus to the number of nucleons forming it.

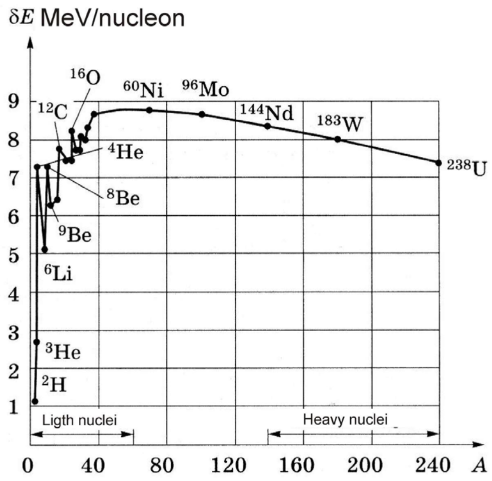

Figure 1: The binding energy per one nucleon of the nucleus.

The dependence of the binding energy (in MeV) per nucleon on the mass number of the nucleus is shown in Figure (1).

From this figure it can be seen that there are two different mechanisms that determine the nuclear forces in light and heavy nuclei differently.

In solving these problems, the first important step is to build a neutron model that makes it possible to predict its main observable properties.

2. Electromagnetic Model of Neutron

2.1 Neutron and the Quark Model

There are several theoretical constructions of the twentieth century that need to be revised due to their incompleteness or disagreement with the measurement data [2]. Apparently, the quark model of elementary particles can be replaced by a description of their excited states [6].

The formation of the quark model in the chain of the science of the structure of matter seems to be quite consistent: all substances consist of molecules and atoms. The central elements of atoms are nuclei. Nuclei consist of protons and neutrons, which in turn consist of quarks.

The quark model assumes that almost all elementary particles consist of quarks. The quark structure of nucleons is of particular interest in this case.

It seems that experts in elementary particle physics initially proceeded from the assumption that at the creation of the world, suitable parameters were individually selected for each elementary particle: charge, spin, mass, magnetic moment, etc.

Gell-Mann simplified this work somewhat. He developed a rule according to which a set of quarks determines the total charge and spin of the former elementary particle. But masses and magnetic moments of these particles do not fall under this rule.

The Gell-Mann quark model assumes that quarks, which make up all elementary particles (with the exception of the lightest), must have a fractional (equal to 1/3 e or 2/3 e) electric charge.

In the 60s, after the formulation of this model, many experimenters tried to find particles with a fractional charge. But without success.

In order to explain this, it was assumed that quarks are characterized by a confinement, i.e. a property that prohibits them from somehow manifesting themselves in a free state. At the same time, it is clear that confinement removes quarks from subordination to the Gilbert principle. In this form, the model of quarks with fractional charges claims to be scientific without confirmation by measurement data.

It should be noted that the quark model successfully describes some experiments on the scattering of particles at high energies, for example, the formation of jets or the feature of scattering of high-energy particles without destruction. However, this does not seem to be enough to recognize the real existence of quarks with a fractional charge.

In the 30s of the last century, theoretical physicists, due to the lack of necessary experimental data, formed the opinion that neutrons, like protons, are elementary particles [7]. In the Gell-Mann quark model, the neutron is also assumed to be an elementary particle in the sense that it consists of a different set of quarks than proton.

However, the fact that neutron is an unstable particle and decays into proton and electron (+ antineutrino) gives reason to attribute it to non-elementary composite particles.

The quark model does not aim to predict the basic properties of neutron, such as its mass, magnetic moment and decay energy. The electromagnetic model of neutron makes it possible to successfully evaluate these parameters [4].

Suppose that neutron, as well as a Bohr hydrogen atom, consists of proton around which electron rotates at a very small distance from it. Near proton, the electron's motion must be relativistic.

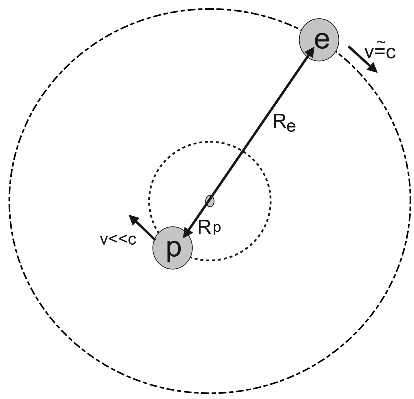

Figure 2: A system consisting of a proton and a heavy (relativistic) electron, revolving around a common center of mass.

2.2 The Interaction of Relativistic Electron with Proton

Consider a composite particle in which an electron having a rest mass me and a charge me is moving around a proton in a circle of radius Re with a speed $v_e \to$ c (Figure (2)).

Since we initially assume that the motion of the electron is likely to be relativistic, it is necessary to take into account the relativistic effect of the growth of its mass:

$$m_e^* = \gamma m_e, \tag{1}$$

Where the relativistic factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{2}$$

and β=v/c.

The rotation of the heavy electron m_e* does not allow to consider the proton as at rest. The proton will also move, revolving around the center of mass common with the heavy electron.

Let's introduce a parameter characterizing the ratio of the mass of a relativistic electron to the mass of proton:

$$\vartheta = \frac{\gamma m_e}{M_p/\sqrt{1-\beta_p^2}}. \tag{3}$$

It follows from the condition of equality of momenta that $\beta_p = \vartheta$ therefore the radii of the orbits of the electron and proton can be written as:

$$R_e = \frac{R_{ep}}{1+\vartheta}, \quad R_p = \frac{R_{ep}\vartheta}{1+\vartheta}. \tag{4}$$

Where $R_{ep}=R_e+R_p$

The relativistic factor characterizing the electron in this case is equal to

$$\gamma = \frac{\vartheta}{\sqrt{1-\vartheta^2}}\frac{M_p}{m_e}. \tag{5}$$

2.2.1 Larmor's Theorem

To describe the characteristic feature of proton motion along a circle of radius Rp, we can use Larmor's theorem [8]. According to this theorem, in a reference frame rotating with proton at a frequency of Ω, a magnetic field is applied to it. This field is determined by its gyromagnetic ratio

$$H_L = \frac{\Omega}{\xi\frac{e}{2M_pc}}. \tag{6}$$

Where ξ =2.79 is the magnetic moment of the proton in units of Bohr magnetons.

As a result of the action of this field, the proton magnetic moment turns out to be oriented perpendicular to the plane of rotation. In other words, we can say that due to the interaction with this field, the rotation of the electron should occur in the plane of the "equator" of proton.

2.2.2 Quantization of Equilibrium Orbit

It can be assumed that, as in the formation of a stable orbit in a hydrogen atom, the orbit of a relativistic electron will be stable if an integer number of de Broglie wavelengths λ_{dB} fits on the circumference of the electron ring $2\pi_{Re}$, that is:

$$2\pi R_e = n\lambda_{dB}. \tag{7}$$

Where n is integer number

And

$$\lambda_{dB} = \frac{2\pi\hbar}{\gamma m_e c}. \tag{8}$$

That is, in accordance with this assumption, the stability condition of the electronic orbit takes the form:

$$\frac{r_c}{R_e} = \frac{\vartheta}{n\sqrt{1-\vartheta^2}}\frac{M_p}{m_e} = \frac{\gamma}{n} \tag{9}$$

Where $r_c = \frac{\hbar}{m_e c}$ is the Compton radius.

2.2.3 The Kinetic Energy of Rotating Electron

The kinetic energy of a relativistic electron is expressed by the equality:

$$\mathcal{E}_{kin}^e = (\gamma - 1)\cdot m_e c^2 \tag{10}$$

Due to the assumption of the electron to be ultrarelativistic

$$\mathcal{E}_{kin}^e \approx \gamma \cdot m_e c^2 \tag{11}$$

In this case, the centrifugal force acts on the electron:

$$\mathcal{F}_1 = \gamma m_e[\omega[\omega, R_e]] = \frac{\gamma m_e c^2}{R_e} \tag{12}$$

The kinetic energy of proton is equal to:

$$\mathcal{E}_{kin}^p = \left(\frac{1}{\sqrt{1-\vartheta^2}} - 1\right)\cdot M_p c^2 \tag{13}$$

An additional contribution to the kinetic energy of electron creates a magnetic field that occurs when it rotates. The energy of this field is equal to

$$\mathcal{E}_\Phi = \frac{\Phi I}{2c}, \tag{14}$$

Due to the fact that the electron motion in the orbit is quantized, the magnetic flux penetrating the ring of radius R_e must be equal to the quantum of the magnetic flux Φ_0:

$$\Phi = \Phi_0 = \frac{2\pi\hbar c}{e}. \tag{15}$$

Since the current in the electronic ring

$$I = \frac{ec}{2\pi R_e}, \tag{16}$$

we have

$$\mathcal{E}_{\Phi_e} = \frac{e^2}{R_e}\frac{1}{2\alpha}\frac{r_c}{R_e} = \frac{1}{2n}\gamma m_e c^2. \tag{17}$$

The force arising at the same time, tending to break the current ring, turns out to be equal

$$\mathcal{F}_2 = \frac{\gamma}{2n}\frac{m_e c^2}{R_e}. \tag{18}$$

The magnetic energy created by the rotation of a proton is much less:

$$\mathcal{E}_{\Phi_p} = \frac{\sqrt{2}\cdot\vartheta^2}{\sqrt{1-\vartheta^2}}\cdot M_p c^2. \quad (19)$$

The force corresponding to this energy is applied to proton and does not directly affect the electron equilibrium orbit.

Thus, the total kinetic energy of the electron, taking into account the energy of the magnetic field that it creates when rotating:

$$\mathcal{E}_{kin}^{\Sigma} = \mathcal{E}_{kin}^{e} + \mathcal{E}_{\Phi_e} = \left(1+\frac{1}{2n}\right)\gamma m_e c^2. \quad (20)$$

2.2.4 The Coulomb interaction in the system of relativistic electron + proton

The energy of Coulomb attraction between a proton and a relativistic electron is [8], §24:

$$\mathcal{E}_C = -\gamma\frac{e^2}{R_{ep}} = -\gamma\frac{\alpha r_c}{R_e(1+\vartheta)} m_e c^2. \quad (21)$$

Where $\alpha = e^2/\hbar c$ is the fine structure constant.

Therefore, the Coulomb attraction force acting between these particles is equal to

$$\mathcal{F}_3 = -\gamma\frac{e^2}{R_{ep}^2} = -\gamma\frac{\alpha}{(1+\vartheta)^2}\frac{r_c}{R_e}\frac{m_e c^2}{R_e}. \quad (22)$$

2.2.5 Interaction of Electron with Magnetic Field of Proton

In the present case a proton possesses two magnetic moments. This is its own internal magnetic moment:

$$\mu_p = \frac{\xi e\hbar}{2M_p c} \quad (23)$$

and the orbital magnetic moment which occurs due to the fact that proton rotates in an orbit of radius R_p:

$$\mu_{0p} = \frac{e\vartheta R_p}{2} \quad (24)$$

Therefore, the energy of interaction of rotating electron with the proton magnetic field consists from two components:

$$\mathcal{E}_\mu = \frac{\gamma e}{2R_e^2}\left(\mu_{0_p} - \mu_p\right). \quad (25)$$

In order for the system energy to be less, the magnetic moments μp and μ0p must be oppositely directed.

The force that acts on the rotating electron can be written as:

$$\mathcal{F}_4 = \gamma e\beta\left(\frac{\mu_{0_p}}{R_e^3} - \frac{\mu_p}{R_{ep}^3}\right) = \gamma e\left(\frac{\mu_{0_p}}{R_e^3} - \frac{\mu_p}{R_e^3(1+\vartheta)^3}\right) = \gamma\frac{m_e c^2}{R_e}\left(\frac{\vartheta^2}{2} - \frac{\xi_p}{(1+\vartheta)^3}\frac{\vartheta}{2n\sqrt{1-\vartheta^2}}\right)\frac{\vartheta}{2n\sqrt{1-\vartheta^2}}\alpha\frac{M_p}{m_e}. \quad (26)$$

The magnetic moment of electron is not considered because, as will be shown below, the generalized momentum (spin) of the electron orbit is equal to zero and there is no direction for the selected orientation of the electron magnetic moment in the system.

2.2.6 Equilibrium Electron Orbit

The equilibrium condition for the electron orbit is:

$$\sum_{i=1}^{4}\mathcal{F}_i = 0. \quad (27)$$

At summing of Equation (12), Equation (22), Equation (18) and Equation (26)

$$\gamma\frac{m_e c^2}{R_e} - \gamma\frac{e^2}{R_{ep}^2} + \gamma\frac{m_e c^2}{2R_e} - \gamma e\left(\frac{\mu_{0p}}{R_e^3} - \frac{\mu_p}{R_{ep}^3}\right) = 0. \quad (28)$$

and after simplifying transformations taking into account Eq.(9) we get:

$$1+\frac{1}{2n} - \left(\frac{\vartheta}{n\sqrt{1-\vartheta^2}}\frac{\alpha M_p}{m_e}\right)\left[\frac{1}{(1+\vartheta)^2} + \frac{\vartheta^2}{2} - \frac{\xi}{2n(1+\vartheta)^3}\frac{\vartheta}{\sqrt{1-\vartheta^2}}\right] = 0. \quad (29)$$

2.3 The Basic State of Neutron

The basic state of this system with the minimal energy is realised at n=1, that is, the length of the electron orbit is equal to the de Broglie wavelength.

We need to find a solution of Equation (29) under this condition:

$$\frac{3}{2} - \alpha\gamma\left[\frac{1}{(1+\vartheta)^2} + \frac{\vartheta^2}{2} - \frac{\xi\gamma}{2(1+\vartheta)^3}\frac{m_e}{M_p}\right] = 0. \quad (30)$$

As the result we have

$$\vartheta = 0:1991 \quad (31)$$

and

$$R_e = 1.03 \cdot 10^{-13} cm \quad (32)$$

2.4 Equilibrium electron orbit. Approximate solution

The complex Equation (30) defining the parameter ϑ can be simplified. Its decomposition gives an approximate expression

$$\vartheta_1 \approx \frac{3\sqrt{\pi}}{2}\frac{m_e}{\alpha M_p} \approx 0.1985. \quad (33)$$

We can introduce the value of the new fundamental length R_*, expressed in terms of the Compton radius r_c or the Bohr radius a_B:

$$R_* = \alpha r_c = \alpha^2 a_B = \frac{e^2}{m_e c^2} = 2.8183 \cdot 10^{-13} cm \qquad (34)$$

The radius of the electron orbit is equal in order of magnitude to the fundamental length R_*:

$$R_e = r_C \frac{\sqrt{1-\vartheta^2}}{\vartheta} \frac{m_e}{M_p} \approx \frac{R_*}{\sqrt{8}} = 9.9 \cdot 10^{-14} cm. \qquad (35)$$

This value is consistent with the estimate obtained earlier [4]:

$$R_e = \frac{\hbar}{c} \sqrt{\frac{\alpha \xi}{2 m_e M_p}} = 9.1 \cdot 10^{-14} cm \qquad (36)$$

2.4.1 Spin of Neutron

The total generalized electron momentum can be written as

$$S_{0e} = \left[R_e \times \gamma \left\{ m_e c - \frac{e}{c} A_e \right\} \right] \qquad (37)$$

Or in the scalar form

$$S_{0e} = \gamma m_e c R_e \qquad (38)$$
$$\left\{ \frac{3}{2} - \alpha\gamma \left(\frac{1}{(1+\vartheta)} - \frac{1-\vartheta^2}{2} + \alpha\gamma \frac{\xi_p}{(1+\vartheta)^2} \frac{m_e}{\alpha M_p} \right) \right\}.$$

Substituting the values of ϑ and Re calculated above into this equality, we come to the conclusion that

$$S_{0e} = 0 \qquad (39)$$

For this reason, the total spin of particles in question is 1/2 because it is created by the spin of proton.

The equality to zero of the spin of this electron ring plays an important role in the formation of the equilibrium state of system. Due to the fact that $S_0 = 0$ the electron's own spin and its magnetic moment are devoid of orientation direction in space and fall out of the balance equations, and therefore out of consideration in this problem altogether.

2.4.2 Mass of Neutron

The mass of a composite particle is determined by the sum of the rest masses of the particles, their relativistic kinetic energy and the mass defect arising from the potential energy of their internal interaction. Calculate these contributions

Kinetic energy of electron and proton

Summing Equations (11),(13),(17) and (19) at n=1 we obtain

$$\mathcal{E}(kin) = \frac{\vartheta}{\sqrt{1-\vartheta^2}} \left[1 + \left(\frac{1}{\sqrt{1-\vartheta^2}} - 1 \right) \frac{\sqrt{1-\vartheta^2}}{\vartheta} + \left(\frac{1}{2} + \sqrt{2}\vartheta \right) \right] \cdot M_p c^2 \qquad (40)$$

Potential energy of electron and proton

Summing Equations (21) and (25) at n=1 we obtain

$$\mathcal{E}(pot) = \frac{\alpha M_p}{m_e} \left[\frac{1}{1+\vartheta} - \frac{\vartheta^2}{2} \left(1 - \frac{1}{(1+\vartheta)^3} \cdot \frac{\xi_p}{\cdot \vartheta \sqrt{1-\vartheta^2}} \right) \right] \left(\frac{\vartheta}{\sqrt{1-\vartheta^2}} \right)^2 \cdot M_p c^2. \qquad (41)$$

The neutron mass

The total mass of proton and electron at taking in to account their energies:

$$\begin{aligned} M_{total} &= m_e + M_p + \frac{\mathcal{E}(kin)}{c^2} - \frac{\mathcal{E}(pot)}{c^2} = \\ &= m_e + M_p + \\ &+ \frac{\vartheta}{\sqrt{1-\vartheta^2}} \left[1 + \left(\frac{1}{\sqrt{1-\vartheta^2}} - 1 \right) \frac{\sqrt{1-\vartheta^2}}{\vartheta} + \left(\frac{1}{2} + \sqrt{2}\vartheta \right) \right] \cdot M_p - \\ &- \frac{\alpha M_p}{m_e} \left[\frac{1}{1+\vartheta} - \frac{\vartheta^2}{2} \left(1 - \frac{1}{(1+\vartheta)^3} \cdot \frac{\xi_p}{\vartheta\sqrt{1-\vartheta^2}} \right) \right] \left(\frac{\vartheta}{\sqrt{1-\vartheta^2}} \right)^2 \cdot M_p \end{aligned} \qquad (42)$$

The sum of kinetic and potential energy thus obtained must correspond to the energy released during the decay of the particle. For the neutron, this estimate is in qualitative agreement with the measured data (Table 2.5.2).

2.4.3 The Neutron Magnetic Moment

The particle magnetic moment is the sum of the proton magnetic moment and magnetic moments of orbital currents of electron and proton.

The total magnetic moment generated by of both circular currents

$$\mu_0 = -\frac{e\beta_e R_e}{2} + \frac{e\beta_p R_p}{2} = \frac{eR_{ep}}{2}\frac{(1-\vartheta^2)}{(1+\vartheta)} = \frac{eR_{ep}}{2}(1-\vartheta). \tag{43}$$

If to express this moment in the magnetons of Bohr μ_B, we get

$$\xi_0 = \frac{\mu_0}{\mu_B} = -\frac{(1-\vartheta^2)\sqrt{1-\vartheta^2}}{\vartheta}. \tag{44}$$

At ϑ = 0:1991 we have

$$\xi_0 \approx -4.7269 \tag{45}$$

Summing it with the proton magnetic moment, we get

$$\xi_{total} = \left[-\frac{(1-\vartheta^2)\sqrt{1-\vartheta^2}}{\vartheta} + 2.79\right] \approx -1.9341. \tag{46}$$

It agrees well with the tabular value

$$\xi_{neutron} = -1.91304273. \tag{47}$$

2.5 The Excited States of Neutron

Just like the Bohr atom, a neutron, in addition to the ground state, can have excited states with n > 1.

2.5.1 The excited state with n=2

Under this condition Equation (29) transforms to:

$$1 + \frac{1}{2\cdot 2} - \left(\frac{\vartheta}{2\sqrt{1-\vartheta^2}}\frac{\alpha M_p}{m_e}\right)\left[\frac{1}{(1+\vartheta)^2}\right] + \tag{48}$$
$$+\left(\frac{\vartheta}{2\sqrt{1-\vartheta^2}}\frac{\alpha M_p}{m_e}\right)\left[\frac{\vartheta^2}{2} - \frac{\xi}{2\cdot 2(1+\vartheta)^3}\frac{\vartheta}{\sqrt{1-\vartheta^2}}\right] = 0.$$

The solution to this equation is

$$\vartheta = 0.236 \tag{49}$$

Table 1: The comparison of calculated particle mass values with measurement Data

n	$\frac{\mathcal{E}_{kin}}{c^2}$	$\frac{\mathcal{E}_{pot}}{c^2}$	M_{total} Eq.(42)	experimental data	$\Delta = \frac{M_{exp}-M_{calc}}{M_{exp}}$
n=1	$702m_e$	$700m_e$	$1839m_e$	$M_{n_0} = 1837m_e$	0.001
n=2	$879m_e$	$778m_e$	$1938m_e$	$M_{\Lambda^0} = 2183m_e$	0.11
n=3	$2103m_e$	$1740m_e$	$2200m_e$	$M_{\Sigma^0} = 2335m_e$	0.06

Table 2: Comparison of calculated values of magnetic moments with measurement data

n	ϑ	μ_0 Eq.(44)	μ_{total} Eq.(46)	experimental. data
n=1	0.1991	-4.727	-1. 9367	$\mu_{n_0} = -1.9130427 \pm 0.0000005$
n=2	0.263	-3.4147	-0.6247	$\mu_{\Lambda^0} = -0.613 \pm 0.004$
n=3	0.479	-1.4121	1.3779	$\mu_{\Sigma^0_{\Sigma\Lambda}} = 1.61 \pm 0.08$

2.5.2 The excited state with n=3

At that the equation is

$$1 + \frac{1}{2\cdot 3} - \left(\frac{\vartheta}{3\sqrt{1-\vartheta^2}}\frac{\alpha M_p}{m_e}\right)\left[\frac{1}{(1+\vartheta)^2}\right] - \tag{50}$$
$$-\left(\frac{\vartheta}{3\sqrt{1-\vartheta^2}}\frac{\alpha M_p}{m_e}\right)\left[\frac{\vartheta^2}{2} - \frac{\xi}{2\cdot 3(1+\vartheta)^3}\frac{\vartheta}{\sqrt{1-\vartheta^2}}\right] = 0$$

and its solution is

$$\vartheta = 0:479: \tag{51}$$

For comparison, the calculated and measured values of masses and magnetic moments of neutron and its excited states are given in Table (2.5.2) and Table (2.5.2).

Based on this comparison, we conclude that neutral Λ- and Σ-hyperons are excited states of neutron [6].

2.6 Discussion

The consent of estimates and measured data indicates that the neutron is not an elementary particle [5]. At that neutron is unique object of microcosm. Its main peculiarity lies in the fact that the proton and electron that compose it are related to each other by a (negative) binding energy. But the neutron mass is greater than the sum of the rest masses of proton and electron despite the presence of a mass defect. This is because proton and electron, forming neutron, are relativistic and their masses are much higher than their rest masses. In result the bound state of neutron disintegrates with the energy releasing.

This structure of neutron must change our approach to the problem of nucleon-nucleon scattering. The nuclear part of an amplitude of the nucleon-nucleon scattering should be the same at all cases, because in fact it is always proton-proton scattering (the only difference is the presence or absence of the Coulomb scattering). It creates the justification for hypothesis of charge independence of the nucleon-nucleon interaction.

The above considered electromagnetic model of neuron is the only theory that predicts the basic properties of the neutron. According to Gilbert's postulate, all other models (and in particular the quark model of neutron) that cannot describe properties of neutron can be regarded as speculative and erroneous. The measurement confirmation for the discussed above electromagnetic model of neutron is the most important, required and completely sufficient argument of its credibility.

Nevertheless, it is important for the understanding of the model to use the standard theoretical apparatus at its construction. It should be noted that for the scientists who are accustomed to the language of relativistic quantum physics, the methodology used for the above estimates does not contribute to the perception of the results at a superficial glance. It is commonly thought that for the reliability, a consideration of an affection of relativism on the electron behavior in the Coulomb field should be carried out within the Dirac theory. However, that is not necessary in the case of calculating of the magnetic moment of the neutron and its decay energy. In this case, all relativistic effects described by the terms with coefficients $(1- v^2/c^2)^{-1/2}$ compensate each other and completely fall out. The neutron considered in our model is the quantum object. Its radius R_0 is proportional to the Planck constant $\hbar$. But it cannot be considered as relativistic particle, because coefficient $(1- v^2/c^2)^{-1/2}$ is not included in the definition of R_0. In the particular case of the calculation of the magnetic moment of the neutron and the energy of its decay, it allows to find an equilibrium of the system from the balance of forces, as it can be made in the case of non-relativistic objects. The another situation arises on the way of an evaluation of the neutron lifetime. A correct estimation of this time even in order of its value do not obtained at that.

3. The One-Electron Bond between Two Protons and the Simplest Molecule

3.1 The one-electron bond between two protons

Let us consider a quantum system consisting of two protons and one electron. If protons are separated by a large distance, this system consists of a hydrogen atom and the proton. If the hydrogen atom is at the origin, then the operator of energy and wave function of the ground state have the form:

$$H_0^{(1)} = -\frac{\hbar^2}{2m}\nabla_r^2 - \frac{e^2}{r}, \qquad \varphi_1 = \frac{1}{\sqrt{\pi a^3}}e^{-\frac{r}{a}} \tag{52}$$

If hydrogen is at point R, then respectively

$$H_0^{(2)} = -\frac{\hbar^2}{2m}\nabla_r^2 - \frac{e^2}{|\vec{R}-\vec{r}|}, \varphi_2 = \frac{1}{\sqrt{\pi a^3}}e^{-\frac{|\vec{R}-\vec{r}|}{a}} \tag{53}$$

In the assumption of fixed protons, the Hamiltonian of the total system has the form:

$$H = -\frac{\hbar^2}{2m}\nabla_r^2 - \frac{e^2}{r} - \frac{e^2}{|\vec{R}-\vec{r}|} + \frac{e^2}{R} \tag{54}$$

At that if one proton removed on infinity, then the energy of the system is equal to the energy of the ground state E_0, and the wave function satisfies the stationary Schrodinger equation:

$$H_0^{(1,2)}\varphi_{1,2} = E_0\varphi_{1,2} \tag{55}$$

We seek a zero-approximation solution in the form of a linear combination of basis functions:

$$\psi = a_1(t)\varphi_1 + a_2(t)\varphi_2 \tag{56}$$

where coefficients $a_1(t)$ and $a_2(t)$ are functions of time, and the desired function satisfies to the energy-dependent Schrodinger equation:

$$i\hbar\frac{d\psi}{dt} = (H_0^{(1,2)} + V_{1,2})\psi, \tag{57}$$

where $V_{1,2}$ is the Coulomb energy of the system in one of two cases.

Hence, using the standard procedure of transformation, we obtain the system of equations

$$\begin{aligned} i\hbar\dot{a}_1 + i\hbar S\dot{a}_2 &= E_0\Big\{(1+Y_{11})a_1 + (S+Y_{12})a_2\Big\} \\ i\hbar S\dot{a}_1 + i\hbar\dot{a}_2 &= E_0\Big\{(S+Y_{21})a_1 + (1+Y_{22})a_2\Big\}, \end{aligned} \tag{58}$$

where we have introduced the notation of the overlap integral of the wave functions

$$S = \int \phi_1^*\phi_2 dv = \int \phi_2^*\phi_1 dv \tag{59}$$

and notations of matrix elements

$$\begin{aligned} Y_{11} &= \frac{1}{E_0}\int \phi_1^* V_1\phi_1 dv \\ Y_{12} &= \frac{1}{E_0}\int \phi_1^* V_2\phi_2 dv \\ Y_{21} &= \frac{1}{E_0}\int \phi_2^* V_1\phi_1 dv \\ Y_{22} &= \frac{1}{E_0}\int \phi_2^* V_2\phi_2 dv \end{aligned} \tag{60}$$

Given the symmetry

$$Y_{11}=Y_{12} \qquad Y_{12}=Y_{21}, \tag{61}$$

after the adding and the subtracting of equations of the system (58), we obtain the system of equations

$$\begin{aligned} i\hbar(1+S)(\dot{a}_1+\dot{a}_2) &= \alpha(a_1+a_2) \\ i\hbar(1-S)(\dot{a}_1-\dot{a}_2) &= \beta(a_1-a_2) \end{aligned} \tag{62}$$

Where

$$\begin{aligned} \alpha &= E_0\Big\{(1+S)+Y_{11}+Y_{12}\Big\} \\ \beta &= E_0\Big\{(1-S)+Y_{11}-Y_{12}\Big\} \end{aligned} \tag{63}$$

As a result, we get two solutions

$$\begin{aligned} a_1 + a_2 &= C_1 exp\left(-i\frac{E_0}{\hbar}t\right)exp\left(-i\frac{\epsilon_1}{\hbar}t\right) \\ a_1 - a_2 &= C_2 exp\left(-i\frac{E_0}{\hbar}t\right)exp\left(-i\frac{\epsilon_2}{\hbar}t\right) \end{aligned} \tag{64}$$

Where

$$\epsilon_1 = E_0 \frac{Y_{11} + Y_{12}}{(1+S)}$$
$$\epsilon_2 = E_0 \frac{Y_{11} - Y_{12}}{(1-S)}. \quad (65)$$

From here

$$a_1 = \frac{1}{2} e^{-i\omega t} \cdot (e^{-i\frac{\epsilon_1}{\hbar}t} + e^{-i\frac{\epsilon_2}{\hbar}t})$$
$$a_2 = \frac{1}{2} e^{-i\omega t} \cdot (e^{-i\frac{\epsilon_1}{\hbar}t} - e^{-i\frac{\epsilon_2}{\hbar}t}) \quad (66)$$

and

$$|a_1|^2 = \frac{1}{2}\left(1 + cos\left(\frac{\epsilon_1 - \epsilon_2}{\hbar} t\right)\right)$$
$$|a_2|^2 = \frac{1}{2}\left(1 - cos\left(\frac{\epsilon_1 - \epsilon_2}{\hbar} t\right)\right) \quad (67)$$

As

$$\epsilon_1 - \epsilon_2 = 2E_0 \frac{Y_{12} - SY_{11}}{1 - S^2} \quad (68)$$

with the initial conditions

$$a_1(0) = 1 \qquad a_2(0) = 0 \quad (69)$$

and

$$C_1 = C_2 = 1 \quad (70)$$

or

$$C_1 = -C_2 = 1 \quad (71)$$

we obtain the oscillating probability of placing of electron near one or other proton:

$$|a_1|^2 = \frac{1}{2}(1 + cos\omega t)$$
$$|a_2|^2 = \frac{1}{2}(1 - cos\omega t) \quad (72)$$

Thus, electron jumps into degenerate system (hydrogen + proton) with frequency ω from one proton to another.

In terms of energy, the frequency ω corresponds to the energy of the tunnel splitting arising due to electron jumping (Figure 3).

As a result, due to the electron exchange, the mutual attraction arises between protons. It decreases their energy on

$$\Delta = \hbar\omega/2 \quad (73)$$

The arising attraction between protons is a purely quantum effect, it does not exist in classical physics.

The tunnel splitting (and the energy of mutual attraction between protons) depends on two parameters:

$$\Delta = |E_0| \cdot \Lambda, \quad (74)$$

where E_0 is energy of the unperturbed state of the system (ie, the electron energy at its association with one of proton, when the second proton removed on infinity),

and function of the mutual distance between the protons Λ.

This function according to Equation (68) has the form:

$$\Lambda = \frac{Y_{12} - SY_{11}}{(1 - S^2)}. \quad (75)$$

It expresses the dependence of the exchange energy on the distance between particles.

The graphic estimation of the exchange splitting ΔƐ indicates that this effect decreases exponentially with increasing a distance between the protons in full compliance with the laws of the particles passing through the tunnel barrier.

3.2 The Molecular Hydrogen ion

The quantum-mechanical model of simplest molecule - the molecular hydrogen ion - was first formulated and solved by Walter Heitler and Fritz London in 1927 [3].

At that, they calculate the Coulomb integral:

$$Y_{11} = \left[1 - (1+x)e^{-2x}\right], \quad (76)$$

the integral of exchange

$$Y_{12} = \left[x(1+x)e^{-x}\right] \quad (77)$$

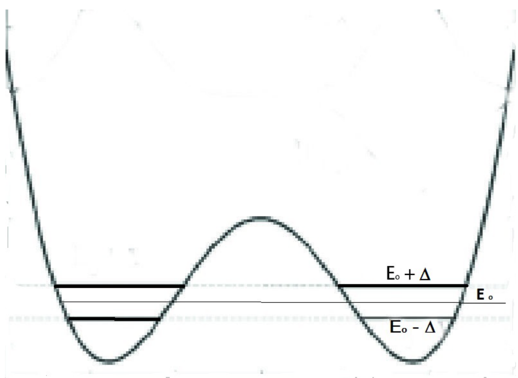

Figure 3: The schematic representation of the potential well with two symmetric states. In the ground state, electron can be either in the right or in the left hole. In the unperturbed state, its wave functions are either φ_1 or φ_2 with the energy E_0. The quantum tunneling transition from one state to another leads to the splitting of energy level and to the lowering of the sublevel on Δ.

and the overlap integral

$$S = \left(1 + x + \frac{x^2}{3}\right) e^{-x}. \quad (78)$$

Where $x = R/a_B$ is the dimensionless distance between the protons.

The potential energy of hydrogen atom

$$\mathcal{E}_0 = -\frac{e^2}{a_B} \tag{79}$$

and with taking into account Equation (76)-Equation (78)

$$\Lambda(x) = e^{-x}\frac{x(1+x) - \left(1+x+\frac{x^2}{3}\right)\left(1-(1+x)e^{-2x}\right)}{1-\left(1+x+\frac{x^2}{3}\right)^2 e^{-2x}} \tag{80}$$

At varying the function Λ(x) we find that at

$$x \simeq 1.3 \tag{81}$$

the energy of the system has a minimum

$$\Lambda_{x=1.3} \simeq 0.43. \tag{82}$$

As a result of permutations of these values we find that in this minimum energy the mutual attraction of protons reaches a maximum value

$$\Delta_{max} \simeq 9.3 \cdot 10^{-12} erg. \tag{83}$$

This result agrees with measurements of only the order of magnitude. The measurements indicate that the equilibrium distance between the protons in the molecular hydrogen ion x ≃ 2 and its breaking energy on proton and hydrogen atom is close to $4{:}3.10^{-12}$ erg.

The remarkable manifestation of an attraction arising between the nuclei at electron exchange is showing himself in the molecular ion of helium. The molecule H_{e2} does not exist. But a neutral helium atom together with a singly ionized atom can form a stable structure - the molecular ion. The above obtained computational evaluation is in accordance with measurement as for both - hydrogen atom and helium atom - the radius of s-shells is equal to a_B, the distance between the nuclei in the molecular ion of helium, as in case of the hydrogen molecular ion, must be near x ≃ 2 and its breaking energy near $4.3.10^{-12}$ erg.

In order to achieve a better agreement between calculated results with measured data, researchers usually produce variation of the Schrodinger equation in the additional parameter- the charge of the electron cloud. At that, one can obtain the quite well consent of the calculations with experiment. But that is beyond the scope of our interest as we was needing the simple consideration of the effect.

4. Deuteron and Other Light Nuclei

4.1 Deuteron

The electromagnetic model of neutron, discussed above, allows us to take a fresh look at the mechanism of the neutron-proton interaction. Neutron as proton surrounded by a electron cloud and a free proton make up together an object similar to a molecular hydrogen ion.

The difference is that in this case the electron is relativistic, and the radius of its orbit is $R \approx 10^{-13}$ cm (Equation (34)).

The electron energy in the composition of neutron in the undisturbed state was calculated earlier (Equation (20)):

$$\mathcal{E}_0 \approx \vartheta\gamma m_e c^2 \tag{84}$$

This function expresses the dependence of the exchange energy on the distance between nucleon. According to Equation (82), it has maximum

$$\Lambda_{max} = 0.43, \tag{85}$$

at the dimensionless distance between protons x = R/Re = 1.3 (Equation (81)).

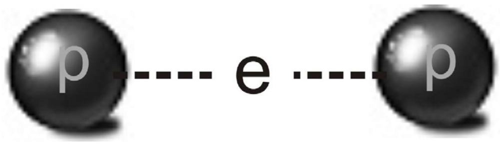

Figure 4: Schematic representation of deuteron. The dotted line schematically shows the possibility of a relativistic electron jumping from one proton to another.

The values of the binding energy between nucleons are usually expressed in terms of the magnitude of the mass defect in atomic units of mass, having the international designation u. With what

$$1u = 1.6605402 \bullet 10^{-24}\text{g}. \tag{86}$$

In these units, the magnitude of the decrease in the energy of two protons exchanging a relativistic electron has the value:

$$\Delta_0 = \Lambda_{max}\mathcal{E}_0 \simeq 10^{-2}u. \tag{87}$$

To compare this energy with the measurement data, it is necessary to calculate the mass defect of particles forming the deuteron

$$\Delta M_D = M_p + M_n - M_d \approx 2.3414 \cdot 10^{-3}u \tag{88}$$

(Where M_p = 1.007276466621u, M_n = 1.00866491560u and M_d = 2.0136u are masses of proton, neutron and deuteron, respectively.

Thus, we can assume that for the deuteron quantum-mechanical rating (Equation (87)), as in the case of molecular hydrogen ion, in order of magnitude is consistent with the experimentally measured the magnitude of the binding energy (Equation (88)), although in both cases a coincidence, without further amendment is not very accurate.

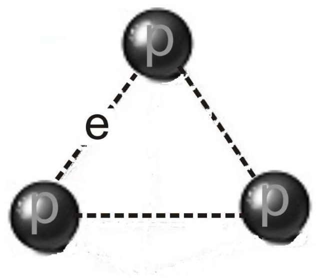

Figure 5: Schematic representation of the $^{3}_{2}He$ core. Dotted lines schematically represent the possibility of a relativistic electron jumping from one proton to another.

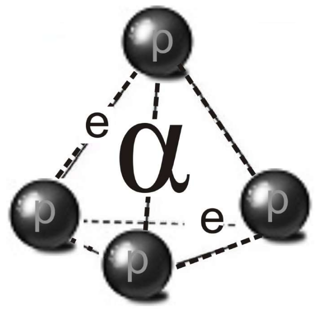

Figure 6: Schematic representation of the core $^{4}_{2}He$. Dotted lines schematically represent the possibility of a relativistic electron jumping from one proton to another.

4.2 Helium Isotopes

Figure (5) shows schematically the energy bonds in the nucleus of $^{3}_{2}He$. From it we can seen that there are three paired interactions of protons. Therefore, it should be assumed that the binding energy of this nucleus should be equal to the triple binding energy of the deuteron (Equation (88)):

$$\delta M_{He3} = 3 \cdot \Delta M_D \approx 7.02 \cdot 10^{-3} u. \quad (89)$$

The experimentally measured mass defect of this nucleus is equal to

$$\Delta M(He3) = 2M_p + M_n - M_{He3} = 8.29 \cdot 10^{-3} u. \quad (90)$$

Thus, the calculated mass defect of this nucleus can be considered quite consistent with its measured value.

As it can be seen from Figure (9), there are bonds are formed by six paired interactions of protons ΔMd, realized by two electrons. For this reason, it can be assumed that the binding energy of the nucleus $^{4}_{2}He$ should be equal to:

$$\delta M_\alpha = 2 \cdot 6 \cdot \Delta M_D \approx 28.1 \cdot 10^{-3} u. \quad (91)$$

The measured mass defect of this nucleus is equal to

$$\Delta M_\alpha = 2M_p + 2M_n - M_\alpha = 30.4 \cdot 10^{-3} u. \quad (92)$$

Such agreement of these values can be considered quite satisfactory.

4.3 Beryllium Isotopes

A comparison of the binding energies of beryllium isotopes points the way to calculating mass defects of heavy nuclei.

If we compare the binding energy of nucleus $^{8}_{4}Be$ with the doubled binding energy of the alpha-particle, we can conclude that this nucleus must be unstable. When it decays into alpha particles, the energy corresponding to the mass defect, which turns out to be negative in this case, should be released:

$$\Delta M(Be8) = 2M_\alpha - M_{Be_8} = -2.29 \cdot 10^{-3} u \quad (95)$$

Indeed, measurements show that the $^{8}_{4}Be$ nucleous is very short-lived. It decays into two alpha-particles, having a lifetime of approximately 10^{-17} sec.

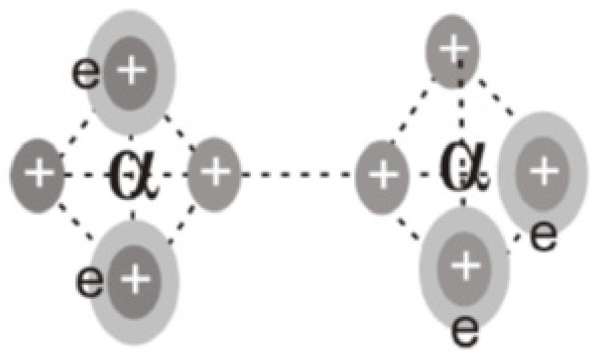

Figure 7: The schematic representation of energy bonds in the Be-8 core. Dotted lines represent the possibility of a relativistic electron jumping between protons.

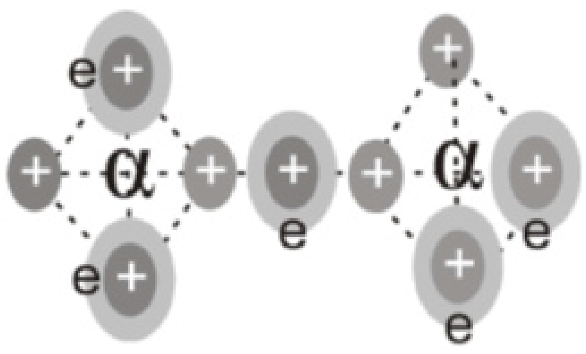

Figure 8: Schematic representation of energy bonds in the Be-9 core. Dotted lines represent the possibility of a relativistic electron jumping between protons.

However, if neutron is attached to the $^{8}_{4}Be$ nucleus to construct the $^{9}_{4}Be$ nucleus (Figure 8), the result is a stable nucleus with a mass defect:

$$\Delta M(Be9) = 4 \cdot M_p + 5 \cdot M_n - M_{Be_9} = 60.25 \cdot 10^{-3} u. \quad (94)$$

This can be explained by the fact that the total mass defect in the structure shown in Figure (8), will increase.

The mass defect of alpha-particle according to Equation (91) is equal to $12 \cdot \Delta M_d$. Two alpha particles respectively create a mass defect $24 \cdot \Delta M_d$. To this value, we need to add a doubled deuteron defect of mass $2 \cdot \Delta M_d$, since the electron of an additional neutron connecting alpha-particles (Figure 8) has the ability to transfer to free protons of neighboring alpha-particles. As a result, we get the total mass defect of $^9_4$Be

$$\delta M(Be9) = 26 \cdot \Delta M_D = 60.88 \cdot 10^{-3} u. \qquad (95)$$

Good agreement of this estimate with the experimental value (Equation (94)) suggests that neutron can bind alpha-particles together, playing the role of a kind of glue.

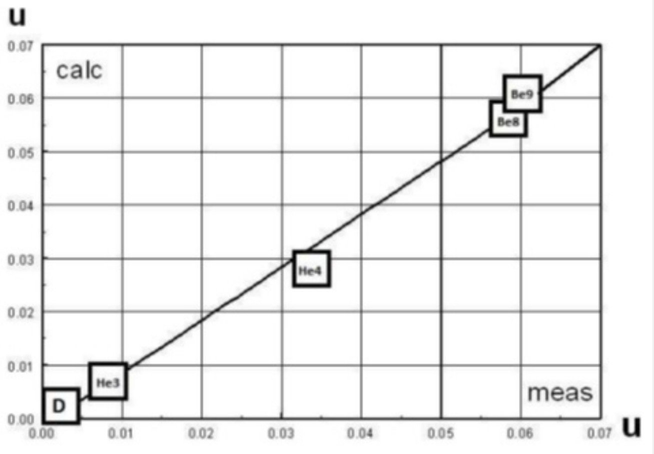

Figure 9: Comparison of calculated values of the mass defect of light nuclei with the measurement data.

Table 3: Comparison of the calculated values of the defect of the mass of light nuclei with the measurement data.

isotope	M u	ΔM $10^{-3}u$	N_d	$\delta M = N_d \cdot \Delta M_D$ $10^{-3}u$	$\frac{\Delta M - \delta M}{\Delta M}$ %
^2_1D	2.01355	2.3414	1	-	
^3_2He	3.01493	8.2878	3	7.0242	15
^4_2He	4.001506179	30.377	12	28.097	7.5
^8_4Be	8.00530510	58.46	24	56.194	3.9
^9_4Be	9.0121822	60.248	26	60.876	1

5. Mass defects of heavy nuclei

Taking into account the scheme of the structure of the nucleus $^9_4$Be (Figure 8), we can by analogy assume that, other stable heavy nuclei can be represented in the form "crystals" , consisting of alpha-particles "glued" each other neutrons (Figure 10).

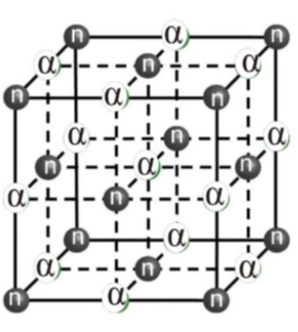

Figure 10: Schematic representation of the "crystal" models of a heavy nucleus in which alpha-particles are "glued"together by neutrons

Since one neutron can "glue" several alpha-particles, in a large "crystal" the number of alpha-particles must be greater or equal than the number of "gluing" neutrons. I.e., the inequality must be fulfilled:

$$N_\alpha = \frac{Z}{2} \geq N_n - Z. \qquad (96)$$

Where
Z is an even number of protons in the nucleus,
$N_\alpha = Z/2$ is the number of alpha-particles formed in the nucleus,
N_n is the number of neutrons.

This assumption is confirmed by experimental data. For heavy stable nuclei, as can be seen from Figure 11, relation N_n-Z / Z/2 is indeed less than (or close) to one.

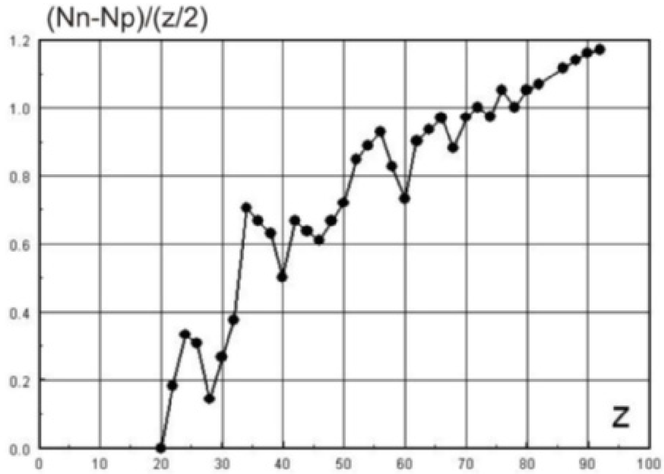

Figure 11: Dependence of the ratio of the excess of the number of neutrons over the number of protons to the value Z/2 for heavy nuclei with an even number of protons.

If we assume that alpha-particles and neutrons form the simplest cubic "crystal"(as shown in Figure (10)), then each alpha-particle will be adjacent to six neutrons. As a result, the mass defect of each alpha-particle will increase by 6 . ΔM_D.

Thus, the total mass defect of one alpha-particle in the "crystal" turns out to be equal to

$$\delta^* M_\alpha = \delta M_\alpha + 6 \cdot \Delta M_D = 18 \cdot \Delta M_D, \qquad (97)$$

and the complete mass defect of the entire nucleus:

$$\delta M = N_\alpha \cdot \delta^* M_\alpha = 9 \cdot Z \cdot \Delta M_D \approx 0.02107 \cdot Z \quad u. \qquad (98)$$

This simple formula predicts the binding energy of heavy nuclei with satisfactory accuracy.

The ratio of the values of mass defects of heavy nuclei calculated by this formula to the experimentally measured values of the defect of the mass of the nucleus

Relationships of values of mass defects of heavy nuclei calculated by this formula to the experimentally measured values of mass defects of these nuclei

$$\frac{\delta M}{\Delta M} = \frac{0.02107 \cdot Z}{Z \cdot M_p + N_n \cdot M_n - M(nucleus)}$$

is shown on Figure (12).

It is very remarkable that on the dependence *δM/ΔM* there is the breaking point separating the falling and growing branches. This fracture occurs on the isotope $^{208}_{82}Pb$. It is the heaviest stable nucleus of the periodic table of elements. It separates the stable isotopes of those elements that have smaller charge numbers and nuclei with a larger Z that experience radioactive decay.

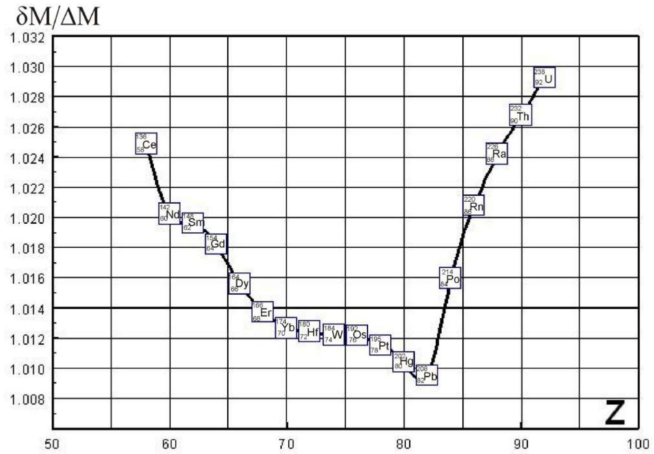

Figure 12: Comparison of calculated values of mass defects of heavy nuclei with the measurement data.

6. Conclusion

The good agreement of the calculated binding energy for many nuclei with the measurement data allows us to assume that the strong interaction is the purely quantum mechanical effect described above.

For the First time, attention to the possibility of explaining nuclear forces based on the effect of electron exchange was apparently drawn by I.E.Tamm [10] back in the 30s of the last century. However, later in nuclear physics, the model of exchange of π-mesons, and then gluons, became predominant. The reason for this understandable. To explain the magnitude and radius of action of nuclear forces, a heavy particle with a small intrinsic wavelength is needed. A non-relativistic electron is not suitable for this. However, on the other hand, the models of π-meson or gluon exchange also did not turn out to be productive. These models could not give a sufficiently accurate quantitative explanation of the binding energy of even light nuclei. Therefore, the above simple and consistent with measurements estimate of this energy is an unambiguous proof that the strong interaction is a manifestation of the quantum-mechanical effect of attraction between protons, which occurs due to the exchange of a relativistic electron.

An important consequence of this model is the ability to determine the cause of instability of some nuclei.

The condition Equation (96) corresponds to the case when neutron "glues" at least two-alpha particles.

In the opposite case, an excess of neutrons forces them to bind to only one alpha-particle, which makes this bond weak and the nucleus prone to decay.

Thus, the condition Equation (96) is actually a condition for the formation of a stable nucleus bond.

It can be converted to the form:

$$\frac{5}{2}Z \geq N_n + Z = A,$$

and, thus, the boundary of the stability of the nuclei is described by the equality:

$$\frac{A}{Z} \leq 2.5\ .$$

Where A is number of nucleons in nucleus.

Really, for some unknown reason, this boundary of stability is slightly shifted. For the heaviest stable nucleus $^{208}_{82}Pb$- this ration A/Z = 2.54 (Figure 13).

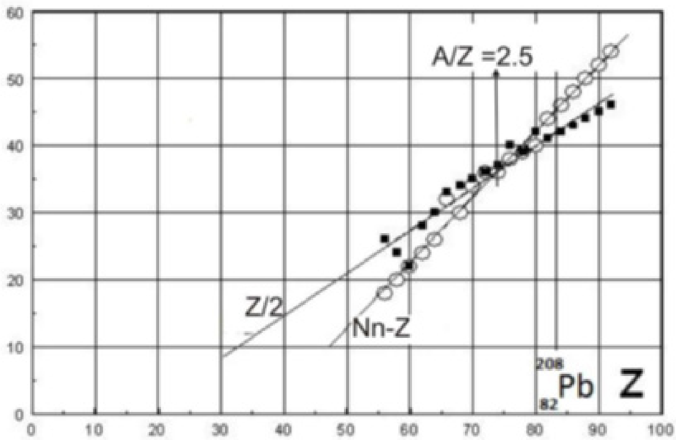

Figure 13: On the question of the boundary separating stable and unstable nuclei, described by the inequality A/Z ≤ 2:5.

It is important that in this model there are no reasons for the appearance of stability islands outside the boundary (Equation (101)) and all the activity to search for them among super heavy nuclei looks devoid of a physical basis.

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