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Research Article

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Aberration of Electric Field and Motion of an Electron

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Abstract

Aberration of electric field is a missing link in physics, which necessitated the concept of special relativity and quantum mechanics, to describe some physical phenomenon. An electron of mass m and charge –e moving with velocity v at angle θ to an electric field E, is subject to aberration of electrid field, as a result of relativity of velocity between the electrical force, transmitted with velodity of light c, and the electron moving with velocity v. Accelerating force, at time t, in accordance with Coulomb's law and Newton's second law of motion, put as F = -(eE/c)(c – v) = m(dv/dt), is less than the electrostatic foce -eE, the difference is radiation reaction force. At the velocity of light v = c, the force F becomes zero and the electron continues to move with speed c as an ultimate limit. Motion of the electron with constant mass m and its radiation power are treated under acceleration with θ = 0 or deceleration with θ = π radians or at constant speed v, in a circle of radiua r, with θ =π/2 radians. It is shown that circular motion of an electron round a central force of attraction, as in the Rutherford's nuclear model of the hydrogen atom, is inherently stable. There is no need of special relativisty for the speed of light to become a limit and quantum mechanics is not necessary in explaining emission of radiation from accelerated charged particles.

Keywords: Aberration Angle, Acceleration, Electric Charge, Field, Mass, Radiation, Velocity

1. Aberration of Light

Aberration of light was discovered, in 1728, by English astronomer, James Bradley [1-3]. It was one of the most significant discoveries in science, but now relegated to the background in favour of constancy of speed of light, by the theory of special relativity [4 -7]. Aberration of light, illustrated in Figure 1, clearly demonstrates the relativity of speed of light, with respect to a moving observer (astronomer).

In Figure 1, an astronomer at P observes a star at S. The observer at P, moving with velocity v at angle θ to the instantaneous line SP. He sees the star in light coming with velocity c, along the direction PN, displaced forwards, through aberration angle α, such that:

$$\sin\alpha = \frac{v}{c}\sin\theta \qquad (1)$$

Equation (1) is Bradley's equation, a universal formula applicable at atomic and astronomical levels. Through this discovery Bradley obtained the first estimate of the speed of light and confirmed that the Earth moves round the Sun. Aberration of light is a clear, but ignored, demonstration of relativity of speed of light with respect to a moving object. Today, aberration of light is hardly mentioned in undergraduate physics because it contradicts the proposition of constancy of speed of light, a cardinal principle of the theory of special relativity.

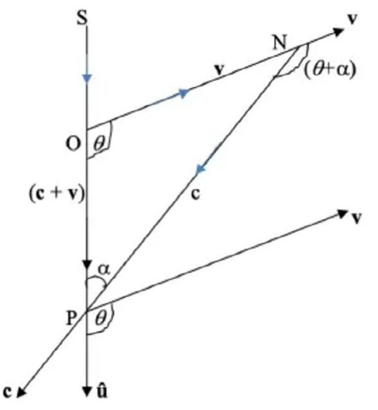

Figure 1: Aberration of light from star along PS as observed along PN by astronomer at P moving with velocity v.

Indeed, if the finite speed of light were that constant, for all moving observers, it would never have been so accurately measured for umpteen times, in so far as all measurements are relative to an accepted reference, which may be a kown speed like that of a rotating disc relative to its centre.

2. Aberration of Electric Field

As an electrical force is transmitted with speed of light c, a charged particle, moving in an electric field, should be subject to aberration of electric field, as depicted in Figures (2) and (3) [8, 9]. This is a phenomenon similar to aberration of light. Aberration of electric field is the missing link in physics today. This paper is to correct the situation in electrodynamics applicable up to the speed of light c, with mass m of a moving particle remaining constant at the rest mass m_o.

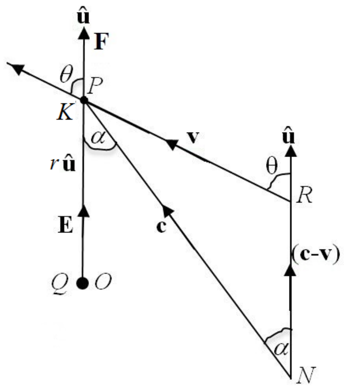

Figure 2: Depicting aberration of electric field due to a particle of charge K moving at a point P with velocity **v** at angle θ to the electric field of intensity **E** due to a stationary source charge Q at a point O.

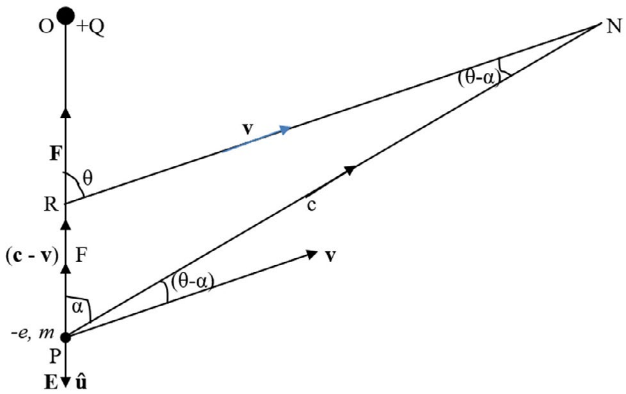

Figure 3: Depicting aberration of electric field due an electron of charge -e and mass m moving withvelocity **v** at angle θ to the direction of accelerating force **F** of source charge Q at O.

Figure 3 is used for the analysis of force, motion and emission of radiation by an electron in an electric field, with reference to Coulomb's law. Bradley's law, for aberration of light in Figure 1, applies equally well in Figures 2 and 3 for aberration of electric field.

3. Coulomb's Law

Coulomb's law of electrostatics, perhaps the most important principle in physics, was enunciated in1785 by French physicist and engineer, Charles-Augustin de Coulomb [10]. The law gives force **F** between two stationary electric charges of magnitudes Q at a point O and K at P separated by distance PO = r, as vector equation:

$$\mathbf{F} = \frac{KQ}{4\pi\varepsilon_o r^2}\hat{\mathbf{u}} = \frac{KQ}{4\pi\varepsilon_o r^2}\frac{\mathbf{c}}{c} = Q\mathbf{E}_r \quad (2)$$

where ε_o is electric permittivity, **û** is unit vector in the direction of force of repulsion, **c** is the velocity of light of magnitude c, with which an electrical force is propagated, and $\mathbf{E_r}$ the electrostatic field intensity due to charge K, at the location of charge Q, as shown in Figure 4.

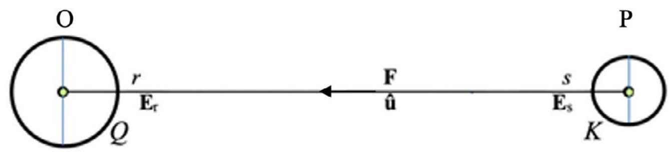

Figure 4: Force F between two stationary electric charges Q and K separated by distance PO = r.

$$\mathbf{E}_r = \frac{K}{4\pi\varepsilon_o r^2}\hat{\mathbf{u}} = \frac{K}{4\pi\varepsilon_o r^2}\frac{\mathbf{c}}{c} \quad (3)$$

Electrical effects, like electromagnetic radiation and electrical force, are propagated in space, along an electric field, with velocity of light c of magnitude c. In the aberration of electric field there is relativity of velocity (**c – v**) between an electrical force propagated with velocity of light **c** and a charged particle, such as an electron, moving with velocity **v**. As such, the electrical force cannot catch up and impact on an electron also moving with velocity of light **c**. The velocity of light, therefore, becomes the ultimate limit to which an electric field can accelerate a charged particle, with emission of radiation and mass of moving particle remaining constant at rest mass.

Resulting from aberration of electric field, for an electron of rest mass mo and with charge K = –e moving at time t with velocity v and acceleration dv/dt in an electric field of intensity E and magnitude E, the accelerating force, is given by:

$$\mathbf{F} = -\frac{eE}{c}(\mathbf{c}-\mathbf{v}) = m_o\frac{d\mathbf{v}}{dt} \quad (4)$$

This force **F** is less than the electrostatic force –e**E** on a stationary electron, relative to the field. The difference is radiation reaction force. The radiation reaction force, in rectilinear motion, is:

$$\mathbf{R}_f = -e\mathbf{E}\frac{v}{c} = -eE\frac{\mathbf{v}}{c} \quad (5)$$

Radiation power, R_p, scalar product -**v.R**$_f$, is:

$$R_p = eE\frac{v^2}{c} \quad (6)$$

In circular motion where velocity **v** is perpendicular to the radial electric field, the radiation power is zero. It makes motion of an electron, round a positively charged nucleus, as in the Rutherford's nuclear model of the atom, without radiation and stable, outside Bohr's quantum mechanics.

4. Velocity of transmission of an electrical force

With reference to Figure 3, the vector **z = (c - v)** is the relative velocity of transmission between the electrical force propagated with velocity of light **c** and the electron moving with velocity **v,** thus:

$$\mathbf{z} = (\mathbf{c} - \mathbf{v}) = -\sqrt{c^2 + v^2 - 2cv\cos(\theta - \alpha)}\,\hat{\mathbf{u}} \qquad (7)$$

where $(\theta - \alpha)$ is the angle between the vectors **c** and **v** and $\hat{\mathbf{u}}$ is a unit vector in the direction of the electric field of intensity **E**. The electron can move with acceleration at $\theta = 0$, with deceleration at $\theta = \pi$ radians, or in a circle with $\theta = \pi/2$ radians.

With $\theta = 0$ there is motion in a straight line with acceleration and equations (1) and (7) give the relative speed of transmission of the electrical force as:

$$z = c - v \qquad (8)$$

If $\theta = \pi$ radians there is motion in a straight line with deceleration and the relative speed of transmission of the force becomes:

$$z = c + v \qquad (9)$$

If $\theta = \pi/2$ radians and noting that $\sin\alpha = v/c$ (equation 1), there is circular motion with constant speed v, giving the relative speed of transmission of the force as:

$$z = \pm\sqrt{c^2 - v^2} = \pm c\sqrt{1 - \frac{v^2}{c^2}} \qquad (10)$$

Equatons (8), (9) and (10) clearly demonstrate the relativity of speed of light with respect to a charged particle moving with speed v. Equations (8) and (9) are familiar in classical mechanics but not allowed in relativistic mechanics, where the speed of light c cannot be added to or subtracted from. The issue is with equation (10), which is a consequence of aberration of electric field and motion perpendicular to an electric field.

5. Accelerating force due to an electric field

From Figure 3, accelerating force **F** at time t, in an electric field of magnitude E, is put as:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) = -\frac{eE}{c}\sqrt{c^2 + v^2 - 2cv\cos(\theta - \alpha)}\,\hat{\mathbf{u}} = m_o\frac{d\mathbf{v}}{dt} \qquad (11)$$

where $\hat{\mathbf{u}}$ is a unit vector in the positive direction of the electric field intensity **E**. For motion in a straight line under acceleration, with $\theta = 0$, equations (1) and (11) give the vector equation:

$$\mathbf{F} = -eE\left(1 - \frac{v}{c}\right)\hat{\mathbf{u}} = m_o\frac{d\mathbf{v}}{dt} \qquad (12)$$

The scalar first order differential equation, for accelerated electron, is:

$$eE\left(1 - \frac{v}{c}\right) = m_o\frac{dv}{dt} \qquad (13)$$

The solution of equation (13) for an electron accelerated from zero initial speed by a uniform electic field of magnitude E, is:

$$v = c\left(1 - e^{\frac{-at}{c}}\right) \qquad (14)$$

where is $a = eE/m_o$ is a constant.
For motion in a straight line under deceleration, with $\theta = \pi$ radians, equations (1) and (11) give the vector equation:

$$\mathbf{F} = -eE\left(1 + \frac{v}{c}\right)\hat{\mathbf{u}} = m_o\frac{d\mathbf{v}}{dt} \qquad (15)$$

The scalar first order differential equation, for a decelerated electron, is:

$$eE\left(1 + \frac{v}{c}\right) = -m_o\frac{dv}{dt} \qquad (16)$$

The solution of equation (16) for deceleration from speed of light c by a uniform field E, is:

$$v = 2ce^{\frac{-at}{c}} - c \qquad (17)$$

where is $a = eE/m_o$ is a constant. In equation (17) the particle is decelerated to a stop in time $t = 0.693c/a$ and then accelerated in the backward direction to reach a maximun speed c. Equations (14) and (17) show that the speed of light c or $-c$ is the maximum attainable as time t $\rightarrow \infty$.

If $\theta = \pi/2$ radians there is motion in a circle of radius r with constant speed v and centripetal acceleration $-v^2/r$. Noting that $\sin\alpha = v/c$, equations (1) and (11) give the vector equation:

$$\mathbf{F} = -eE\sqrt{1 - \frac{v^2}{c^2}}\,\hat{\mathbf{u}} = -m_0\frac{v^2}{c}\hat{\mathbf{u}} \qquad (18)$$

The scalar equation is:

$$eE\sqrt{1 - \frac{v^2}{c^2}} = m_0\frac{v^2}{c} \qquad (19)$$

Equation (19) may be written as:

$$eE = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{v^2}{r} = m_v\frac{v^2}{r} \qquad (20)$$

Equation (20) is what obtains in relativistic electrodynamics, where eE is supposed to be a constant independent of speed of an electron in an electrid field of magnitude E. Equation (20) gives:

$$m_v = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o \qquad (21)$$

where m_v is the relativistic mass and γ the Lorentz factor.

Equation (19) is correct and applicable in circular motion only. Equation (20), leading to equation (21), is the relativistic mass-velocity formula. It is an inversion of a mathematical equation, putting γm_o instead of E/γ, for the accelerating electric field to vary with speed.

Applying equation (21) in rectilinear motion is an expensive mistake in physics. Lorentz factor γ has nothing to do with mass. It is the result of motion of a chaged particle, perpendicular to an electric field, as in circular revolution of an electron round a positive nucleus.

6. Radiation power

The difference between the accelerating force **F**, on a moving particle, in equation (11), and electrostatic force or impressed force –**eE**, is the radiation reaction force, a vector $\mathbf{R}_f$ given by:

$$\mathbf{R}_f = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) + e\mathbf{E} \qquad (22)$$

The radiation power Rp is the scalar product –v.Rf, thus:

$$R_p = -\mathbf{v}.\mathbf{R}_f = -\frac{eE}{c}(\mathbf{c} - \mathbf{v}).\mathbf{v} - e\mathbf{E}.\mathbf{v} \qquad (23)$$

With reference to Figure 3, radiation power R_p is expressed in terms of the angles θ and α, as:

$$R_p = \frac{eEv^2}{c} - eEv\cos(\theta - \alpha) + eEv\cos\theta \qquad (24)$$

Equation (24) shows that readiation power is eEv^2/c under acceleration with $\theta = 0$ or under deceleration with $\theta = \pi$ radians. For $\theta = \pi/2$ radians, there is circular motion without radiation.

7. Results and discussion

• Equations (8), (9) and (10) show that the speed of transmission of an electrical force, same as the speed of light, relative to an observer, can be subtracted from or added to, contrary to the relativistic principle of constancy of the speed of light for all observers, statioary or moving. So, a cardinal principle of the theory of special relativity becomes questionsble.
• Equation (21), the relativistic mass-velocity formula, applicable in circular motion only, is a physical misnterpretation of a mathematical equation, not representing a physical reality.
• The ubiquitous Lorentz factor γ, in equation (21), has nothing to do with mass. It is the result of motion of a charged particle perpendicular to an electric field.
• There is no radiation reaction force in relativistic mechanics. With no radiation reaction force, there should be no radiation. But there must be radiation when an electron is accelerated with a component of its velocity in the direction of an electric field.
• An important result is contained in equation (24). Here, if $\theta = \pi/2$ radians, there is circular motion, round a central force of attraction, with zero radiation power. This makes Ruthrford's nuclear model of the hydrogen atom stable. Radiation takes place where the electron is dislodged from a circular orbit. It then revolves in an unclosed elliptic orbit with emission of radiation at the frequency of revolution, before reverting to a circular orbit.

8. Conclusion

By invoking aberration of electric field, the paper has succeeded in showing that an electron is accelerated, by an electric field, to the speed of light, as a limit, at constant mass and with emission of radiation. So, there should be no need of special relativity to explain the speed of light as an ultimate limit or quantum mechanics to obtain radiation from an accelerated electron.

References

1. J. Bradley. (1728). Phil. Trans. Royal Soc. 35, 406.
2. Bradley, J. (1729). IV. A letter from the Reverend Mr. James Bradley Savilian Professor of Astronomy at Oxford, and FRS to Dr. Edmond Halley Astronom. Reg. &c. giving an account of a new discovered motion of the fix'd stars. Philosophical Transactions of the Royal Society of London, 35(406), 637-661.
3. Bradley, J. (1747). I. A letter to the Right honourable George Earl of Macclesfield concerning an apparent motion observed in some of the fixed stars; by James Bradley DDFR S. Philosophical Transactions of the Royal Society of London, 45(485), 1-41.
4. A. Einstein. (1905). "On the Electrodynamics of Moving Bodies", Ann. Phys., 17, 891.
5. A. Einstein & H.A. Lorentz. (1923). The Principles of Relativity, Matheun, London.
6. W. Rosser. (1967). Introduction to Special Relativity. Butterworth & Co. Ltd., London.
7. W. Rindler. (1991). Introduction to Special Relativity. Oxford University Press, Oxford.
8. M. Abdullahi (2014): https://www.academia.edu/7691834/Motion_of_an_Electron_in_Classical_and_Relativistic_Electrodynamics_and_a_Radiative_Electrodynamics .
9. M.D. Abdullahi (2019): https://www.academia.edu/39281167/ABERRATION_OF_ELECTRIC_FIELD_AND_ACCELERATED_MOTION_OF_AN_ELECTRON_WITH_CONSTANT_MASS.
10. Roller, D. E., & Roller, D. H. D. B. (1954). The Development of the Concept of Electric Charge: electricity from the Greeks to Coulomb. Harvard University Press.