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Review Article

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A Review of Lawson Criterion for Nuclear Fusion

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Abstract

In the pursuit for abundant clean energy from nuclear fusion, the Lawson Criterion is cited by fusion researchers and experimentalists as the condition to exceed for fusion of atomic nuclei to occur. In his 1955 paper J.D. Lawson analyzed a fusion plasma conforming to thermodynamic principles at steadystate but with stated omissions and simplifications. Modern fusion researchers developed many incantations of the Lawson Criterion urging their brand leads to fusion ignition, necessarily to sell a rational fusion hypothesis and attract research funding. Others have shown how confined fusion in the laboratory cannot satisfy a positive energy balance due to energy losses from the system. A literature search for "fusion energy balance" shows a suspicious absence of this sanity-check to verify the energy condition was met. This paper applies conservation of energy from classical thermodynamics to a fusion plasma and summarizes eleven modern Lawson-like interpretations in a uniform way, doing so shows the requirements for fusion ignition are not met. Indeed despite billions of speculative dollars spent, sustained laboratory fusion has not been demonstrated in any experimental apparatus built and tested to date worldwide. Heed the warning, in 1955 John D. Lawson wrote: "To conclude we emphasize that these conditions, though necessary are far from sufficient. The working cycle that has been assumed is very optimistic."

The Fusion Record

The first demonstration of a large terrestrial fusion reaction was the thermonuclear detonation of Ivy Mike in the Enewetak Attol, Pacific Ocean November 1 1952. A common measure of power yield from a nuclear process is the gain factor ratio Q, the power produced to the power input. The best performing Magnetic Confinement Fusion (MCF) reactor to date is the JET Tokamak in the United Kingdom in 1997 with a gain Q = 0.67 (16 MW/24 MW). For Inertial Confinement Fusion (ICF) the National Ignition Facility (NIF) reported a shot test on 28 September 2013 exceeding the breakeven gain factor of Q = 1. A fusion process in our sun reliably fuses Hydrogen nuclei producing 1366 W/m2, the accepted value of the Solar Constant at 1AU (Astronomical Unit) [1].

Despite billions of dollars in public funds spent on fusion facilities super computers and experimentation and more venture capital infusion start-ups, sustained fusion has defied experimenters in any laboratory setting on the planet. In full view of the record, fusion pioneers like the Lawrence Livermore National Laboratory NIF exaggerate results in order to mask the failure record of confined nuclear fusioni The NIF project director boasted that at 5:15 a.m. on 28 September 2013 the facility achieved an energy gain of 1.4 later disputed by astute observers as actually 0.0077 [2].

For the first time in the history of controlled fusion research, a DT plasma has produced more energy (14 kJ) than was supplied to it (10 kJ). That achievement has led to the first-ever positive fuel-energy balance achieved in a laboratory (defined as fusion energy exceeding input energy to the fuel) [3].

A new renaissance in physics questions the validity of theoretical physics in astronomy, cosmology and quantum foundations when researchers "act based on the conviction that mathematical structures don't correspond to measurable entities" [4]. At her blog Backreaction Sabine Hossenfelder begs a return to foundations in quantum mechanics leaving shut-up and calculate for religious evangelists of beautiful math.v In this paper we review popular fusion success criteria and their disagreement with well established fundamentals in both thermodynamics and plasma physics using quantifiable calculations. This paper is not about modeling the transport properties of physical phenomena such as laser preheating, Rayleigh- Taylor and other instabilities, or the hydrodynamics of multi-state phenomena. We simply seek what fundamentals of themodynamic principles say about fusion ignition for an ICF target.

Gain Factor Q

A fusion reactor containing a fusion plasma must recirculate the products of reaction in a thermal process to produce useful net energy. For fusion energy to be useful as a power generating resource, the reaction rate must be high enough and the process recirculation requirement low enough to maintain the fusion reaction. The process must also sustain the reaction at steady-state, when the energy gain rate dW/dt is zero and still drive turbo machinery. Q is the gain factor for the fusion reaction, in terms of power generated Q = power produced / power input >> 1 . We will return to the gain factor but first it is helpful to identify these energy ratios from a early analysis of a fusion plasma process.

J.D. Lawson Criterion

To understand the Lawson Criterion it is useful to revisit the origin of the term. In his classified 1955 paper J.D. Lawson derived conditions for sustaining nuclear fusion through the simplified power balance of a fusion plasma assumed at steady-state [6]. Lawson calculated the energy components of the power balance and specified the internal energy as a function of confinement time, particle density and temperature. Lawson reasoned two cases, the plasma in a star where the reactants and disintegration products do not escape, or in a proposed "pulsed system" where plasma fuel reactants instantaneously rise in temperature from an external energy source and disintegration products escape the system as energy losses. Lawson then developed a criteria for fusion to occur in a "useful reactor" hosting a DD or DT (Deuterium and Tritium) hot-dense plasma.

During this development Lawson states his assumptions and makes deliberate omissions as follows. He makes explicit assumptions:
- Radiation is lost in Bremsstrahlung radiation
- Ion and electron temperatures are the same
- A fixed number of particles at steady-state

Some deliberate omissions are made for simplification:
- Thermal conduction through the plasma or apparatus
- Neutron energy is not reabsorbed but lost from the apparatus

Some power terms are neglected altogether:
- Power inputs to the sample from external energy input
- Losses from mechanical PdV work done by the sample gas

Fusion Energy

Lawson gave the energy release from fusion as

$$P_R = n_1 n_2 <\sigma v> (T)\ E \quad [\mathrm{MeV/cc\text{-}s}]$$

and equivalent to modern derivationsvii where the nuclei count density n1 = n2 = n/2, and for DT n1n2 =n2/4. The product of nuclei relative velocity and cross section <σv> (T) is averaged over the Maxwellian velocity distribution at absolute temperature T [K], E is the energy released by one reaction. Lawson cites the total energy density contributing to a fusion source is 17.6 MeV, for alpha nuclei 3.5 MeV, for neutrons 14.1 MeV. Neutrons comprise eighty percent of the fusion energy but escape the system, only the alphas contribute to plasma heating a known condition and accounted for in calculations since Lawson's time (see Atzeni and Meyer-Ter-Vehn, 2004, p79) [8].

Plasma Internal Energy

In a pulsed fusion energy system Lawson reasoned the heating energy required to heat the gaseous fuel is 3nkT where n is the nuclei count density for electrons and ions uniformly and accounts for the 3 factor rather than 3/2, k the Boltzmann constant, T is the absolute temperature [K]. For a confinement time 't' Lawson argued the power to heat the confined plasma is 3nkT/t. As used in this paper in consistent units

$$W = 3nkT\ [\mathrm{MeV/cc}].$$

Bremsstrahlung Energy

The energy lost by radiation is given by the power radiated per unit volume as

$$PB = 1.4 \times 10^{-34}\ n2\ T^{1/2}\ [\mathrm{Watts/cc}],$$

n is the nuclei count density, T is absolute temperature [K] with care given to units conversion.

Steady-State Energy Balance

After assumptions and simplifications are made, Lawson infers the energy balance as steady-state. Lawson does not compute the internal energy from an energy balance but rather specifies the plasma internal energy as 3nkT, PR is the fusion power due to fusion products, PB is the power lost by Bremsstrahlung radiation. From Lawson's paper in the original nomenclature and consistent units

$$\begin{array}{ll} 3nkT & [\mathrm{MeV/cc}] \\ P_R = 1/4\ n^2 <\sigma v>(T)\ E & [\mathrm{MeV/cc\text{-}s}] \\ P_B = 1.4 \times 10^{-34}\ n^2\ T^{1/2} & [\mathrm{MeV/cc\text{-}s}]. \end{array}$$

Since neutrons are lost from the system, he accounts for alpha products alone (3.5 MeV) in PR, t is the confinement time and repeated here

$$3nkT = {}_tP_R - {}_tP_B.$$

Original Lawson Criterion

Lawson reasoned a useful energy parameter 'R' to be “energy released in the hot gas to the energy supplied” meaning the ratio of alpha fusion products to energy supplied from internal plasma energy plus Bremsstrahlung radiation:

$$R = {}_tP_R / (3nkT + {}_tP_B)$$

where
3nkT= plasma internal energy
PR = alpha fusion product power
PB = Bremsstrahlung radiation power
t = confinement time [s]
n = nuclei count density [1/cc]
k = Boltzman constant
T = absolute temperature [K].

‘R’ is NOT the "Lawson Criterion" so often found in simplifications of the term, and not the definition of fusion gain factor ‘Q’ the quotient of power produced to power input. In 1955 Lawson recognized the fusion reaction does not heat the gas directly noting "disintegration products escape". He reasoned the gas initially must be heated through energy recirculation by the experimental apparatus.

In 1955 John Lawson defined a heating "efficiency" ŋ (Eta) as the portion of total energy recirculated to heat the plasma. Lawson reasoned the more energy required to heat the plasma in proportion to the total system energy, the less favorable fusion conditions will be. A high value of ŋ means the fusion process demands a large portion of the total system energy to sustain a fusion reaction, a low value means little of the total system energy is necessary to sustain the reaction and more useful energy available as energy production. Lawson chose ŋ = 1/3 as a plausible value the system apparatus could achieve to recirculate energy to sustain the plasma, values below this ratio are desirable, values above it undesirable. As Lawson defined it the energy recirculated is the plasma internal energy plus the Bremsstrahlung radiation loss ‘$3nkT + {}_tP_B$’.

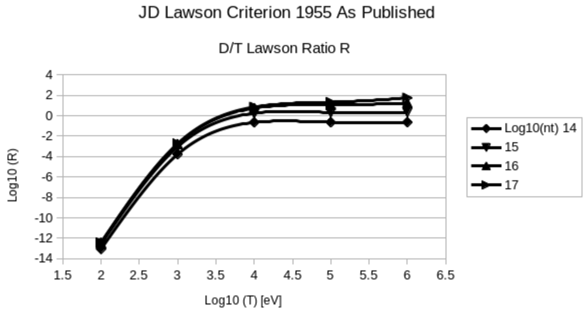
In 1955 Lawson cautioned all these conditions though necessary are far from sufficient where he wrote "The working cycle which has been assumed is very optimistic". Lawson reasoned the quotient of the energy supplied to the total energy of the system is 1/(R+1). The Lawson Criterion says this ratio must be less than the heating efficiency η:

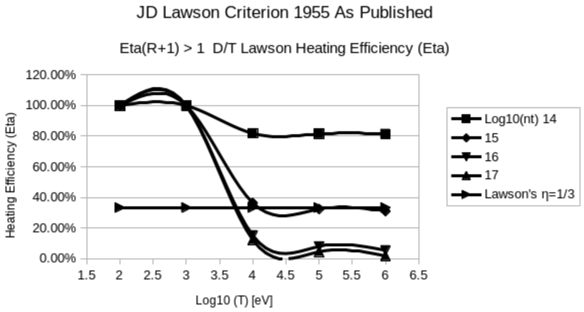
$$1/(R+1) < ŋ \text{ or the Lawson Criterion } 1 < ŋ(R+1).$$

Lawson develops the dimensionless energy ratio ‘R’ in terms of confinement parameter ‘nt’ and absolute temperature T or R(nt, T), he charts the value of R for DT DD reactions over temperatures and confinement values:

$$2 < \text{Log10 (T)} < 6 \text{ [eV]}$$
$$14 < \text{Log10 (nt)} < 17 \text{ [s/cc].}$$

As a familiar starting point the original physical properties and notation in the 1955 paperix are used to compute Lawson's original chart for energy ratio R, and the corresponding heating efficiency ŋ. The charts match Lawson's original confirming the units used and computations are the same as those in Lawson's 1955 paper. In 1955 Lawson considered only alpha products in the reaction value for 'E', the matching charts confirm this. Like Lawson's charts these show ratio R and efficiency η (Eta) for Deuterium/Tritium (DT) on Log scales for values of confinement parameter (nt). For Log10(T) = 6 and Log10(nt) = 17 the chart shows ‘R’ the ratio of energy released to energy supplied approaches Log10(R) = 2 or 100. When Log10(T) > 4 (1E4 eV) and confinement parameter Log10(nt) > 15 (1E15 s/cc) energy ratio R shows the energy released exceeds the energy supplied.





For a fusion process with temperature Log10(T) = 4 and confinement parameter Log10(nt) =14 the heating efficiency η is 80% of the total system energy. For fusion to occur the energy recirculated must be 80% of the total energy, in this example the requirement is more severe than the 33% value suggested by Lawson. Note for a confinement parameter Log10(nt) > 15 the recirculated energy meets Lawson's 33% and useful energy returned is much higher, the prospect for fusion is greater. This is the original Lawson criterion. To summarize:

1/(R+1) is (energy supplied) / (total energy) and
1/(R+1) < ŋ this ratio is less than heating efficiency η.

The Fusion Power Balance

For the remainder of this paper the notation is expanded to a complete power balance for an ICF equimolar DT target. A fast ignition approach is taken where the target fuel density remains constant within the hot central core and the cold surrounding fuel, but both pressure and temperature jump at the hot-cold boundary (Atzeni et al., 2004, Figure 4.1). This is the isochoric assembly discussed in references on fusion ignition (Pfalzner, 2006). Applying first-law fundamentals to a fusion plasma we have a power balance at the hot central core shown in Figure 1, all terms are power densities in units of [MeV/cc-s] (Eq. 4.1, Atzeni et al., 2004).

$$dW/dt = Pr + Pe - Pb - Pl \qquad \text{Eqs. 4.1.}$$

The nomenclature becomes:
dW/dt = plasma internal energy
Pa = alpha fusion product power
Pn = neutron fusion product power
Pr = Pa + Pn
Pe = external input power

Pb = Bremsstrahlung radiation power
Pl = power losses
Pp = nkT/t
t = confinement time (seconds)
n = nuclei count density (cc)
k = Boltzman constant
T = absolute temperature.

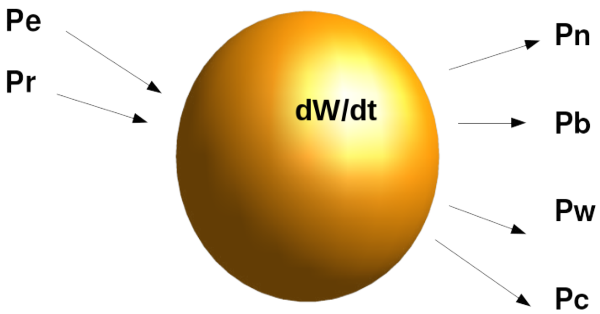


Figure 1 Power Balance Hot Core

The charged fusion products released consist of alpha nuclei and neutrons Pr = Pa + Pn. The power losses in such a system are accounted for as Pl = Pn + Pw + Pc where

Pw = PdV work rate in isochoric (constant density, variable pressure, temperature) expansion
Pc = thermal conduction power loss.

For consistency with Lawson's work the fusion energy source and cross sections are the same values as Lawson defined them in 1955, the power losses are developed in references on nuclear fusion by Friedberg or Atzeni. Here are power losses developed by Atzeni and Meyer-Ter-Vehn:

$Pc = 3c_eA_eT_h^{7/2} / \ln\Lambda R_h^2$	Thermal Conduction	(Eq. 4.10)
$Pw = 3\, \Gamma_B\, \rho_h\, T_h\, u\, /R_h$	Mechanical Work	(Eq. 4.13).

Thermal Conduction

Now to evaluate power losses due to thermal conduction, specifically electron diffusion at the hot target core. The power flowing through a unit surface is $-\chi_e$ gradT_e where χ_e is the electron conductivity, gradTe is the gradient of the electron temperature on the hot core surface and the negative sign the direction of power flow. The heat flux is shown proportional to the gradient of the internal energy density U, itself approximated as a function of temperature $U(T)$ and temperature as $T(r, t)$ (page197 reference)

$$\mathbf{q} = -\chi(T)\ gradT_e.$$

The reference shows for an ideal gas of electrons with velocity v_{av} and temperature dependence due to the Coulomb cross-section σ_c the conductivity takes the form $\chi_e = \chi_{e0}\, T_e^{\ 5/2}$. When a dimensional argument is made, grad$T_e = T_h/R_h$ where R_h is the radius of the hot core. For a classical DT plasma $\chi_e = A_e\, T_e^{\ 5/2} / \ln\Lambda$ where $A_e = 9.5$ x 10^{19} ergs s^{-1} cm^{-1} $keV^{-7/2}$, Te is the electron temperature, $\ln\Lambda$ is the Coulomb logarithm. For a surface area S and volume V the thermal conduction power takes the form

$$Pc = -\,\chi_e\, gradT_e\, S/V \sim 3c_eA_e\, T_h^{7/2}/ \ln\Lambda\, /\, R_h^2 \qquad [\text{MeV/cc-s}]$$

where c_e is a coefficient close to unity and the Coulomb logarithm is given by

$$\ln \Lambda_e = 7.1 - 0.5 \ln n_e + \ln T_e \quad (T_e >= 10\text{eV})$$
$$\ln \Lambda_i = 9.2 - 0.5 \ln n_e + 1.5 \ln T_i \quad (T_i <= 10\ \text{AkeV}),$$

(Eq. 10.136 reference)

and ion conductivity is neglected. x Table 10.4 of the reference shows computed values from Eq. 10.136, these values are used in the computations for the thermal conduction Pc. The relation is independent of density in agreement with the isochoric case in development of mechanical work next.

Mechanical Work

The hot spherical core exchanges energy with the surrounding fuel through mechanical pdV work, fuel matter at pressure p on a volume changing by an amount dV contributes energy dE = pdV. The reference shows for a homogeneous fuel sphere the contribution to the power balance from work is $Pw = (1/V)(dE_h/dt) = (p_h/V)\,(dV/dt) = (p_h\, S/V\,)u$ where S/V is the surface to volume ratio $3/R_h$ and u is the velocity of the sphere surface. If the ideal gas equation of state is $p = \Gamma B\, \rho T$ where Γ_B is the gas constant (Γ_B = 7.66 x 10^{14} $_{erg/g}$ keV for DT), the power due to work becomes

$$Pw = 3\, \Gamma_B\, \rho_h T_h u/R_h \qquad [\text{MeV/cc-s}].$$

For the isochoric case where the pressure in the hot core is much higher than the surrounding fuel, a shock is driven into the cold fuel, the velocity u for the material behind a shock gives

$$u = (3p_h/4\rho_c)^{\ 1/2} = (3\Gamma_B T_h\, \rho_h/\rho_c)^{1/2}\,.$$

Finally the power density for an isochoric ignition can be written with Am = 5.5 x 10^{22} cm^3 s^{-3} keV-3/2 as

$$Pw = A_m \rho_h T_h^{3/2}/R_{h.} \qquad [\text{MeV/cc-s}].$$

External Energy Source

Energy input from an external source is now developed for the fusion initiation process. If an external energy source such as an ion beam or laser delivers energy E to the ICF target of radius Rc in shot time ts , the power density delivered assuming 100% efficiency is

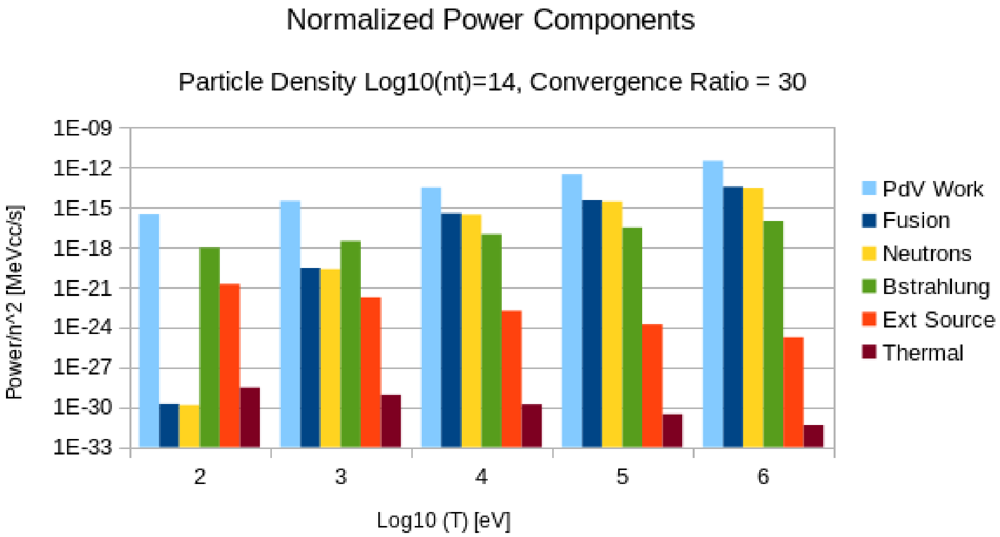
$$Pe = E/(4/3\pi R_c^3)/t_s \;.$$

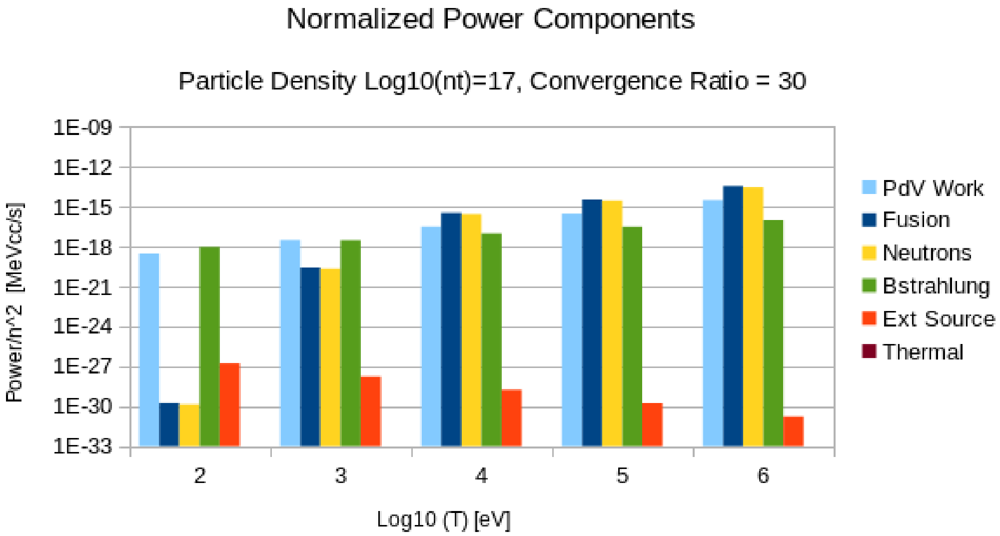
For a 1.8MJ laser shot delivered to a 2mm diameter spherical target in 20ns the power density is 2.149x10^{16} Joule/cc-s or 1.341x10^{23} MeV/cc-s.

Power Balance Components

Each power component from the previous section are calculated separately, finally the power balance is computed from Eq. 4.1 and this resultant is the new transient internal energy. To compare with Lawson's original work the power values are normalized to the nuclei density squared (n^2). As in Lawson's original paper these power terms are now functions of absolute temperature (T) and confinement parameter (nt) alone.

The next charts show the contribution of each power term relative to others for two values of the confinement parameter Log(nt) = 14 and 17 and a convergence ratio of the target shell R_c/R_h = 30. Notice the magnitude of losses from neutrons PdV work and thermal conductivity, these dominate the energy balance yet are completely omitted in steady-state assessments of the fusion reaction in literature.





Transient Fusion Criterion

The original Lawson Criterion neglected energy losses and assumed simplifications previously, here we include those omissions in a power balance for a pulsed system as proposed in Lawson's day. Here the modern pulsed system is laser ICF with a 2-mm spherical target (Rc=1mm) of DT fuel modeled with equimolar molecular weight 5.03 [gm/Mol] and convergence ratio 30. The complete power balance is

dW/dt = Pr + Pe – Pb – Pl (Eq. 1)
Pl = Pn + Pw + Pc. (Eq. 2)

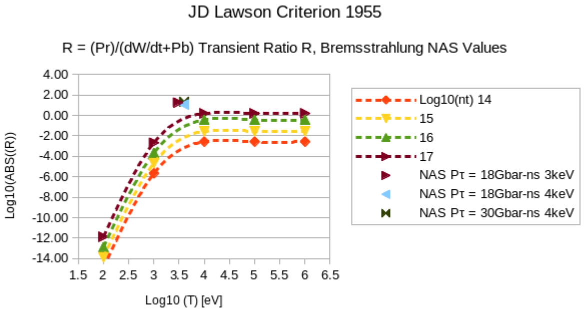
The power balance is fully populated including an external source and losses from neutrons, thermal conduction and mechanical work. In this form Pr is the total particle energy 17.6 MeV, Pn the neutron loss 14.1 MeV is accounted for in system losses separately.

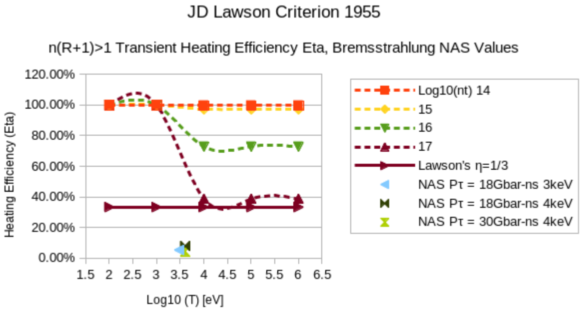
Initiating fusion ignition using a pulsed energy source (laser or ion beam) is a transient event, the power balance is not steady-state and non-zero. Modern interpretations of fusion ignition stray far from first-law fundamentals that in one case a converging computer algorithm is criteria enoughxi, in another counting neutrons is the fusion criteria. As a sanity check for conditions necessary and sufficient for fusion, the internal energy dW/dt is calculated from the complete power relation Eq. 1.

Lawson reasoned (previous section) the ratio R consisted of energy components that a fusion apparatus could recirculate to heat the plasma, he reasoned neutrons escape the system and losses were omitted from consideration. For a fixed volume where dW/dt is substituted for 3nkT/t

R = Pa / (dW/dt + Pb) (Eq. 3)
ŋ(R+1) > 1 (Eq. 4).

As the charts below show, when omissions to Lawson's power balance and in particular the transient internal energy dW/dt is calculated, there is no case where the power released exceeds power supplied and R ~ 1. The fusion plasma is loosing energy to losses, neutrons and Bremsstrahlung faster than the fusion reaction rate and external sources can replenish the plasma. The power balance is mostly negative shown with dashed curves and fusion under these conditions is not viable.



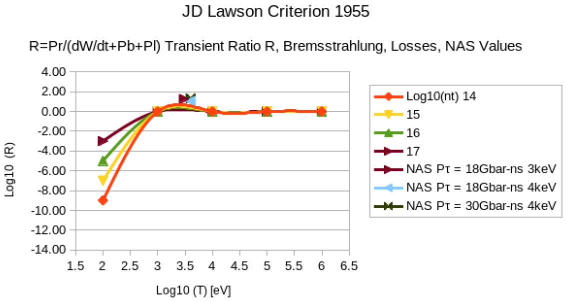


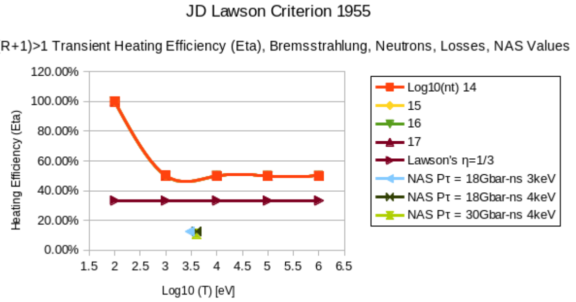
Note: Efficiency Eta 'η' in the Lawson Criteria 'η(R+1)>1' is shown as 'n(R+1)>1' due to charting software limitations.

The ignition criteria given in fusion references state that dW/dt is large and positive, a simple treatment from classical thermodynamics show this condition is not met [7]. The heating efficiency exceeds Lawson's suggested ŋ = 0.33 for any value of temperature or confinement parameter (nt), by this measure the necessary conditions for fusion are both unlikely and insufficient. As a bounding case let us assume all losses due to neutrons Bremsstrahlung and PdV work somehow contribute to plasma heating regeneration, then replace the alpha fusion Pa with total fusion Pr and include all losses as contributors to the fusion process:

R = (Pr) / (dW/dt + Pb + Pl) (Eq. 5)
ŋ(R+1) > 1. (Eq. 6)

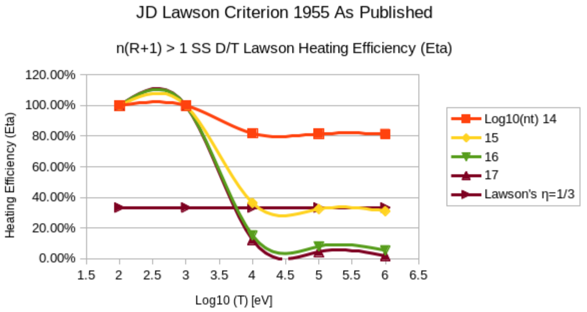
The next charts show this transient condition, note R becomes positive meaning the internal plasma energy is increasing suggesting a viable fusion process. Still the heating efficiency required exceeds 50%, meaning more than half the total system energy is required to sustain the fusion process.

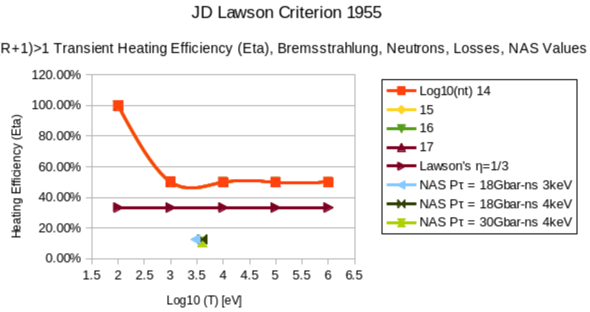




The charts also show the equivalent values of R, ŋ calculated from from empirical values of Pτ (pressure and confinement time) reported by the National Academy of Sciences, NAS Review of NIF Campaign in 2013.xiii Comparing the empirical values to the first law constraints suggests the heating efficiency attained by experiment is much better than our bounding case allows but doubtful.

Next, charts for the original 1955 Lawson heating efficiency and the chart including transient heating effect for the bounding case compared side-by-side. The bounding case shows 50% of the total energy must be recycled to sustain the fusion reaction but not achievable with the current apparatus.





Agreement with Previous Research

In his 1995 thesis Todd Rider shows how a fusion plasma in thermodynamic nonequilibrium cannot produce net power and the recirculation efficiencies required for recycling enough power to sustain fusion are not yet available[9]. Rider shows for a pulsed system not in equilibrium that fusion is not possible. From the Rider thesis abstract:

"For virtually all possible types of fusion reactors in which the major particle species are significantly non-Maxwellian or are at radically different mean energies, this minimum recirculating power is substantially larger than the fusion power. Barring the discovery of methods for recycling the power at exceedingly high efficiencies, grossly nonequilibrium reactors will not be able to produce net power."

In the 2000 paper Energy Balance of Controlled Thermonuclear Fusion by Hashmi and Staudenmaier the authors derive the fusion energy balance for both magnetized and non-magnetized fusion plasmas not from thermodynamics but from basic physics [10]. The authors carefully take account for binary interactions during disassociation and reabsorbtion of bound-bound, ionized and free-free transitions between particles in a fusion plasma process. They compute the probability of binary interactions accounting for the size of particle cross-section. They formulate the energy balance of fusion considering these interactions and show the energy losses due to radiation make the energy balance negative. From the paper, power losses given by Prad = Pb + Pc + Pexc + Prec (Eqs 23) are due to bremsstrahlung, cyclotron, and excitation radiation respectively. Their results show for a nonmagnitized plasma such as ICF, the energy balance is already crippled by excitation radiation Pexc alone. From the paper Table IV shows the ratio of ionization N* that makes the energy balance negative, for the temperature regime considered there is no temperature where the sustainable reaction criteria is satisfied.

From the Hashmi and Staudenmaier abstract:

"On the grounds of basic physics, a complete energy balance of magnetized and non-magnetized plasmas is presented for pulsed, stationary and self-sustaining operations by taking into account the energy release by reactions of light nuclei as well as different kinds of diffusive (conduction) and radiative (bremsstrahlung, cyclotron or synchrotron radiation and excitation radiation) energy losses. Already the energy losses by radiation make the energy balance negative. Hence, a fusion reactor ─ an energy producing device ─ seems to be beyond the realms of realization."

As the department lead from the Princeton Plasma Physics Laboratory, Daniel Jassby writes in Voodoo Fusion Energy:

"A tepid plasma of deuterium cannot produce measurable levels of fusion neutrons because one or more of the ion temperature, ion density or plasma volume is too small."

The present work agrees with these previous treatments of fusion in Inertial Confinement Fusion, that a fusion plasma is unlikely [11].

Summary of Lawson-like Criterion

As we have shown, some ICF researchers stress the importance of each term in an power balance, as this excerpt from researchers at Los Alamos National Laboratory shows:

“By focusing on the temperature rate equations and assuming that the electron and ion temperatures are the same in a homogeneous fusion fuel contained in a spherical shell, the net rate of temperature change depends on the work rate (dV/dt) due to compression, the fusion energy self-heating, the thermal conduction (both electron and ion), the Bremsstrahlung (or free-free) emission, the inverse Compton cooling, the synchrotron radiation, and the re-radiation of the shell material [12].”

Other researchers search for a new parameter space that seeks to make ICF fusion ignition likely, like this excerpt from Lawrence

Livermore:

"Such a survey revealed that there is a new region in the parameter space for thermonuclear fusion, which exists at lower initial density r0 and implosion velocity v0 than necessary for ICF [13]."

Given less than favorable conditions for fusion predicted by a comprehensive power balance, some researchers re-interpret the 1955 Lawson Criterion to serve a new fusion hypothesis more favorable to fusion and ignition. A Lawson-like criterion may simplify interpretation for both lay-people and venture capitalists, the true path to fusion remains cloaked in first-law fundamentals. It is useful for analysts to check physics fundamentals for a sanity-check using real numbers, this paper strives to achieve that.

Comparison Methodology

Next, some recent and popular Lawson-like criteria are cast in a consistent way for comparison as power ratio R and heating efficiency η. The energy terms and thus R, η are again functions of the fusion process temperature 'T' and the confinement parameter 'nt'. R and η are graphed over the region of interest

2 < Log10(T) < 6 T in [eV],

14 < Log10(nt) < 17 nt in [1/cc-s].

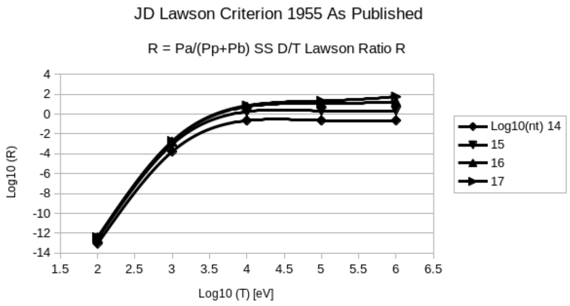
For each Lawson-like criterion reported by the researchers these steps were followed:

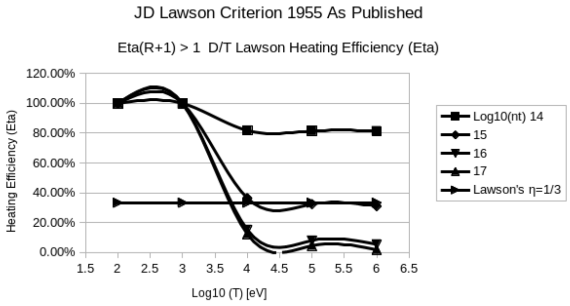
1. Formulate the power balance (if applicable)
2. Identify energy released and energy supplied terms
3. Calculate the power ratio R = power produced / power input
4. Calculate 1/R+1 is the power input to system total power
5. Chart R and the Lawson Criterion η(R+1) > 1.

Again some charts also show the empirical value of R, η achieved by the premier National Ignition Facility as reported in the 2013 National Academy of Sciences Review [14]. The reported NAS values in Pτ units (pressure and confinement time) are converted by simple calculation to the energy ratio R.

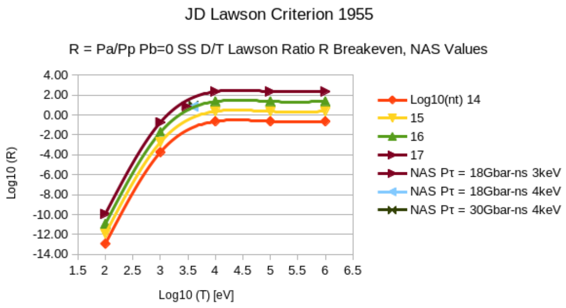
We begin by repeating the original 1955 Lawson Criterion revisited for an ICF target. In the Lawson paper a steady-state reaction was assumed for DD and DT fuels and the values for R charted, only the result for DT fuel is reproduced here. Like the original Lawson paper these results suggest the energy ratio R is quite optimistic and efficiency necessary reasonable, particularly at higher temperatures.

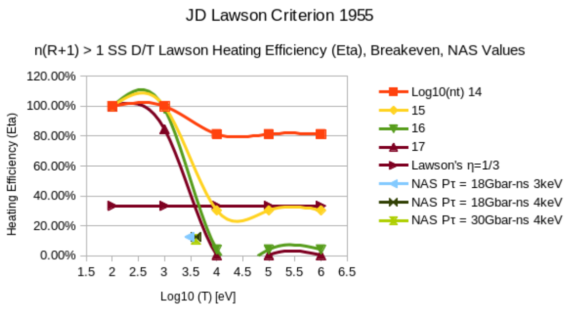
Lawson Criterion 1955
R = Pa / (Pp + Pb)





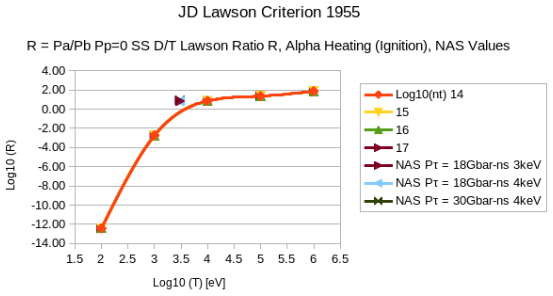
Lawson Criterion 1955 – Breakeven
R = Pa / Pp

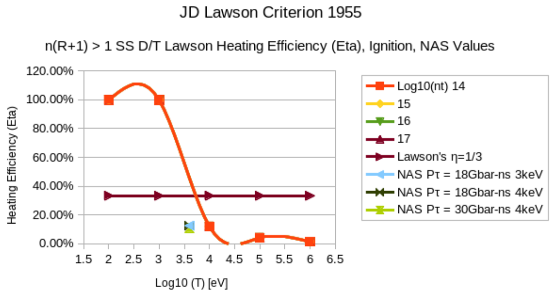




Lawson Criterion 1955 – Alpha Self-heating (Pfalzner [xx])

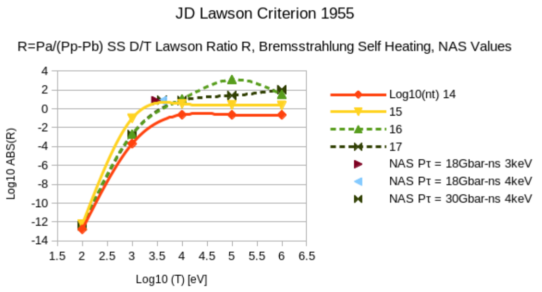
R = Pa / Pb

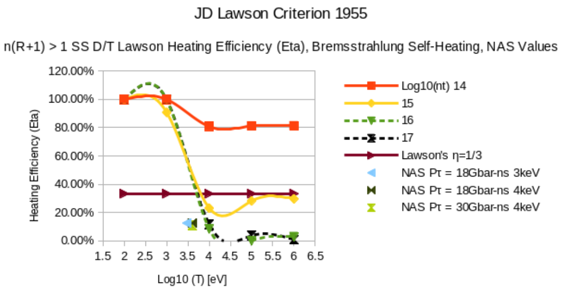




Lawson Criterion – with Bremsstrahlung Self-heating[1]

R = Pa / (Pp - Pb)

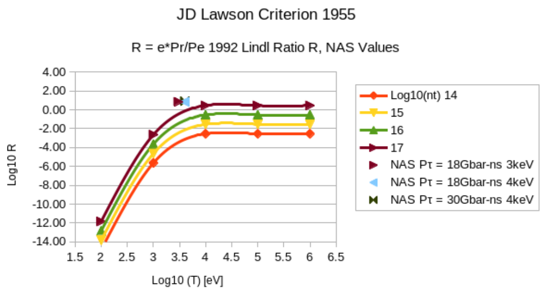


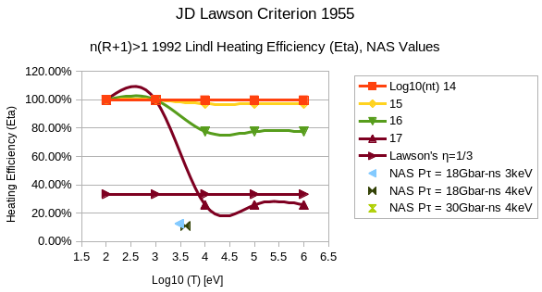


Where the dominant term produces a negative ratio R, the absolute value is taken to produce a logarithmic scale and the curve is dashed. To the degree Bremsstrahlung Pb could be captured for selfheating the term Pb is included, the solution is academic but suggests an invalid assumption when Pb exceeds the plasma internal energy Pp.

Lindl LLNL Physics Today 1992 [xxi]

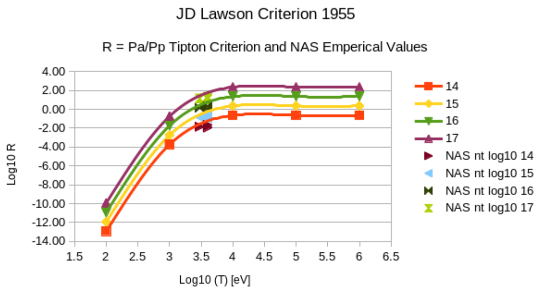
R = εPr / Pe

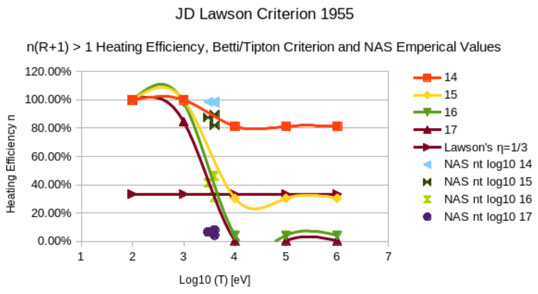




Betti/Tipton [xxii] [xxiii]

R = Pa / Pp

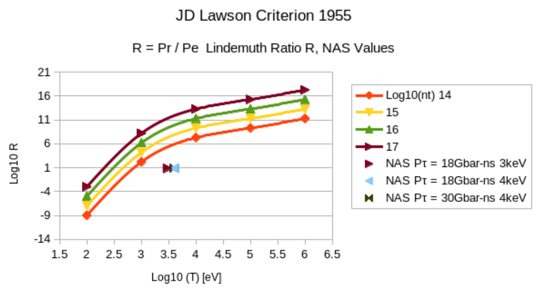


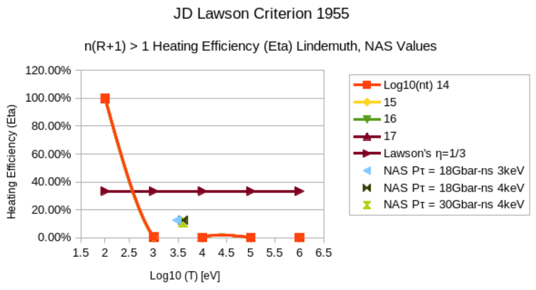


[1] Like alpha self-heating assumes Bremsstrahlung used in regeneration, not a system loss.

Lindemuth and Kirkpatrick [xxiv]

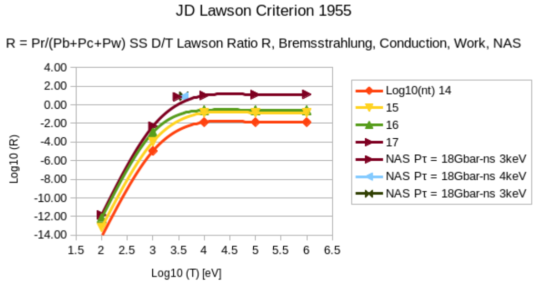
$$R = Pr / Pe$$

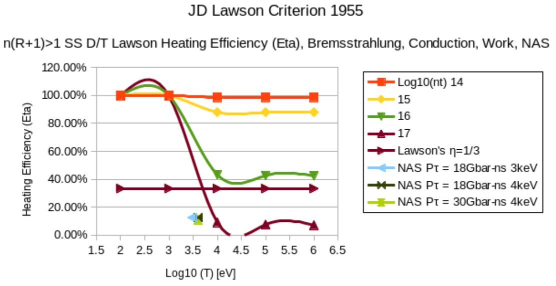




Atzeni and Meyer-Ter-Vehn [xxv]

$$R = Pr / (Pb + Pc + Pw)$$

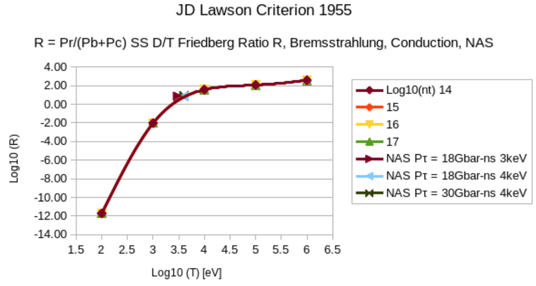


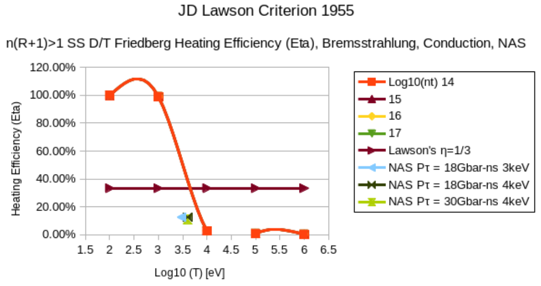


In this development by Atzeni and Meyer-Ter-Vehn a departure from the Lawson convention assumes Bremsstrahlung thermal conduction and mechanical work could be recovered if they appear in the denominator as energy supplied.

Friedberg [xxvi]

$$R = Pr / (Pb + Pc)$$

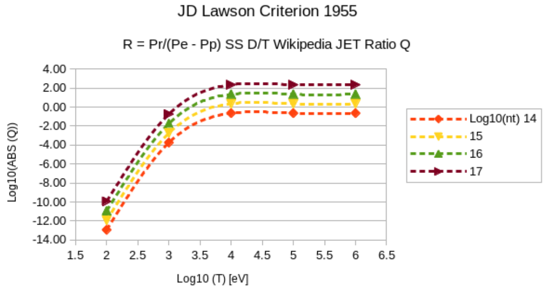


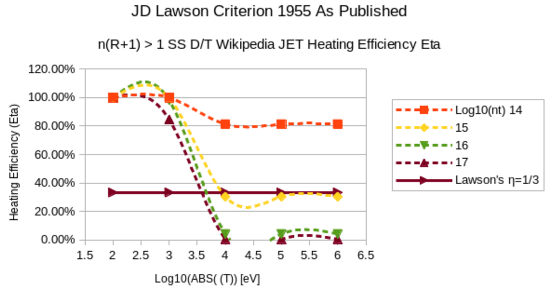


Again a departure from the Lawson convention of energy released to energy supplied, to the degree that Bremsstrahlung and thermal conduction loss could be recovered those contributions appear in the denominator as energy supplied.

Wikipedia: JET Tokamak [xxvii]

$Q = Pr/(Pe - Pp)$

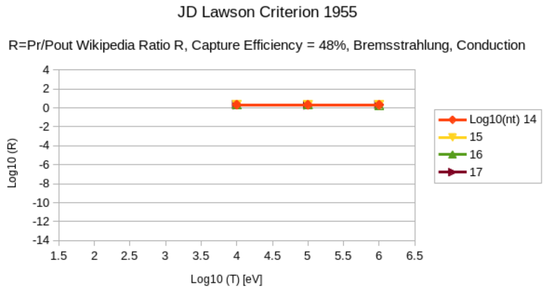


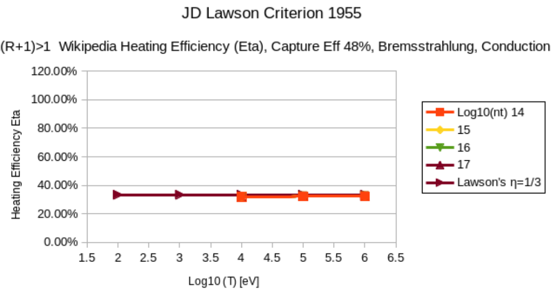


Here Gain factor Q and R are treated as equivalent for the sake of charting the ratio R and η. For the confinement parameter Log10(nt) > 15 the results look quite promising except the equivalent power balance Pp = Pe – Pr (power accumulated = net fusion power input) is not meaningful.

Wikipedia: Fusion Energy [xxviii]

$R = Pr/\varepsilon(Pr - Pb – Pc)$ where $\varepsilon = 48\%$





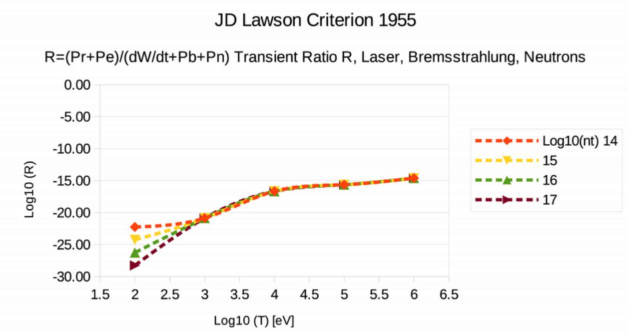
These curves show only the reduced domain where the power ratios remain positive and valid.

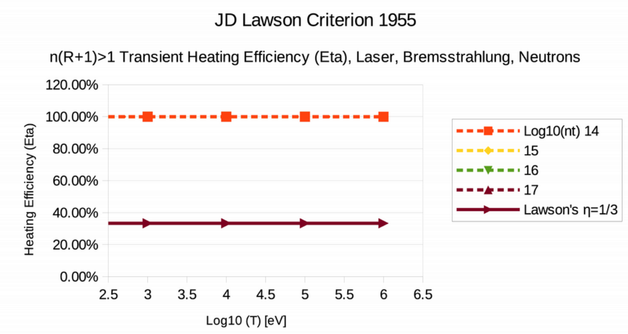
Interpreting Results

The fusion criterion proposed from different researchers remain quite optimistic for the prospect of steady-state fusion particularly at higher particle concentrations and temperatures. However when accounting for system losses and omissions the transition to steady-state through a transient barrier remains which no process accounted for (here) can cross. When a complete power balance is taken and the transient conditions leading to fusion are accounted for, the prospects for fusion are quite bleak. Even at the highest confinement parameter (nt) and temperature (10E17 1/cc-s and 10E6 eV) the heating efficiency demands 50% of total system energy. As a final bounding case lets assume both Bremsstrahlung and neutrons can be recirculated to heat the plasma and a 1.8MJ laser external source is used, the charts below show this case. Again the power balance is negative and a dashed curve is used.

Bounding Case: Bremsstrahlung and neutrons recirculated

$R= (Pr+Pe)/(dW/dt+Pb+Pn)$





With neutrons included for regeneration the system losses remain so great the power ratio remains negative and the heating efficiency must be 100%. In other interpretations of the Lawson criterion researchers have ignored the power balance and defy first-law fundamentals but the computation models developed here reveal the transition barrier persists.

Conclusions

These charts are a reminder that regardless of creative fusion criteria for predicting fusion ignition, basic physics and fundamental thermodynamics still apply and should guide any new fusion postulate. Modern derivations of fusion ignition criteria assume plasma internal energy accumulation is positive and much greater than zero or dW/dt >> 0. Computations show the energy balance is mostly negative, applying a Lawson-like criterion to this energy state is inappropriate. Assuming a bounding case shows the heating efficiency must be at best 50% of the total system energy to sustain fusion, however this bounding case assumes re-circulation of energies from neutrons, Bremsstrahlung, and system losses not achievable with present experimental apparatus. Computations applying basic thermodynamic principles show the conditions required to initiate fusion remain severe and do not predict sustained fusion is possible. Despite the plethora of tortured Lawson-like criteria in technical literature and convergent computer code schemes predicting fusion ignition, sustained fusion has not happened in any Laboratory [15-28].

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