

A Physical Theory based on the Barycenter Frame of Reference II: Principles of Particle Dynamics

Zhong Cheng Liang*

College of Electronic and Optical Engineering, Nanjing University of Posts and Telecommunications, China

*Corresponding Author

Zhong Cheng Liang, College of Electronic and Optical Engineering, Nanjing University of Posts and Telecommunications, China.

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Abstract

This paper extends the field theory of elastic particle fluids based on the barycenter reference frame and constructs the theoretical foundation of new particle dynamics. According to the particle flow field theory, the complete interaction contains the forces of gradient, curl, and divergence. The principle of action proposed in this paper affirms that the motion of a particle is driven by a force field, and the unified form of the force includes attraction and repulsion. In the force field, the motion of the particles follows the modified energy theorem and angular momentum theorem, as well as the newly discovered curlity theorem. Energy, curlity, and angular momentum are conserved in dynamically balanced systems, and their variations obey universal quantization rules. In a kernel field, the motion of particles follows the generalized Newton's law ($\mathbf{F} = m\mathbf{a}$, dynamic equation), and elliptical orbit is the special solution of the equation at the steady state $\mathbf{F} = 0$. The eigenstate orbits of a multi-particle system are petal-shaped, and the orbital energy spectrum (energy eigenvalues) is determined by the square of the curlity. A mathematical model of particle orbits is proposed, the structure of electron shells and the spectrum of the hydrogen atom are explained, and the relationship between the nuclear spin and the chirality of the electron orbits is also elucidated. The self-consistency and completeness of the theory show that the flow-field force is a unified form of particle interaction, that the principle of measurement is a first principle integrating the foundations of relativity and quantum theories, and that the theorems of particle dynamics are universal physical laws for the macroscopic and microscopic worlds.

Keywords: Particle Dynamics, Elastic Particles, Barycenter Frame of Reference, Flow Field Forces, Measurement Principle, Dynamical Theorems, Orbital Quantization

1. Introduction

An appropriate frame of reference must be chosen to describe the motion of an object, and the adoption of different frames of reference often leads to different theoretical frameworks. It is well known that Ptolemy's astronomy used a geocentric frame of reference, Copernican astronomy used a heliocentric frame of reference, and the currently popular Newtonian mechanics framework is an inertial frame of reference [1]. To address the problem of reference frame of classical electrodynamics, Einstein invented the spacetime frame of reference, thus creating the theory of relativity [2]. However, quantum theory, as one of the pillars of modern physics, lacks a unified reference frame, so that the controversy over the foundations of quantum mechanics lasted for one century.

The focus of the controversy over the foundations of quantum mechanics involves the interpretation of the nature of particles [3]. Whether the quantum is a wave or a particle is considered by classical mechanics to be two opposing propositions that cannot be true at the same time. However, quantum mechanics assumes that quanta are entities with wave-particle duality, and so the issue of the quantum frame of reference is ignored and covered up. Despite the great success of modern quantum theory, the fundamental questions about the nature of particles and the frame of reference have never been well resolved. In recent years, the author created a physical theory based on the model of elastic particles (real particles); more recently, the author moved this theory to the barycenter (center-of-mass) frame of reference [4-11]. The particle field theory based on the barycenter frame of reference gives a unified form of forces and reveals the universal laws of the interaction of objects [6,9-11]. On this basis, this paper further elucidates the measurement principle of the

barycenter frame theory, puts forward the action principle of particle dynamics, solves the dynamic equilibrium equation, and reveals the fundamental laws of the motion of particles.

Classical mechanics, electrodynamics, relativistic mechanics, and quantum mechanics are four different theoretical frameworks. Starting from the first principle thinking, the elastic particle theory based on the barycenter frame of reference integrates several theories into a unique theoretical paradigm and discourse system. In order to facilitate and improve communication, readers need to be aware of novel concepts as well as updated terminology to mitigate conceptual conflicts caused by incommensurability [12].

2. Basic Principles

2.1. Measurement Principle

The principle of measurement is expressed as follows: in the barycenter reference system, the center of mass of a flow field has uncertainty, and there exists a limit to the precision of the measurement of physical quantities, which is constrained by physical relations.

The limiting precision of a physical quantity is called the scale, and the physical quantity expressed in terms of the scale is called an actual quantity. The actual quantity q is defined as the product of a digit factor \tilde{q} and a scale factor q_s

$$q = \tilde{q} \cdot q_s \quad (0 < \{|q|, q_s\} < \infty). \quad (1)$$

The digit \tilde{q} is the numerical value of the physical quantity, and the scale q_s is the identity and measure of the physical quantity. To differentiate, digit factors are marked with the symbol “~” and scale factors are labeled with the subscript “s”. If there is a physical relation $z = f(x, y)$ between the actual quantities $\{x, y, z\}$, then the following constraint must exist

$$z = f(x, y) = \tilde{z} \cdot z_s \Leftrightarrow \tilde{z} = f(\tilde{x}, \tilde{y}) \wedge z_s = f_s, \quad (2)$$

where $\tilde{z} = f(\tilde{x}, \tilde{y})$ is called relation invariance, and $z_s = f_s$ is called scale covariance. Relation invariance and scale covariance determine unique arithmetic rules for actual quantities [10,11].

The scale of an actual quantity can be understood as the unit of the physical quantity. For example, r_s is the unit of spatial length and t_s is the unit of time. Classical physics uses units as the measure of physical quantities, known as the unit system; the barycenter frame theory uses scales as the measure of physical quantities, called the scale system. The difference between the two is that units cannot vary continuously whereas scales can vary continuously. The mathematical idea of scale covariance is to treat the units as continuous variables, and to study the low-dimensional physical world in a high-dimensional mathematical space. The International System of Units (SIU) has seven independent basic units. Constrained by physical relationships, the scale system has only three independent scales. The independent scales are called scale bases, and the other scales can be derived from the scale bases.

Scales are essential factors of physical quantities, and digits without scales have no physical meaning. Scales not only represent measurement precision and physical metrics, but also imply unified scientific ideas. For example, $\{r_s, t_s\}$ can be understood as space quantum and time quantum, and $\{r_s^{-1}, t_s^{-1}\}$ can be interpreted as space curvature and time curvature. The Measurement Principle integrates the core ideas of quantum and relativity theories and unifies the theoretical foundations of physics. The measurement principle has been called the principle of measurement relativity and the principle of objectivity [13, 4-9].

2.2. Flow Field Fundamentals

2.2.1. Convolution Field

According to the flow field theory, there exist a field of mass density $\rho(\mathbf{r}, t)$, a field of momentum density $\mathbf{j}(\mathbf{r}, t)$, and a field of velocity $\mathbf{v}(\mathbf{r}, t) = \mathbf{j}(\mathbf{r}, t)/\rho(\mathbf{r}, t)$ in the particle space [10,11]. From this one can compute the mass convolution $\Phi(\mathbf{r}, t)$ and the momentum convolution $\mathbf{A}(\mathbf{r}, t)$

$$\Phi(\mathbf{r}, t) := -\frac{1}{\phi} \iiint_V \frac{\rho(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV'; \quad \Phi_s = c^2, \quad \phi_s = \frac{m_s}{c^2 r_s}. \quad (3a)$$

$$\mathbf{A}(\mathbf{r}, t) := \alpha \iiint_V \frac{\mathbf{j}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV'; \quad A_s = c, \quad \alpha_s = \frac{r_s}{m_s}. \quad (3b)$$

Where c is the signal speed used for time synchronization. $\phi = 4\pi\phi_s$ and $\alpha = \alpha_s / (4\pi)$ are coefficients of mass convolution and momentum convolution, respectively, with the constraint $\phi\alpha = \phi_s \alpha_s = c^{-2}$. The particle dynamic theory stipulates using the speed of light as the scale of velocity, that is, $v_s = c = 29979246$ m/s. The convolution field is also known as the potential field, with the mass convolution being the scalar potential and the momentum convolution being the vector potential.

2.2.2. Action Field

The first-order spatial derivative of convolution is called the action field. The action field consists of a gradient \mathbf{G} , a curl \mathbf{C} and a divergence D , which are defined as

$$\mathbf{G} = -\nabla\phi, \quad \mathbf{C} = \nabla \times \mathbf{A}, \quad D = \nabla \cdot \mathbf{A}. \quad (4)$$

The gradient \mathbf{G} represents the gravitational acceleration and its scale is $G_s = v_s \nu_s$, where $\nu_s = t_s^{-1}$ is the scale of frequency. The curl \mathbf{C} and divergence D both have the scale of frequency and respectively represent the rotational and vibrational frequencies of the field elements. Frequency is a more practical concept than time in the barycenter system.

2.2.3. Force and Acceleration Fields

A field element (fieldon) of density $\rho(\mathbf{r}, t)$ is subjected to a force field $\mathbf{f}(\mathbf{r}, t)$

$$\mathbf{f}(\mathbf{r}, t) = \rho \frac{D\mathbf{v}}{Dt} = \rho \mathbf{a} = \mathbf{f}_G + \mathbf{f}_C + \mathbf{f}_D; \quad a_s = v_s \nu_s, \quad f_s = \rho_s a_s. \quad (5)$$

where $\mathbf{v}(\mathbf{r}, t)$ and $\mathbf{a}(\mathbf{r}, t)$ are respectively the velocity and acceleration of fieldons, and $D\mathbf{v}/Dt$ is the motion derivatives (material derivative) of the velocity. $\mathbf{f}_G, \mathbf{f}_C, \mathbf{f}_D$ are the densities of gradient force, curl force, and divergence force, respectively, which have the form

$$\mathbf{f}_G = \rho \mathbf{a}_G = \rho \mathbf{G}, \quad \mathbf{f}_C = \rho \mathbf{a}_C = \rho \mathbf{v} \times \mathbf{C}, \quad \mathbf{f}_D = \rho \mathbf{a}_D = \rho D\mathbf{v}. \quad (6)$$

$\mathbf{a}_G, \mathbf{a}_C, \mathbf{a}_D$ are the accelerations produced by the forces of gradient, curl and divergence, respectively. In the flow field theory, all the above physical quantities are Eulerian variables.

2.3. Action Principle

For a real particle of the mass m , if its position vector is expressed as a function of time, $\mathbf{r}(t)$, then the velocity and acceleration are respectively in the form

$$\mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt}, \quad v_s = r_s \nu_s; \quad \mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt}, \quad a_s = v_s \nu_s. \quad (7)$$

The real particle represented by the vector $m\mathbf{r}(t)$ is a point mass in classical mechanics, which is called particle for short. The model of point mass does not take into account the volume and shape of the real particles.

According to equation (5), the combined force $\mathbf{F}(t)$ acting on the particle consists of three terms

$$\begin{aligned} \mathbf{F} &= m \frac{d\mathbf{v}}{dt} = m\mathbf{a} = \mathbf{F}_G + \mathbf{F}_C + \mathbf{F}_D, \quad F_s = m_s a_s; \\ \mathbf{F}_G &= m\mathbf{a}_G, \quad \mathbf{F}_C = m\mathbf{a}_C, \quad \mathbf{F}_D = m\mathbf{a}_D; \\ \mathbf{a}_G &= \mathbf{G}, \quad \mathbf{a}_C = \mathbf{v} \times \mathbf{C}, \quad \mathbf{a}_D = D\mathbf{v}. \end{aligned} \quad (8)$$

The action principle of dynamics is stated as follows: the particle is driven by the flow field, and the complete flow field action includes the gradient force \mathbf{F}_G , the curl force \mathbf{F}_C , and the divergence force \mathbf{F}_D , which cause the particle to produce a gradient acceleration \mathbf{a}_G , a curl acceleration \mathbf{a}_C , and a divergence acceleration \mathbf{a}_D , respectively.

2.4. Attraction and Repulsion

The divergence D is a scalar and the curl C is a vector. Taking the divergence as the scale of the frequency, that is, letting

$$D = \nu_s = 1/t_s, \quad \tilde{D} = 1, \quad (9)$$

the digit of the curl can thus be expressed as

$$\boldsymbol{\omega} := \tilde{\boldsymbol{C}} = \frac{C_1 \mathbf{i} + C_2 \mathbf{j} + C_3 \mathbf{k}}{D} = \omega_1 \mathbf{i} + \omega_2 \mathbf{j} + \omega_3 \mathbf{k}, \quad (10)$$

$$\omega_i = \tilde{C}_i = \frac{C_i}{D} \quad (i = 1, 2, 3).$$

Remember that $\boldsymbol{\omega}$ is a digital curl (with the digital notation omitted).

The combined acceleration of a particle can be expressed in the following form

$$\mathbf{a} = \mathbf{a}_G + \mathbf{a}_C + \mathbf{a}_D = \mathbf{G} + \mathbf{R} \cdot \mathbf{v}, \quad (11)$$

where \mathbf{G} is the attraction acceleration and $\mathbf{R} \cdot \mathbf{v}$ is the repulsion acceleration. \mathbf{R} is called the repulsion matrix and is of the form

$$\mathbf{R} = \begin{pmatrix} D & C_3 & -C_2 \\ -C_3 & D & C_1 \\ C_2 & -C_1 & D \end{pmatrix} = \begin{pmatrix} 1 & \omega_3 & -\omega_2 \\ -\omega_3 & 1 & \omega_1 \\ \omega_2 & -\omega_1 & 1 \end{pmatrix} D. \quad (12)$$

As long as $D \neq 0$, then $\det \mathbf{R} \neq 0$.

Equation (11) is equivalent to $\mathbf{F} = m\mathbf{a}$ and is called the dynamical equation of particles. It is clear that the acceleration of repulsion is zero at velocity $\mathbf{v} = 0$. Therefore, the motion itself is the causes of repulsion. The acceleration comes from the affine transformation of the velocity, which shows the non-inertial motion or nonlinear characteristics. Because $\mathbf{a}(-\mathbf{v}) \neq \mathbf{a}(\mathbf{v})$, this is called the chirality of acceleration. The chirality is an important feature that distinguishes the barycenter frames from the inertial frames. When $\mathbf{a} = 0$ and $\mathbf{v} \neq 0$, the attractive force is equal to the repulsive force, and the particle is in the state of dynamic equilibrium. The state of dynamic equilibrium is called a steady state as well.

2.5. Modes and States

An object (a real particle) has three modes of motion: translational, rotational, and vibrational. The translation mode is the displacement of the object's barycenter, the rotation mode is a fixed-point rotation of the particles inside the object relative to the barycenter, and the vibration mode is the elastic oscillation of the particles inside the object relative to the barycenter. Each mode of motion in three-dimensional space has three degrees of freedom, and the three modes have a total of nine motional degrees of freedom.

The translation mode of the object's barycenter (*i.e.*, point mass) is called the orbital motion. A steady orbit can be further decomposed into three states: circulatory, pulsational, and nutational. The circulation state is the revolution of the point-mass around a planar circle, the pulsation state is the radial oscillation with respect to the center of circle, and the nutation state is the normal wobbling with respect to the plane of circle.

2.6. System and Constraints

The moving particle $m\mathbf{r}$ and the action field $\{\mathbf{G}, \mathbf{C}, D\}$ form a dynamics system, and the equation of dynamics is equivalent to Newton's second law $\mathbf{F} = m\mathbf{a}$. Solving $\{\mathbf{r}, \mathbf{v}, \mathbf{a}, \mathbf{G}, \mathbf{C}, D\}$ under constraints is the basic problem of particle dynamics. The constraints of the dynamics system include mathematical and physical constraints. The mathematical constraints contain the scale covariance and finiteness conditions, and the physical constraints contain the energy theorem, the angular momentum theorem, the curlity theorem, as well as the conservation theorems of the three quantities.

A system with a combined force $\mathbf{F} = 0$ is a steady system and the equation of steady system is

$$\mathbf{a} = \mathbf{G} + \mathbf{R} \cdot \mathbf{v} = \mathbf{0}. \quad (13)$$

The equation of steady system is called eigen equation and the solution of the eigen equation is called eigen state. The steady system requires the action field to be a steady field, so $\{\mathbf{G}, \mathbf{C}, D\}$ is independent of time. Particles in steady systems have stable orbits and periods, such as the rotation of the Moon orbiting the Earth and the Earth orbiting the Sun.

2.7. Energy Theorem

The motion energy of the particle is defined as

$$E := \frac{1}{2}m(\mathbf{v} \cdot \mathbf{v}) = \frac{1}{2}mv^2, \quad E_s = m_s v_s^2. \quad (14)$$

The motion energy is different from the kinetic energy of a point mass. The motion energy contains the contribution of the field and is not owned by the particle alone.

The change rate of the motion energy is defined as the motion power

$$P := \frac{dE}{dt} = m\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} = m\mathbf{v} \cdot \mathbf{a} = \mathbf{v} \cdot \mathbf{F} = P_G + P_C + P_D, \quad P_s = E_s v_s. \quad (15)$$

$$\begin{cases} P_G = \mathbf{v} \cdot \mathbf{F}_G = m\mathbf{v} \cdot \mathbf{G} \\ P_C = \mathbf{v} \cdot \mathbf{F}_C = m\mathbf{v} \cdot (\mathbf{v} \times \mathbf{C}) \\ P_D = \mathbf{v} \cdot \mathbf{F}_D = m\mathbf{v} \cdot (\mathbf{v}D) = mDv^2 \end{cases}$$

Above equation indicates that the motion power equals the scalar product of velocity and force, which is called the energy theorem. P_G, P_C, P_D are the gradient power, the curl power, and the divergence power, respectively. For a steady system with $\mathbf{F} = 0$, we have

$$P = \frac{dE}{dt} = \mathbf{v} \cdot \mathbf{F} = 0, \quad E = \frac{1}{2}mv^2 = \text{constant}. \quad (16)$$

Above equation shows that the total power of the steady system is zero and the motion energy is a constant, which is called the energy conservation theorem. The theorem is comparable to the law of energy conservation for a system composed of point masses, but the motion energy contains the orbital energies of circulation, pulsation and nutation. The theory of barycenter system no longer uses the concept of potential energy of the inertial system.

The energy conservation theorem shows that the particle energy at an eigenstate has a stable value, and that a jump in the energy is possible only when the external perturbation reaches E_s . This is the generalized rule of energy quantization.

2.8. Angular Momentum Theorem

The angular momentum of the particle is defined as

$$\mathbf{M} := m\mathbf{r} \times \mathbf{v}, \quad \mathbf{M}_l := m\mathbf{r} \times (-\mathbf{v}) = -\mathbf{M}; \quad M_s = m_s v_s r_s. \quad (17)$$

We specify \mathbf{M} as the right-handed angular momentum and \mathbf{M}_l as the left-handed angular momentum. The antisymmetry $\mathbf{M}_l = -\mathbf{M}$ is called the chirality of angular momentum. Chirality is an important addition of the barycenter frame theory to the inertial frame theory. Unless otherwise stated, the analysis in this article is in terms of right-handed angular momentum.

The change rate of angular momentum is defined as the force torque \mathbf{T}

$$\begin{aligned}
\mathbf{T} &:= \frac{d\mathbf{M}}{dt} = m \left(\frac{d\mathbf{r}}{dt} \times \mathbf{v} + \mathbf{r} \times \frac{d\mathbf{v}}{dt} \right) \\
&= \mathbf{r} \times \mathbf{F} = \mathbf{T}_G + \mathbf{T}_C + \mathbf{T}_D, \quad T_s = E_s. \\
\begin{cases} \mathbf{T}_G = \mathbf{r} \times \mathbf{F}_G = m\mathbf{r} \times \mathbf{G} \\ \mathbf{T}_C = \mathbf{r} \times \mathbf{F}_C = m\mathbf{r} \times (\mathbf{v} \times \mathbf{C}) \\ \mathbf{T}_D = \mathbf{r} \times \mathbf{F}_D = m\mathbf{r} \times \mathbf{v}D \end{cases}
\end{aligned} \tag{18}$$

It indicates that the force torque equals the vector product of the position vector and the force, which is known as the angular momentum theorem. $\mathbf{T}_G, \mathbf{T}_C, \mathbf{T}_D$ are the gradient torque, the curl torque, and the divergence torque, respectively. For a steady system with $\mathbf{F} = 0$, we have

$$\mathbf{T} = \frac{d\mathbf{M}}{dt} = \mathbf{r} \times \mathbf{F} = \mathbf{0}, \quad \mathbf{M} = \text{constant vector}. \tag{19}$$

Above equation shows that the combined torque of the steady system is zero and the angular momentum is a constant vector, which is called the theorem of angular momentum conservation. The theorem also shows that the angular momentum of the eigenstate particle has a stable value, but when the external perturbation reaches M_s , the angular momentum of the particle may undergo a jump. This is the generalized rule of angular momentum quantization.

2.9. Curlity Theorem

The curlity of the particle is defined as

$$\mathbf{Z} := m\mathbf{r} \times \mathbf{C}, \quad Z_s = m_s r_s v_s = m_s v_s. \tag{20}$$

The curlity of a barycenter system has the same scale as the momentum ($m\mathbf{v}$) of an inertial system, but they have completely different physical meanings.

The self-consistency of a physical theory requires that the change rate of the curlity equals the acting force, *i.e.*

$$\begin{aligned}
\mathbf{F} &= \frac{d\mathbf{Z}}{dt} = m \left(\frac{d\mathbf{r}}{dt} \times \mathbf{C} + \mathbf{r} \times \frac{d\mathbf{C}}{dt} \right) = \mathbf{F}_C + \mathbf{F}_R, \quad F_s = Z_s v_s = m_s a_s; \\
\mathbf{F}_C &= m \frac{d\mathbf{r}}{dt} \times \mathbf{C} = m\mathbf{v} \times \mathbf{C}, \quad \mathbf{F}_R = m\mathbf{r} \times \frac{d\mathbf{C}}{dt} = \mathbf{F}_G + \mathbf{F}_D.
\end{aligned} \tag{21}$$

Above equation is called the curlity theorem. The theorem shows that the acting force \mathbf{F} contains the curl force \mathbf{F}_C and the radial force \mathbf{F}_R , and \mathbf{F}_R in turn contains the gradient force \mathbf{F}_G and the divergence force \mathbf{F}_D .

For a steady system with $\mathbf{F} = 0$, we have

$$\mathbf{F} = \frac{d\mathbf{Z}}{dt} = \mathbf{0}, \quad \mathbf{Z} = \text{constant vector}. \tag{22}$$

Above equation shows that the curlity of the steady system is a constant vector, which is called the theorem of curlity conservation. The theorem also shows that the curlity of an eigenstate particle has a stable value and a jump in the curlity is possible only when the external perturbation reaches Z_s . This is called the rule of curlity quantization.

Self-consistency also requires that the following closure conditions be met between the energy and curlity.

$$E = \frac{1}{2m} (\mathbf{Z} \cdot \mathbf{Z}) = \frac{Z^2}{2m}. \tag{23}$$

Curlity is a unique physical quantity of the elastic particle theory. Curlity theorem, curlity conservation theorem, and curlity quantization are laws specific to the barycenter systems. The closure relationship between energy and curlity makes particle dynamics a theoretical system of consistence and completeness.

3. Steady Fields

3.1. Acting Kernel

Consider a uniform rigid sphere with radius r_0 , density ρ_0 , and mass m_0 . There is a fixed axis passing through the center of the sphere, and the sphere rotates uniformly around the fixed axis at an angular frequency ω_0 . This sphere is called the acting kernel, and the rotation of the kernel around the axis is called kernel spin. The mass of the acting kernel generates a gradient field \mathbf{G}' , the kernel spin generates a curl field \mathbf{C}' , and the relative movement of the particle with the acting kernel generates a divergence field D' . With the direction of the kernel spin as a reference, the orbital motion of the particle is chiral. This article studies the orbital motion of particles in a kernel field, without considering the spin of the orbital particles themselves.

A kernel coordinate system $[OX'_1X'_2X'_3]$ is established with unit vectors $(\mathbf{i}', \mathbf{j}', \mathbf{k}')$ for the coordinate axes. The coordinate origin O is at the center of kernel and X'_3 coincides with the kernel spin axis. The coordinate plane $X'_3 = 0$ is called the kernel equatorial plane and is denoted by π_e . The spherical coordinate parameter with OX'_3 as the polar axis is (r', θ', φ') , and the unit vector in the direction of the increasing parameter is $(\mathbf{e}'_r, \mathbf{e}'_\theta, \mathbf{e}'_\varphi)$. The relationship between the basis vectors $(\mathbf{i}', \mathbf{j}', \mathbf{k}')$ and the $(\mathbf{e}'_r, \mathbf{e}'_\theta, \mathbf{e}'_\varphi)$ is

$$\begin{cases} \mathbf{e}'_r = \sin\theta' \cos\varphi' \mathbf{i}' + \sin\theta' \sin\varphi' \mathbf{j}' + \cos\theta' \mathbf{k}' \\ \mathbf{e}'_\theta = \cos\theta' \cos\varphi' \mathbf{i}' + \cos\theta' \sin\varphi' \mathbf{j}' - \sin\theta' \mathbf{k}' \\ \mathbf{e}'_\varphi = -\sin\varphi' \mathbf{i}' + \cos\varphi' \mathbf{j}' \end{cases}$$

The moving particles and the acting kernel form a dynamic system with the barycenter always deviating from the origin O . The kernel coordinate system is the observer's frame of reference and the origin is the observation site. The transfer from the observer's frame to the barycenter frame requires only a scale transformation, not a coordinate transformation [11].

3.2. Gradient Field

The mass convolution outside the acting kernel can be calculated from the flow field theory

$$\Phi'(\mathbf{r}') = -\frac{1}{\phi} \iiint_{V'} \frac{\rho_0(\mathbf{r}^*)}{|\mathbf{r}' - \mathbf{r}^*|} dV^* = -\frac{m_0}{\phi r'} \quad (r' > r_0), \quad (24)$$

where \mathbf{r}^* , V^* are integral variables. The mass convolution is equivalent to the gravitational potential (electrostatic potential). From this the gradient outside the kernel can be calculated as

$$\mathbf{G}'(\mathbf{r}') = -G'(r') \mathbf{e}'_r, \quad G'(r') = \frac{m_0}{\phi r'^2}, \quad G_s = c v_s. \quad (25)$$

where the convolution factor $\phi = 1/g$ and g is the universal gravitational constant. The essence of the gradient is the gravitational acceleration. The gradient can be expressed in Cartesian terms as

$$\mathbf{G}'(\mathbf{r}') = -G'(r')(\sin\theta' \cos\varphi' \mathbf{i}' + \sin\theta' \sin\varphi' \mathbf{j}' + \cos\theta' \mathbf{k}'). \quad (26)$$

3.3. Divergence Field

The flow field theory gives the divergence as

$$D'(\mathbf{r}', t) = \frac{1}{c^2} \frac{\partial \Phi'}{\partial t} + D'_0(t), \quad v_s = D'. \quad (27)$$

The mass convolution of the steady field is independent of time, *i.e.*, $\partial \Phi' / \partial t = 0$, so the divergence D' of the steady field is a system constant. The divergence represents the frequency of relative vibration between particles and acting kernel.

3.4. Curl Field

The momentum convolution of the steady field is

$$\mathbf{A}'(\mathbf{r}') = \alpha \iiint_{V'} \frac{\mathbf{j}_0(\mathbf{r}^*)}{|\mathbf{r}' - \mathbf{r}^*|} dV^*. \quad (28)$$

Momentum convolution is equivalent to the static magnetic potential. Referring to the calculation of the magnetic potential of a uniformly charged sphere rotating on a fixed axis, we give here the momentum convolution outside the acting kernel [14].

$$\mathbf{A}'(\mathbf{r}') = \frac{\alpha_s m_0 \omega_0 r_0^2}{20\pi} \frac{r_0^2}{r'^2} \sin\theta' \mathbf{e}'_\varphi \quad (r' > r_0). \quad (29)$$

From this we calculate the curl

$$\mathbf{C}'(\mathbf{r}') = B'(r')(2\cos\theta' \mathbf{e}'_r + \sin\theta' \mathbf{e}'_\theta); \quad (30)$$

$$B'(r') = \frac{\alpha_s m_0 \omega_0 r_0^2}{20\pi} \frac{r_0^2}{r'^3}, \quad B_s = v_s = D'.$$

Therefore, the digital curl can be expressed in Cartesian basis as

$$\boldsymbol{\omega}' = \tilde{B}'(2\cos\theta' \mathbf{e}'_r + \sin\theta' \mathbf{e}'_\theta) = \omega'_1 \mathbf{i}' + \omega'_2 \mathbf{j}' + \omega'_3 \mathbf{k}'. \quad (31)$$

$$\begin{cases} \omega'_1 = 3\tilde{B}' \sin\theta' \cos\theta' \cos\varphi' \\ \omega'_2 = 3\tilde{B}' \sin\theta' \cos\theta' \sin\varphi' \\ \omega'_3 = \tilde{B}'(3\cos^2\theta' - 1) \end{cases}$$

4. Steady Orbits

4.1. Orbital Constraints

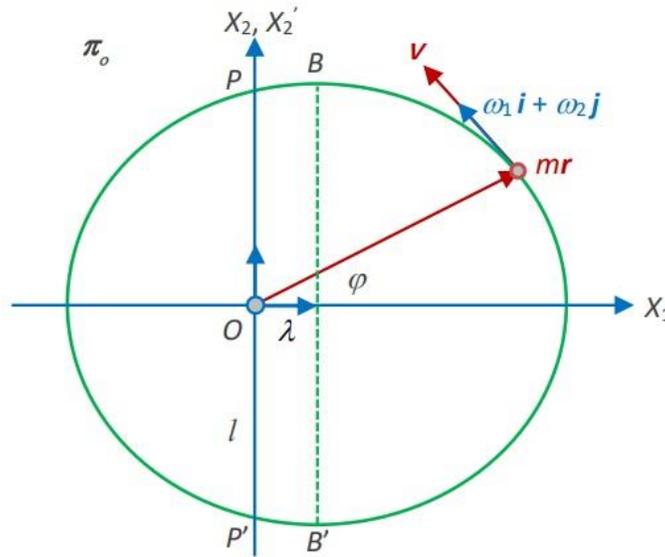


Figure 1: Orbital Coordinate System

The gradient generated by the acting kernel is a centrosymmetric field and the curl is an axisymmetric field. It is known that particles have stable elliptical orbits in the kernel field and the center of kernel O is a focus of the ellipse. As shown in Figure 1, the orbital plane is π_o and its normal unit vector is \mathbf{k} . The line l is the intersection of the orbital plane with the equatorial plane, and the angle β is the orbital inclination between the two planes of π_e and π_o . Establish an orbital coordinate system $[OX_1 X_2 X_3]$ with the axial unit vector being $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ and the orbital plane coinciding with the coordinate plane $X_3 = 0$. In addition, let the X_2 axis coincide with the intersection l and the X_1 axis coincide with the major axis of the ellipse. The spherical coordinate parameters with X_3 as the polar axis are (r, θ, φ) . With the origin at the kernel center, the orbital coordinate system also belongs to the observer frame of reference.

The basic constraints on the equilibrium orbits are finiteness and periodicity

$$0 < r < \infty, \quad r(\varphi) = r(\varphi + 2\pi k), \quad k = 1, 2, 3, \dots \quad (32)$$

In the orbital coordinate system, the vectors $\{\mathbf{r}, \mathbf{G}\}$ are co-linear and $\{\mathbf{r}, \mathbf{v}, \mathbf{G}\}$ are coplanar. Since the position vector of the barycenter system has uncertainty, we cancel the co-linear condition and keep the coplanar condition

$$\mathbf{r} \times \mathbf{G} \neq \mathbf{0}, \quad (\mathbf{r} \times \mathbf{G}) \cdot \mathbf{v} = 0. \quad (33)$$

Then, the vectors $\{\mathbf{r}, \mathbf{v}, \mathbf{G}, \mathbf{C}\}$ in the orbital coordinates can be expressed as

$$\mathbf{r} = x_1 \mathbf{i} + x_2 \mathbf{j}, \quad \mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j}, \quad \mathbf{G} = -(G_1 \mathbf{i} + G_2 \mathbf{j}), \quad \mathbf{C} = \omega_1 \mathbf{i} + \omega_2 \mathbf{j} + \omega_3 \mathbf{k}. \quad (34)$$

From this, we can calculate the acceleration

$$\begin{aligned} \mathbf{a} &= \mathbf{G} + \mathbf{v} \times \mathbf{C} + \mathbf{v}D \\ &= (C_3 v_2 + v_1 D - G_1) \mathbf{i} + (D v_2 - C_3 v_1 - G_2) \mathbf{j} + (C_2 v_1 - C_1 v_2) \mathbf{k}. \end{aligned} \quad (35)$$

4.2. Steady States

Set equation of (35) be zero ($\mathbf{a} = 0$), we obtain the motion equation for steady states

$$\begin{cases} D v_1 + C_3 v_2 = G_1 \\ -C_3 v_1 + D v_2 = G_2 \\ C_2 v_1 - C_1 v_2 = 0 \end{cases} \quad (36)$$

This is a set of algebraic equations for velocity. Solving the first and third equations of (36) obtains a set of velocities

$$v_1 = \frac{G_1 C_1}{D C_1 + C_2 C_3}, \quad v_2 = \frac{G_1 C_2}{D C_1 + C_2 C_3}.$$

Solving the second and third equations of (36) yields another set of velocities

$$v_1 = \frac{G_2 C_1}{D C_2 - C_1 C_3}, \quad v_2 = \frac{G_2 C_2}{D C_2 - C_1 C_3}.$$

Combining the above two equations gives the steady solution for the velocity

$$v_1 = r C_1 = r \omega_1 D, \quad v_2 = r C_2 = r \omega_2 D. \quad (37)$$

where r is the length of the position vector

$$r = \frac{G_1}{D C_1 + C_2 C_3} = \frac{G_2}{D C_2 - C_1 C_3}. \quad (38)$$

The finiteness condition requires the exclusion of the divergence point $r \rightarrow \infty$, *i.e.*

$$D C_2 - C_1 C_3 = (\omega_2 - \omega_1 \omega_3) D^2 \neq 0 \implies D \neq 0, \quad \omega_2 - \omega_1 \omega_3 \neq 0. \quad (39)$$

Solving r and D from the two equations of (38) yields

$$r = \frac{G_1 C_2 - G_2 C_1}{(C_1^2 + C_2^2) C_3}, \quad D = \left(\frac{G_1 C_1 + G_2 C_2}{G_1 C_2 - G_2 C_1} \right) C_3. \quad (40)$$

It can be seen that $r = 0$ is a point of spatial singularity that causes the divergence to infinity ($D \rightarrow \infty$). The condition that excludes $r = 0$ is

$$G_1 C_2 - G_2 C_1 = (G_1 \omega_2 - G_2 \omega_1) D \neq 0, \implies G_1 \omega_2 - G_2 \omega_1 \neq 0. \quad (41)$$

4.3. Scale Systems

The scales of actual quantities are non-zero finite values representing the limiting precision of physical quantities. The use of a scale system directly eliminates singularities and guarantees the finiteness of the steady solution. Actual quantities have three independent scale bases. The different scale bases form a scale system, denoted by $[s_1, s_2, s_3]$. The scale bases must be physical quantities of practical

significance, *e.g.*, the scale bases of the $[m\lambda c]$ system are

$$m_s = m, \quad r_s = \lambda, \quad v_s = c. \quad (42)$$

Where m is the mass of the particle and c is the speed of waves. The meaning of λ is to be determined. Other scales can be derived based on the scale covariance

$$v_s = v_s/r_s = c/\lambda = D, \quad G_s = v_s v_s = cD, \quad (43)$$

$$E_s = m_s v_s^2 = mc^2, \quad Z_s = m_s v_s = mc, \quad M_s = m_s r_s v_s = m\lambda c.$$

Derived scales vary according to the scale bases and have only measurement meaning, no physical meanings. For example, divergence $D = c/\lambda$ is the frequency unit adopted by the system and does not represent the vibration frequency of the wave source. Another example, $E_s = mc^2$ is a unit of energy, not a mutual conversion between energy and mass. The operation of actual quantities facilitates physical analysis while avoiding tedious numerical calculations.

4.4. Velocity and Gradient

The velocity of the steady state (Eq. 37) can be further analyzed by applying the rules of actual quantities as follows

$$v_i = r\omega_i D = (\tilde{r}\omega_i) \cdot (\lambda D) = \tilde{v}_i \cdot c, \quad \tilde{v}_i = \tilde{r}\omega_i \quad (i = 1,2). \quad (44)$$

$$\tilde{v} = \pm \sqrt{\tilde{v}_1^2 + \tilde{v}_2^2} = \tilde{r}\Omega, \quad \Omega = \pm \sqrt{\omega_1^2 + \omega_2^2}.$$

where Ω is the digit of the rotational angular frequency. Taking the direction of kernel spin as a reference, $\{\tilde{v} > 0, \Omega > 0\}$ is specified as a right-handed state and $\{\tilde{v} < 0, \Omega < 0\}$ as a left-handed state.

The gradient can be calculated by the first two equations of (36)

$$G_1 = Dv_1 + C_3 v_2 = \tilde{v}_1 \cdot (cD) + \tilde{v}_2 \omega_3 \cdot (cD) = (\tilde{v}_1 + \tilde{v}_2 \omega_3) \cdot G_s,$$

$$G_2 = Dv_2 - C_3 v_1 = \tilde{v}_2 \cdot (cD) - \tilde{v}_1 \omega_3 \cdot (cD) = (\tilde{v}_2 - \tilde{v}_1 \omega_3) \cdot G_s.$$

Therefore

$$\tilde{G}_1 = \tilde{v}_1 + \tilde{v}_2 \omega_3, \quad \tilde{G}_2 = \tilde{v}_2 - \tilde{v}_1 \omega_3; \quad (45)$$

$$\tilde{G} = \sqrt{\tilde{G}_1^2 + \tilde{G}_2^2} = \sqrt{(\tilde{v}_1^2 + \tilde{v}_2^2)(1 + \omega_3^2)} = \tilde{r}|\Omega| \sqrt{1 + \omega_3^2}.$$

Substituting the gradient into the equation (41) yields another constraint for steady solution

$$\tilde{G}_1 \omega_2 - \tilde{G}_2 \omega_1 = \tilde{r}\Omega^2 \omega_3 \neq 0 \quad \Rightarrow \quad \omega_3 \neq 0. \quad (46)$$

4.5. Energy and Power

The motion energy of the particle is

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\tilde{v}^2 \cdot (mc^2) = \frac{1}{2}\tilde{v}^2 \cdot E_s, \quad \tilde{E} = \frac{1}{2}\tilde{v}^2 = \frac{1}{2}(\tilde{r}\Omega)^2. \quad (47)$$

Omitting the derivation process, the power is given as follows

$$P_G = m\mathbf{v} \cdot \mathbf{G} = -\tilde{v}^2 \cdot P_s,$$

$$P_C = m\mathbf{v} \cdot (\mathbf{v} \times \mathbf{C}) \equiv 0. \quad (48)$$

$$P_D = m\mathbf{v} \cdot (\mathbf{v}D) = \tilde{v}^2 \cdot P_s,$$

This verifies that the total power of the steady system is zero. $P_c = 0$ indicates that the curl energy is not dissipated, and $P_g = -P_d$ indicates that the gradient energy and the divergence energy compensate for each other, so the total system energy is conserved.

4.6. Momentum and Torque

The angular momentum of the particle is given by

$$\mathbf{M} = m\mathbf{r} \times \mathbf{v} = m(x_1v_2 - x_2v_1)\mathbf{k} = M_3\mathbf{k} \quad (49)$$

$$M_1 = 0, \quad M_2 = 0, \quad M_3 = M = (\tilde{x}_1\tilde{v}_2 - \tilde{x}_2\tilde{v}_1) \cdot M_s.$$

Omitting the steps of derivation, we give the force torque as follows

$$\mathbf{T}_G = m\mathbf{r} \times \mathbf{G} = [(\tilde{x}_2\tilde{v}_1 - \tilde{x}_1\tilde{v}_2) + (\tilde{x}_1\tilde{v}_1 + \tilde{x}_2\tilde{v}_2)\omega_3]\mathbf{k} \cdot E_s$$

$$\mathbf{T}_C = m\mathbf{r} \times (\mathbf{v} \times \mathbf{C}) = -(\tilde{x}_1\tilde{v}_1 + \tilde{x}_2\tilde{v}_2)\omega_3\mathbf{k} \cdot E_s \quad (50)$$

$$\mathbf{T}_D = m\mathbf{r} \times (\mathbf{v}D) = -(\tilde{x}_2\tilde{v}_1 - \tilde{x}_1\tilde{v}_2)\mathbf{k} \cdot E_s$$

It can be seen that the total torque of the steady system is zero.

4.7. Orbital Geometry

In Figure 1, the intersection of the ellipse with the minor axis is marked as $B(B')$, and that of the ellipse with the X_2 axis is marked as $P(P')$. The geometric parameters of the ellipse include the semimajor axis a , the semiminor axis b , the semifocal distance $d = \lambda$, and the semilatus rectum $p = b^2/a$. The orbital parameters can be determined using the conservation theorem of the angular momentum.

At the point $B(B')$, the position vector is $\mathbf{r}_B = (d, \pm b)$, and the velocity vector is $\mathbf{v}_B = (\mp v_{1B}, 0)$. According to equation (49), the angular momentum at $B(B')$ can be calculated as

$$M = mbv_{1B} = (\tilde{b}\tilde{v}_{1B}) \cdot (m_s r_s v_s), \quad \tilde{M} = \tilde{b}\tilde{v}_{1B}. \quad (51)$$

At the point $P(P')$, the position vector is $\mathbf{r}_P = (0, \pm p)$, and the velocity vector is $\mathbf{v}_P = (\mp v_{1P}, -v_{2P})$, and the angular momentum is equal to

$$M = mpv_{1P} = (\tilde{p}\tilde{v}_{1P}) \cdot (m_s r_s v_s), \quad \tilde{M} = \tilde{p}\tilde{v}_{1P}. \quad (52)$$

Introducing the elliptic angle γ to characterize the orbit, we have

$$\tan\gamma := \frac{\tilde{v}_{1B}}{\tilde{v}_{1P}} = \frac{p}{b} = \frac{b}{a}. \quad (53)$$

The semimajor axis a and the semifocal distance d can be expressed in terms of the elliptic angle γ as

$$\begin{aligned} a &= b \cot\gamma = (\tilde{b} \cot\gamma) \cdot r_s \\ d &= \sqrt{a^2 - b^2} = b\sqrt{\cot^2\gamma - 1} = \tilde{d} \cdot r_s, \end{aligned} \quad (54)$$

$$\tilde{d} = \tilde{b}\sqrt{\cot^2\gamma - 1}.$$

Using the semifocal distance as a scale for length, that is, letting

$$\tilde{d} = 1, \quad r_s = d = \lambda. \quad (55)$$

At this point, the shape of the ellipse is completely determined by γ

$$\tilde{b} = \frac{1}{\sqrt{\cot^2\gamma - 1}} = \frac{\tan\gamma}{\sqrt{1 - \tan^2\gamma}}, \quad (56a)$$

$$\tilde{p} = \tilde{b} \tan\gamma = \frac{\tan^2\gamma}{\sqrt{1 - \tan^2\gamma}}, \quad (56b)$$

$$\tilde{a} = \frac{\tilde{b}^2}{\tilde{p}} = \frac{1}{\sqrt{1 - \tan^2\gamma}}, \quad (56c)$$

$$e = \frac{d}{a} = \frac{1}{\tilde{a}} = \sqrt{1 - \tan^2\gamma}. \quad (56d)$$

In three-dimensional space, $r_s = d = \lambda$ represents a unit sphere, which is the uncertain range of the barycenter and is the limiting precision of the spatial length. Theoretically, the mass of the system is concentrated in this unit sphere. This sphere is the low limit of unmeasurable space, which can be called a “black hole”.

4.8. Eigen Parameters

Above we have proved that elliptical orbits are special solution of the equation of steady state. The steady states can be expressed in terms of the digital curl ω . The general form of the digital curl with the orbital coordinates is given by

$$\omega = \omega_1 \mathbf{i} + \omega_2 \mathbf{j} + \omega_3 \mathbf{k} = \omega(r)(\sin\theta\cos\varphi \mathbf{i} + \sin\theta\sin\varphi \mathbf{j} + \cos\theta \mathbf{k}) \quad (57)$$

where $\omega(r)$ is a function of r and (θ, φ) are the polar and azimuthal angles of the curl vector, respectively. We need replace $\omega(r)$ and (θ, φ) with orbital invariants.

First express (θ, φ) in terms of (β, γ) by a coordinate transformation. The kernel coordinate system $[OX'_1 X'_2 X'_3]$ is rotated counterclockwise by an angle of β about the X'_2 axis, so that X'_3 coincides with X_3 , and the equatorial plane π_e coincides with the orbital plane π_o , thus the coordinate system becomes $[OX'_1 X'_2 X_3]$. It is then rotated counterclockwise by an angle of γ around the X_3 axis so that X'_1 coincides with X_1 and X'_2 coincides with X_2 . After two rotations the basis vectors change from $(\mathbf{i}', \mathbf{j}', \mathbf{k}')$ to $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ with the transformation relation

$$(\mathbf{i}, \mathbf{j}, \mathbf{k}) = (\mathbf{i}', \mathbf{j}', \mathbf{k}') \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (58)$$

The transformation of the curl vector from $[OX'_1 X'_2 X'_3]$ to $[OX_1 X_2 X_3]$ is

$$\begin{aligned} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} &= \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \omega'_1 \\ \omega'_2 \\ \omega'_3 \end{pmatrix} \\ &= \begin{pmatrix} (\omega'_1 \cos\beta + \omega'_3 \sin\beta) \cos\gamma - \omega'_2 \sin\gamma \\ (\omega'_1 \cos\beta + \omega'_3 \sin\beta) \sin\gamma + \omega'_2 \cos\gamma \\ -\omega'_1 \sin\beta + \omega'_3 \cos\beta \end{pmatrix}. \end{aligned} \quad (59)$$

On the equatorial plane, $\theta' = \pi/2$, we find $\omega' = (0, 0, -\tilde{B}')$ according to equation (31). Substituting this into the above equation gives the curl expressed in terms of (β, γ)

$$\boldsymbol{\omega} = \omega(r)(\sin\beta\cos\gamma\mathbf{i} + \sin\beta\sin\gamma\mathbf{j} + \cos\beta\mathbf{k}), \quad \omega(r) = -\dot{B}(r). \quad (60)$$

Since the angular frequency Ω of the particle is an orbital invariant

$$\Omega = \pm\sqrt{\omega_1^2 + \omega_2^2} = \omega(r)\sin\beta, \quad (61)$$

The curl can finally be expressed in terms of invariants as

$$\boldsymbol{\omega} = \Omega(\cos\gamma\mathbf{i} + \sin\gamma\mathbf{j} + \cot\beta\mathbf{k}) \quad (62)$$

The above equation is a steady solution expressed in terms of digitized curl, and $\{\Omega, \beta, \gamma\}$ are the orbital frequency, orbital inclination, and elliptic angle, respectively, which are called orbital eigen parameters.

The velocity, curl, gradient, and energy of the particle can be calculated according to the following equations

$$\tilde{v} = \tilde{r}\Omega, \quad \Omega = \pm\sqrt{\omega_1^2 + \omega_2^2} \quad (63a)$$

$$\omega = \pm\sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2} = \Omega\sqrt{1 + \cot^2\beta} \quad (63b)$$

$$\tilde{G} = \tilde{r}|\Omega|\sqrt{1 + \omega_3^2} = \tilde{r}|\Omega|\sqrt{1 + (\Omega\cot\beta)^2} \quad (63c)$$

$$\tilde{E} = \frac{1}{2}\tilde{v}^2 = \frac{1}{2}(\tilde{r}\Omega)^2 \quad (63d)$$

4.9. Lunar Orbit

The Moon moves periodically in the Earth's action field, and the Moon and Earth form a dynamic equilibrium system. Table 1 lists the data of lunar orbit quoted from Wikipedia.

Moon's mass	$m = 7.346 \times 10^{22}$ kg
Orbital period (sidereal)	$\tau = 27.321661$ day = 2354146 s
Semi-major axis	$a = 384399$ km
Eccentricity	$e = 0.0549$
Inclination	$\beta = 23.44^\circ + 5.145^\circ = 0.498902$ rad

Table 1: Characteristics of the Lunar Orbit

We select the scale bases of the Moon-Earth system as

$$m_s = m = 7.346 \times 10^{22} \text{ kg}, \quad v_s = c = 299792458 \text{ m/s}, \quad (64)$$

$$r_s = \lambda = ae = 2.11035 \times 10^7 \text{ m};$$

with the derived scales

$$\begin{aligned} v_s &= D = c/\lambda = 14.2058 \text{ Hz}, \quad G_s = cD = 4.2588 \times 10^9 \text{ m/s}^2, \\ E_s &= mc^2 = 6.60226 \times 10^{39} \text{ J}. \end{aligned} \quad (65)$$

The angular frequency and radius of the moon's orbit are

$$\Omega = \frac{2\pi}{\tau D} = 1.8788 \times 10^{-7}, \quad \tilde{r} = \tilde{a} = \frac{1}{e} = \frac{1}{0.0549} = 18.2149. \quad (66)$$

The other orbital parameters are calculated as follows

$$\begin{aligned} \omega &= \Omega \sqrt{1 + \cot^2 \beta} = 3.92675 \times 10^{-7} \\ \tilde{G} &= \tilde{r} \Omega \sqrt{1 + (\Omega \cot \beta)^2} = 3.42222 \times 10^{-6}, \quad G = \tilde{G} \cdot G_s = 14574.5 \text{ m/s}^2. \\ \tilde{E} &= \frac{1}{2} (\tilde{r} \Omega)^2 = 5.8558 \times 10^{-12}, \quad E = \tilde{E} \cdot E_s = 3.86615 \times 10^{28} \text{ J}. \end{aligned} \quad (67)$$

The lunar orbit is quantized. If the energy of external disturbances reaches E_s , the lunar orbit will undergo a discontinuous leap. This conclusion likewise supports the prediction that the Earth's orbit may undergo a catastrophic jump [15,16].

5. Multi-Particle Systems

5.1. Orbital Energy

For multi-particle systems, we adopt a $[m\lambda D]$ scale system. Its scale bases and derived scales are

$$\begin{aligned} m_s &= m, \quad r_s = \lambda, \quad v_s = D; \\ v_s &= \lambda D, \quad Z_s = m\lambda D, \quad E_s = m(\lambda D)^2. \end{aligned} \quad (68)$$

In this system, the scale base D is the angular frequency of particle rotation, and the derived scale $v_s = \lambda D$ is no longer the constant of wave speed.

The curlity of the particle is calculated according to equation (20)

$$\begin{aligned} \mathbf{Z} &= m\mathbf{r} \times \mathbf{C} = (\tilde{Z}_1 \mathbf{i} + \tilde{Z}_2 \mathbf{j} + \tilde{Z}_3 \mathbf{k}) \cdot (m\lambda D) = (n_1 \mathbf{i} - n_2 \mathbf{j} + n_3 \mathbf{k}) \cdot Z_s; \\ \tilde{Z}_1 &= n_1 = \tilde{x}_2 \omega_3, \quad \tilde{Z}_2 = -n_2 = -\tilde{x}_1 \omega_3, \quad \tilde{Z}_3 = n_3 = \tilde{x}_1 \omega_2 - \tilde{x}_2 \omega_1. \end{aligned} \quad (69)$$

Conservation of curlity requires that $\{n_1, n_2, n_3\}$ are invariants.

The length of the curlity is

$$Z = \sqrt{Z_1^2 + Z_2^2 + Z_3^2} = \tilde{Z} \cdot Z_s, \quad \tilde{Z} = \sqrt{n_1^2 + n_2^2 + n_3^2}. \quad (70)$$

According to the closure condition (Eq. 23), we obtain the equation for the orbital energy

$$\begin{aligned} E &= \frac{Z^2}{2m} = \frac{1}{2} (n_1^2 + n_2^2 + n_3^2) \cdot E_s = E_1 + E_2 + E_3, \\ \tilde{E}_1 &= \frac{1}{2} n_1^2, \quad \tilde{E}_2 = \frac{1}{2} n_2^2, \quad \tilde{E}_3 = \frac{1}{2} n_3^2. \end{aligned} \quad (71)$$

The above formula is called the orbital energy spectrum. E_1, E_2, E_3 are called the circulation energy, the pulsation energy, and the nutation energy, respectively, according to the following model.

5.2. Orbit Modeling

The interaction of multiple particles leads to orbital deformation. A steady planar orbit can be considered as a synthesis of rounded circulation and radial pulsation, and is expressed by the parametric equation

$$r(\tau) = a + \lambda \sin(n_2\tau) \quad (n_2 = 1, 2, 3, \dots). \quad (72)$$

where $\tau = tD = \tilde{t}$ is the digital time and n_2 is the times of pulsation within one revolution. The first term at the right end of the equation represents the circulation, and a is the radius of circulation. The second term represents the radial pulsation, and λ is the amplitude of pulsation. The synthesis of circulation and pulsation is a petal-shaped orbit, and the ellipse is a special case of the petal orbits with $n_2 = 1$ [15,16].

The digital form of the equation of the petal orbits is

$$\tilde{r}(\tau) = n_1 + \sin(n_2\tau), \quad (n_1 = \tilde{a} = 2, 3, 4, \dots; \quad n_2 = 1, 2, 3, \dots). \quad (73)$$

The n_1 is called the radius number, and n_2 the petal number. It has $n_1 \geq n_2$ since the radius of circulation is greater than the amplitude of pulsation. The circulation energy E_1 and the pulsation energy E_2 of the petal orbits are

$$E_1 = \frac{1}{2} m(aD)^2 = \frac{1}{2} \tilde{a}^2 \cdot (m\lambda^2 D^2) = \frac{1}{2} n_1^2 \cdot E_s, \quad (74)$$

$$E_2 = \frac{1}{2} m(n_2\lambda D)^2 = \frac{1}{2} n_2^2 \cdot (m\lambda^2 D^2) = \frac{1}{2} n_2^2 \cdot E_s.$$

It indicates that the circulation energy is greater than the pulsation energy when the radius number is greater than the petal number.

Figure 2 shows four petal orbits with $n_1 = 6$, $n_2 = 1, 2, 3, 4$. The dashed line in the figure is the circle of $\{n_1 = 6, n_2 = 0\}$, and the solid line is the petal orbit relative to that circle.

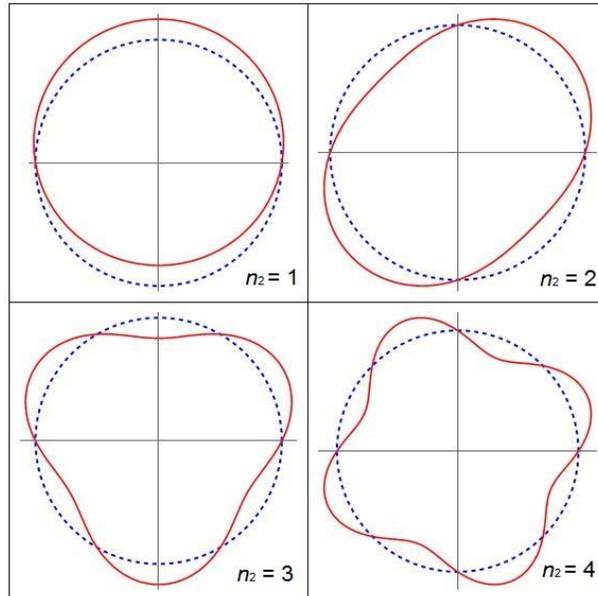


Figure 2: Examples of Planar Orbits. $n_1=6$; $n_2=1,2,3,4$

5.3. Spatial Orbits

The orbits of a multi-particle system are not limited to the plane. The trajectory of each particle has not only radial pulsation in the orbital plane, but also normal nutation perpendicular to the orbital plane. The spatial orbits of the particles can be expressed by a set of parametric equations

$$\begin{cases} X(\tau) = [n_1 + \sin(n_2\tau)]\cos(\tau + \pi n_3/n_1) \\ Y(\tau) = [n_1 + \sin(n_2\tau)]\sin(\tau + \pi n_3/n_1) \\ Z(\tau) = n_3 \sin(\tau + \pi n_2/n_1) \end{cases} \quad (75)$$

(n_1, n_2, n_3) are collectively referred to as orbital numbers, which represent the radius of circulation, the frequency of radial pulsation, and the amplitude of normal nutation, respectively. Spatial orbits satisfy the conditions of periodicity and uniqueness when the orbital numbers are taken as integers. The periodicity condition requires that the orbits must be closed for one cycle of circulation (2π radians), which is a fundamental characteristic of the steady state. The uniqueness condition requires that each set of orbital numbers corresponds to only one orbit, which is the essence of the Pauli exclusion principle.

According to equations (49), (69) and (71), we can find a relationship between angular momentum and nutation energy

$$\tilde{M} = \tilde{x}_1 \tilde{v}_2 - \tilde{x}_2 \tilde{v}_1 = \tilde{r}(\tilde{x}_1 \omega_2 - \tilde{x}_2 \omega_1) = \tilde{r} n_3 = \pm \tilde{r} \sqrt{2\tilde{E}_3}. \quad (76)$$

Therefore, $n_3 > 0$ and $n_3 < 0$ represent the right-handed and left-handed angular momentum, respectively. Because of the conservation of angular momentum, the nutation frequency must equal the circulation frequency, so that the nutation amplitude n_3 represents an invariable orbital inclination β

$$\tan\beta = \frac{n_3}{n_1} \quad \left(-\frac{\pi}{2} < \beta < \frac{\pi}{2}\right). \quad (77)$$

We refer to β as the slant angle and n_3 as the slant number. The smaller the n_3 the lower the nutation energy, so that low-energy orbits are concentrated near the equatorial plane of the acting kernel. Planetary orbits in our solar system approach a plane, and that plane is the equatorial plane of the solar spin.

The symbol $O(n_1, n_2, n_3)$ is used to denote the spatial orbits, and Figure 3 shows the six orbits $O(6,3,n_3)$ with $n_3 = \pm 1, \pm 2, \pm 3$.

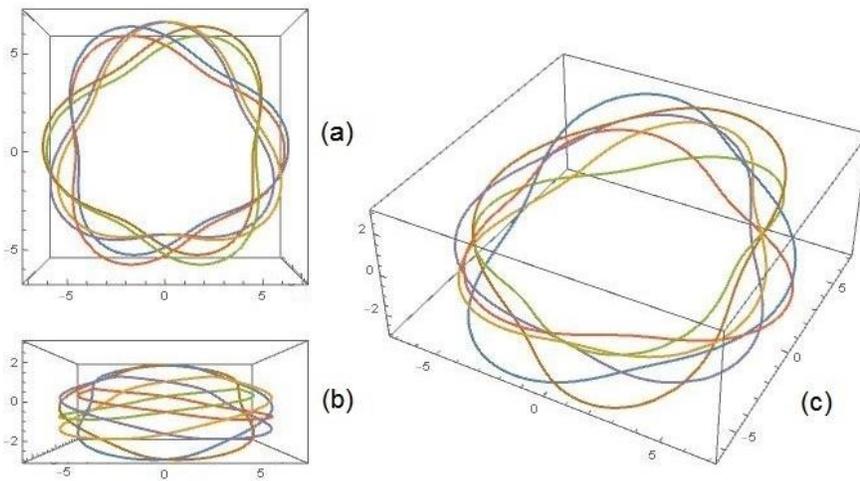


Figure 3: Six Spatial Orbits $O(6,3,n_3)$ with $n_3 = \pm 1, \pm 2, \pm 3$
(a) top view, (b) side view, (c) panoramic view

5.4. Electron Shells

An atom is a multi-particle system with electrons rotating around a nucleus. The space of electron orbits with the same radius number n_1 is called an electron shell. The petal number allowed in each shell is $n_2 \leq n_1 - 1$, and the slant number allowed is $|n_3| \leq n_2$. Therefore, the digits allowed for the orbital number is

$$n_1 = 2, 3, 4, \dots; \quad n_2 = 1, 2, 3, \dots, (n_1 - 1); \quad n_3 = \pm 1, \pm 2, \pm 3, \dots, \pm n_2. \quad (78)$$

For a definite n_2 , there are $2n_2 - 1$ right-handed orbits, plus left-handed orbits, the maximum number of allowable orbits in the n_1 -the shell is

$$N(n_1) = 2 \sum_{n_2=1}^{n_1-1} (2n_2 - 1) = 2(n_1 - 1)^2. \quad (79)$$

In atomic physics, the quantum numbers of electron are $n = n_1 - 1, l = n_2 - 1, m = \pm l$ [17]. n, l, m are called the principal quantum number, the angular quantum number and the magnetic quantum number, respectively. The $n = 1, 2, 3, 4, \dots$ shells are labeled with K, L, M, N, \dots . The same shell is divided into n subshells with $l = 0, 1, 2, 3, \dots, (n - 1)$ and labeled with s, p, d, f, \dots , respectively. The magnetic quantum number m of the l -th subshell has $2l + 1$ values, so the n -th shell possesses n^2 energy levels. Assuming that the electron has the spin quantum number $m_s = \pm 1/2$, quantum mechanics extends the number of electrons that the n -th shell can accommodate to $2n^2$. This result is consistent with the conclusions of formula (79). The difference is that the electronic states in our theory need only three orbital numbers to describe them, and also excludes the circular orbit of $n_2 = 0$ and the equatorial plane orbit of $n_3 = 0$.

In the atomic structure theory, the phenomenon in which the energy of the electrons in the higher shells is less than the energy of the electrons in the lower shells is called the energy staggering. For example, the energy levels of the $4s$ and $3d$ orbits are inverted. The modern theory explains the energy level staggering phenomenon as the effect of orbital penetration. According to the orbital energy spectrum (Eq. 71), the electron energy within a subshell is jointly determined by n_2 and n_3 . For example, the highest energy level of $3d$ orbit is $\tilde{E}(4,3,3)=34/2$ and the next highest is $\tilde{E}(4,3,2)=29/2$, while the actual energy level of $4s$ orbit is $\tilde{E}(5,1,1)=27/2$, which shows that there is an inversion of the energy levels of the $3d$ and $4s$ orbits. However, the highest energy level of the $2p$ orbit is $\tilde{E}(3,2,2)=17/2$ and the energy level of the $3s$ orbit is $\tilde{E}(4,1,1)=18/2$, so there is no staggering of the energy levels of the $2p$ and $3s$ orbits.

5.5. Hydrogen Spectrum

Optical spectrum is the main basis for the study of atomic structure. The hydrogen atom is a dynamic system consisting of one proton and one electron and has the simplest spectral structure. The spectra can be studied using the $[mah]$ scale system, whose scale bases are

$$m_s = m_e, \quad r_s = a, \quad M_s = h. \quad (80)$$

Where $m_e = 9.10938371 \times 10^{-31}$ kg is the electron mass and $h = 6.62607015 \times 10^{-34}$ J·Hz⁻¹ is the Planck constant, and a is the electron orbital radius. Then we have derived scales

$$v_s = \frac{M_s}{m_s r_s} = \frac{h}{m_e a}, \quad v_s = \frac{v_s}{r_s} = \frac{h}{m_e a^2} = D, \quad E_s = m_s v_s^2 = \frac{h^2}{m_e a^2} = hD. \quad (81)$$

It can be seen that the energy scale E_s varies with orbital radius a or divergence D . Let $a_k = ka_1$, thus

$$E_s(k) = hD_k = \frac{h^2}{m_e a_k^2} = \frac{h^2}{m_e (ka_1)^2} = \frac{E_s(1)}{k^2}, \quad (82)$$

$$E_s(1) = \frac{h^2}{m_e a_1^2} \quad (k = 1, 2, 3, \dots).$$

Where a_1 is the minimum orbital radius and $E_s(1)$ is the lowest orbital energy corresponding to the minimum radius a_1 .

The energy difference between the electron's n -th orbit and the m -th orbit ($n > m$) is

$$\Delta E_{nm} = E_s(m) - E_s(n) = h(D_m - D_n) = hD_{nm}. \quad (83)$$

If ΔE_{nm} is released as a light wave, the corresponding divergence difference D_{nm} is the frequency of the emitted light

$$D_{nm} = \frac{\Delta E_{nm}}{h} = \frac{E_s(1)}{h} \left(\frac{1}{m^2} - \frac{1}{n^2} \right), \quad (1 \leq m < n) \quad (84)$$

In the spectral theory of the hydrogen atom, D_{n1} is the Lyman series, D_{n2} the Balmain series, D_{n3} the Paschen series, and so on.

Comparison with the spectral terms of the hydrogen atom shows that the Rydberg constant is

$$R = \frac{E_s(1)}{hc} = \frac{h}{m_e c a_1^2}. \quad (85)$$

It is known that $R = 10973731.6 \text{ m}^{-1}$, from which the minimum orbital radius of the electron in the hydrogen atom is calculated

$$a_1 = \sqrt{\frac{h}{m_e c R}} = 4.40214479 \times 10^{-10} \text{ m}. \quad (86)$$

Based on this result, Planck's constant h can be interpreted as the angular momentum of the electron's smallest orbit.

5.6. Orbit Chirality

The chirality of the orbital angular momentum is an important feature of the barycenter frame theory. Outside an acting kernel, the mass of kernel generates a gradient field, the spin of kernel generates a curl field, and the relative motion of the kernel and the particle generates a divergence field. The kernel spin provides not only the centrifugal force for the motion of particle around the kernel, but also a spatial reference direction, which is the fundamental cause of the orbital chirality. Chirality allows the orbital angular momentum \vec{M} (or the slant number n_3) to take negative integers. Thus, particles can rotate counterclockwise or clockwise in the same action field. Having considered the kernel spin, it is no longer necessary to assume that the particle has the spin property.

Quantum mechanics assumes that the electron has an intrinsic spin and that the total angular momentum is the vector sum of the orbital angular momentum and the spin angular momentum. The state of the electron is described by four quantum numbers (n, l, m, m_s) , where n represents the orbital radius, l the orbital angular momentum, m the spatial orientation of the orbital angular momentum, and $m_s = \pm 1/2$ the angular momentum of the electronic spin. The barycenter frame theory replaces electron spin with kernel spin and describes the state of the electron with only three orbital numbers (n_1, n_2, n_3) . The orbital numbers are independent of each other and equal to exactly the three degrees of freedom of the orbital motion, with n_3 being both positive and negative to express the chiral nature of the angular momentum.

There are three types of degeneracy of the orbital energy levels in the barycenter system. The first category is the chiral degeneracy due to the antisymmetry of n_3 , e.g., $\tilde{E}(2,1,1) = \tilde{E}(2,1,-1) = 6/2$. The second category is the symmetric degeneracy resulting from exchanging the positions of n_2 and n_3 , e.g., $\tilde{E}(6,1,5) = \tilde{E}(6,5,1) = 62/2$. The last category is the asymmetric degeneracy with different radius number of n_1 , e.g., $\tilde{E}(6,1,5) = \tilde{E}(7,2,3) = 62/2$. The probability of chiral degeneracy is the largest, the that of symmetric degeneracy is the second largest, and the that of asymmetric degeneracy is the smallest. In the presence of an external field, the energy levels of chiral degeneracy are most likely to split, thus leading to the fine structure of the spectrum.

Particles of different masses have similar energy level structures in the same kernel field, but their actual energy levels do not coincide due to different energy scales. Only particles of the same mass can occupy both orbits of the chiral degeneracy energy levels. Electrons are same-mass particles, so electrons in an atom can be in both right-handed and left-handed orbits. The reason that a pair of counter-rotating planets has not yet been found in nature is that the probability of the planets of the same mass and energy occurring in the same stellar field is negligible.

6. Conclusion

The particle dynamics theory based on the barycenter frame of reference establishes the dynamic equations from the complete flow field forces. The equations contain both attractive and repulsive interactions: the attractive force comes from the gradient field and the repulsive force comes from the coupling of the particle motion to the curl field. The motion of the particles follows Newton's second law, the energy theorem, the curlity theorem and the angular momentum theorem. Energy, curlity and angular momentum are conserved in dynamically balanced systems, and their variations satisfy generalized quantization rules. Due to the introduction of the concept of scale, the theory of barycenter reference frame is applicable to objects of any size, thus unifying the foundations of Newtonian mechanics, electrodynamics, relativistic mechanics, and quantum mechanics within a framework of the hyper-classical or post-modern physics.

By solving the dynamic equilibrium equations, it is proved that the elliptic orbit is the special solution to the equation of steady state. According to the particle orbital model, the spatial orbits of the multi-particle system are petal-shaped and contains three degrees of freedom: rounded circulation, radial pulsation, and normal nutation. The state of the particle system is characterized by the curlity, and the orbital energy is determined by the square of the curlity. It is an urgent and challenging task to study the structure of different

materials by applying the particle dynamics theory. The consistency and completeness of the theory of the barycenter reference frames show that the flow-field force is a unified form of particle interaction, that the measurement principle is the first principle that integrates the foundations of relativity and quantum theory, and that the theorems of particle dynamics are universal laws for both macroscopic and microscopic worlds.

References

1. Jin, S. N., Ma, Y. L. (2002). *Theoretical Mechanics*. Higher Education Press.
2. Einstein, A. (2025). *The meaning of relativity*. Princeton, NJ: Princeton University Press.
3. Qian, B. C. (2006). Quantum Mechanics. *Higher Education Press*, 5:191.
4. Liang, Z. C. (2020). Outline of real physics. *Glob. J. Sci. Front. Res. A*, 20(3), 9-27.
5. Liang, Z. C. (2019, November). Modeling of real particles. In *Journal of Physics: Conference Series* (Vol. 1391, No. 1, p. 012026). IOP Publishing.
6. Liang, Z. C. (2019). The origin of gravitation and electromagnetism. *Theoretical Physics*, 4(2), 85-102.
7. Liang, Z. C. (2019). Motion, energy, and state of body particle system. *Theoretical Physics*, 4(2), 67.
8. Liang, Z. C. (2019). Cluster ensemble statistics of body particle system. *New Horiz. Math. Phys*, 3, 53.
9. Liang, Z. C. (2021). Dark matter and real-particle field theory. *Glob. J. Sci. Front. Res. A*, 21(6), 27-39.
10. Liang, Z. C. (2024). A fluid field theory based on barycenter reference systems.
11. Liang, Z. C. (2024). Physical theories based on the center-of-mass reference frame: 1. Particle flow field principle.
12. Kuhn, T. S. (1970). *The Structure of Scientific Revolutions*, 2nd enl. ed. University of Chicago Press.
13. Liang, Z. (2015). *Physical principles of finite particle system*. Scientific Research Publishing, Inc. USA.
14. Zanke, M. Rotating Charged Sphere - Problem with Suggested Solution.
15. Liang, Z. C., & Xie, L. H. (2023). Wave or particle? Quantum behavior of solar system. *Glob. J. Sci. Front. Res*, 23(9).
16. Liang, Z. C., & Chen, N. N. (2023). The distortion and quantization of the Earth's orbit. *J. Phys. Chem. Biophys*, 13, 361.
17. Zhu, S. L. (1979). *Atomic Physics*. Higher Education Press.

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