ISSN: 2834-4928

Review Article

Journal of Electrical Electronics Engineering

A Lowest Boundary for the Density of Twin Primes Maintains Their Cardinality

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Submitted: 2026, Aug 03; **Accepted:** 2026, Sep 15; **Published:** Under Process

Citation: Chauvet, J. (2026). A Lowest Boundary for the Density of Twin Primes Maintains Their Cardinality. *J Electrical Electron Eng, 5*(5), 01-04.

Abstract

We address here the TPC (Twin Primes Conjecture). Considering the unique growing list of twin primes as elements of a dynamical system with invariant transformation law onN, we infer a lower boundary for their density along the real axis incompatible with their finiteness. Although non constructive, since we do not prove a general formula for finding twin primes pairs arbitrarily great, this proof relies for closure on the tertium non datur principle with regard to their minimal density, and thus their cardinality at infinity.

Proof

Proving the TPC is equivalent to proving that the set of twin primes pairs cannot be finite. The hypothetical existence of such a finite set suggests that there exists a greatest element in the list, possibly equal to the greatest one calculated algorithmically at the date of reading this, or any greater otherwise [1].

We adress this issue by assuming the existence of a density function R: $t \to \delta(t)$ such that the number of prime couples$(p, p\text{-}2)$between two real numbers a and b is given by the following integral :

$$\int_a^b \delta(t)\,dt$$

If we assume p_L to be the greatest prime such that p_L-2 is also prime, we then must also assume that we should have for any $p_L < \epsilon$:

$$\int_\epsilon^\infty \delta(t)\,dt = 0$$

More precisely, given that the density function has to be strictly positive $(0 \le \delta(t) \le 1)$, the latter should hold for any two a and b such that $p_L < a < b \le \infty$, so we have :

$$(*) \quad \delta(t) = 0 \quad \text{with} \quad p_L < t \le \infty$$

In order to demonstrate the nature of the density δ, we consider the following recursive dynamical system based on the Eratosthene's algorithm :

$$p_{k+1} = \min\left(\overline{p_0\mathbb{N} \cup p_1\mathbb{N} \cup \ldots \cup p_k\mathbb{N}} \setminus \{1\}\right)$$

In the latter, p_k is the $(k+1)^{th}$ greatest prime in the list of primes **IP**, and pN the sets containing every multiples of p. Hence, $p_{(k+1)}$ is the next prime in the list obtained by merging all prime multiples up to p_k, then inverting the set into the complementary in N, then removing

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the singleton 1 off of it, then selecting the next $p_{(k+1)}$ with the minimum-seeking function min( ) as the least number of it. Through infinite recursion, this algorithm straightforwardly yields the entire set of primes **IP**, which is know to be infinite by proof.

Using this constructive method, we then have to demonstrate that the following :

$$(**) \quad \frac{1}{2} \geq \delta(t) \geq \frac{1}{p_k\#} \quad \text{holds for any} \quad k \in \mathbb{N}$$
$$p_k \leq t < p_{k+1}$$

To prove that (*) and (**) are incompatible, and hence that the TPC is true.

Let $A_k = p_0\mathbb{N} \cup p_1\mathbb{N} \cup \ldots \cup p_k\mathbb{N}$ (with IP $=\{p_0, p_1, p_2, p_3, \ldots\} = \{2,3,5,7,\ldots\}$) be the set containing all primes and all their multiples up the k^{th} prime of the list of primes. We easily see that any natural number of the form $n\, p_k\# := n\, p_0 p_1 p_2 \ldots p_k$ with n any positive integer is in A_k. As a consequence, if we define the complementary set $\overline{A_k}$ such that :

$$\overline{A_k} \cup A_k = \mathbb{N} \quad \text{and} \quad \overline{A_k} \cap A_k = \varnothing$$

All couples of the form $(n\, p_k\# - 1,\ n\, p_k\# + 1)$ with n any positive integer are in turn automatically in $\overline{A_k}$. This defines the lower boundary $\delta(t) \geq \frac{1}{p_k\#}$ for $t \in [\, p_k, p_{k+1}[$.

The upper boundary for the density is directly deduced according to the same logic from the initial state $\overline{A_0}$ which in particular is the set of all odd numbers, i.e. of the form 2q+1with q some positive integer, and which has regular density 1/2. Since $p_0\# = 1 \times 2 = 2$, then :

$$\frac{1}{2} \geq \delta(t) \geq \frac{1}{2}$$
$$\text{for} \quad 2 \leq t < 3$$

Literally speaking, the latter means that right after $p_1 = 3$, all possible primes are both odd and own a twin, making $p_2 = 5$an automatic next choice. On this basis, the following holds naturally as well :

$$\frac{1}{2} \geq \delta(t) \geq \frac{1}{p_k\#}$$
$$\text{for} \quad 3 \leq t < p_{k+1}$$

Hence proving the TPC.

Remark 1
It is straightforward to demonstrate that :

$$\int_a^b \delta(t)\, dt = \sum_{n=\lfloor a \rfloor}^{\lfloor b \rfloor} \left((\pi(n) - \pi(n-1))(\pi(n-2) - \pi(n-3)) \right)$$

Where π is the prime counting function, that is to say [2], the function that is equal to the number of primes inferior or equal to its argument, and $\lfloor x \rfloor$ is the floor function, which is such that $\lfloor \ldots d_3 d_2 d_1 . d_{-1} d_{-2} d_{-3} \ldots \rfloor = \ldots d_3 d_2 d_1$ with $d_i = 0,1,2,\ldots,9$.

Indeed, since $\pi(p) = \pi(p\text{-}1) + 1$ for p is prime, but $\pi(\overline{p}) = \pi(\overline{p} - 1) + 0$ otherwise, the product above is always such that :

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	$P = \pi(n)-\pi(n\text{-}1)$	$Q = \pi(n\text{-}2) - \pi(n\text{-}3)$	PQ
n is prime, n-2 is not	1	0	0
n-2 is prime, n is not	0	1	0
Neither are primes	0	0	0
Both are primes	1	1	1

Whenever $n > 3$.

Remark 2

The density $\delta(t)$ for an affine progression of the form $u_n = \alpha n + \beta$ with $(\alpha,\beta) \in \mathbb{N} \times \mathbb{N}$ is $\frac{1}{\alpha}$ and we have :

$$N_{[a,b]} = \int_a^b \frac{1}{\alpha} dt = \frac{1}{\alpha}\int_a^b dt = \frac{b-a}{\alpha} \quad \text{with} \quad b > a$$

$N_{[a,b]}$ is a positive integer equal to C whenever $b\text{-}a = C\alpha$.

When the latter does not hold, the rest by division yields the remaining distance to the next element of u.

Remark 3

The special limit $\int_\epsilon^{+\infty} 0^+ dt = 0^+ \int_\epsilon^{+\infty} dt = 0^+(\infty - \epsilon) = +\infty$ means that a positive constant arbitrarily small taken as the density function δ shall yield an infinite amount of twin primes. Under this scope, the following is therefore true for all k:

$$\frac{1}{2} \geq \delta(t) \geq \frac{1}{p_k\,\#} > 0^+$$

Indeed, since the terms $\frac{1}{p_k\,\#}$ are all rational numbers with bijective relation toNaccording to k, hence forming a countable set of rationals, the latter holds for infinitely many instances of 0^+ taken as any real number (that is to say, from the uncountable set R) between 0 and $\lim_{k\to+\infty} \frac{1}{p_k\,\#}$ [3].

Although we do not know the pointwise variations of $\delta(t)$, we are allowed to consider those boundaries "tile-after-tile" of respective length $[p_k, p_{k+1}]$, so as to obtain :

$$\frac{1}{2}\int_{p_k}^{p_{k+1}} dt \geq \int_{p_k}^{p_{k+1}} \delta(t)\,dt \geq \frac{1}{p_k\,\#}\int_{p_k}^{p_{k+1}} dt > 0^+ \int_{p_k}^{p_{k+1}} dt$$

$$\Leftrightarrow \frac{p_{k+1}-p_k}{2} \geq \int_{p_k}^{p_{k+1}} \delta(t)\,dt \geq \frac{p_{k+1}-p_k}{p_k\,\#} > 0^+(p_{k+1}-p_k)$$

$$\Leftrightarrow \sum_{k=0}^{N} \frac{p_{k+1}-p_k}{2} \geq \sum_{k=0}^{N}\int_{p_k}^{p_{k+1}} \delta(t)\,dt \geq \sum_{k=0}^{N} \frac{p_{k+1}-p_k}{p_k\,\#} > \sum_{k=0}^{N} 0^+(p_{k+1}-p_k)$$

For N an arbitrarily great positive integer, the lowest boundary becomes :

$$\sum_{k=0}^{N} 0^+(p_{k+1}-p_k) = 0^+(p_{N+1}-p_N+p_N-\ldots-p_{k-1}+p_{k-1}-p_{k-2}+p_{k-2}-\ldots-2) = 0^+(p_{N+1}-2)$$

Which in turn yields the following inequality as proof of the TPC :

$$\int_2^{+\infty} \delta(t)\,dt > \int_2^{+\infty} 0^+\,dt$$

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References

1. https://en.wikipedia.org/wiki/Twin_Prime_Search.
2. https://en.wikipedia.org/wiki/Prime-counting_function.
3. https://en.wikipedia.org/wiki/Countable_set.

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