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When Intrinsic Randomness Could Come From the Finite/Infinite Transition

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Abstract

Quantum physics is non-causal, and randomness is so-called "intrinsic". We propose no less than an 18th interpretation of it through non-Archimedean geometry to bring back causality, respect of the Kolmogorov axioms and the existence of hidden variables. For these latter ones, we show that they cannot be in any Hilbert space and hence could not be detected in any traditional experiment. We end through proposing two experiments which would prove the non-Archimedean nature of our universe. The first one consists in a new disruptive type of quantum radar. The second one explains how viscosity naturally occurs in fluid mechanics whereas Boltzmann's approach only considers elastic shocks at the molecular scale.

Introduction

Quantum physics is non-causal [1]. The randomness at stake is said to be intrinsic [2]. And the big problem that this raises in the end is when treating entanglement: a subfield of quantum physics in which there was a dispute between, say, Einstein and de Broglie, on the one hand, and the Copenhagen School about the existence or not of hidden variables on the other. The orthodoxy has experimentally concluded that such hidden variables do not exist, but everyone knows the extensive use that quantum physics makes of mathematics and we could expect, at least, some kind of coherence between mathematics and physics. Unfortunately, this is not the case! The probabilities calculated in quantum physics do not follow the Kolmogorov axioms [2]. In the meanwhile, this is the math which allowed discovering the neutrino out of the Dirac equation. We therefore touch here to the deep foundations.

I do not say that quantum physics is contradictory, but what I deeply think is that the non-existence of hidden variables, so many times proved experimentally by quantum physicists, is kind of and I am just proposing here to surmount it through a new interpretation of quantum physics to make it disappear, without contradicting the orthodox experimental point of view, but for sure by contradicting its very philosophical point of view.

My approach is mathematical. In part 2, I shall explain that in the very axioms of quantum physics, the axiom of choice sits at the heart of them. In part 3, I shall introduce relativity; not relativity like in physics, but the relativity of scales and lengths. I shall draw from this a new model of universe, much more general than the one considered until now. I shall draw a new model of particles in part 4 while part 5 will be dedicated to (retrieved!) causality. I shall propose an application for a quantum radar not necessitating any echo in order to detect a target in part 6 while I shall elaborate on the viscosity in the Navier-Stokes equations as a result of the infinite transition in part 7. Part 8 will be dedicated to a philosophical discussion about the infinitesimal scale and its observability and I shall make in it a parallel with chemistry. It will then be easy to conclude in part 9 what the very goal of this paper is. Intrinsic randomness is only an impression. The underlying world is, to some extent, fully deterministic, like at our macroscopic level. And I shall unite at once the modern mathematical view of what randomness is and is known since 1983 together with the intrinsic randomness notion defended by the Copenhagen School [3].

The foundations of quantum physics

General considerations

I shall here just refer to [2]. The axiomatization of quantum physics requires 5 axioms which I describe very shortly.

- A Hilbert space is associated to any quantum system and its representation is made through a norm 1 vector in this space
- A Hermitian operator is associated to any physical magnitude and any measurement sums up to the projection onto the Eigenspace which gives as a result real numbers
- The Schrödinger equation for the evolution of the system (for non-relativistic systems)
- 2 or more quantum systems are represented through the tensor product of their Hilbert spaces
- Under such circumstances, if the representation vector cannot be decomposed into a tensor product of vectors, the particles are said to be entangled and display "weird" behavior

One thing, very important, is worth to be noted and this thing is well known from the mathematicians. Indeed, we all remember the famous 5 axioms of Euclidean geometry. But we also all know that in 1899, Hilbert, through a careful look at the foundations of geometry [4], proved that what Euclid considered has "common notions” had very often to be considered as axioms. And it is a fact that Euclidean geometry in general needs about 20 axioms.

Now, in the same way, quantum physics refers to Hilbert spaces. But such spaces are just some assumptions about the underlying geometry of the quantum world. And, as just said, we need about 20 axioms in order to build Hilbert Spaces, so that quantum physics is based on about 25 axioms.

This does not simplify things. Indeed, the basis of the mathematics, the Zermelo Fraenkel + Choice set of axioms, is already not known to be non-contradictory. Just imagine if instead of 9 axioms that you have 25. The risk of having contradictions dramatically increases! Moreover, any new axiom is a requirement for a specific property, bringing us always more away from the largest generality.

The axiom of choice

Let us look at quantum physics under a more mathematical light and for our very purpose, we are going to consider an interesting case which is overwhelmingly used. Let us consider the position operator which generally is noted |**r**›. This operator is known to have a continuous spectrum and in the general case, its spectrum is $\mathbb{R}$ itself. Now, if we make a measurement of the position of, say, a particle, we shall get a real result, say, $\mathbf{r_0} \in \mathbb{R}^3$, where $\mathbf{r_0}$ is an element of the spectrum of |**r**›. Let us raise the following question. What is the mathematical process at stake in the “choice” of $\mathbf{r_0}$? We used the term “choice” on purpose for the following reason. Let us refer to the axiom of choice in mathematics which says that for any non-empty set X there exists a “choice” function f such that.

$$f : 2^X - \varnothing \to X \text{ and } \forall x, f(x) \in x$$

Now, the reader will consider as trivial that what quantum mathematicians call the axiom of choice, not only is used in quantum physics, but is even a physical process in the physical world: *any measurement of the position of a particle in quantum physics consists in applying a choice function to the set of possibilities for this position and this choice function occurs in a set of cardinal* $\aleph_2$ *which implies that we need more than the axiom of dependent choice (which, by the way, would be enough to ensure a basis in any Hilbert space).*

So, quantum physicists use the axiom of choice with its full power and not any weaker version of it. Here, of course, the writing of the choice function would rather be

$$f : 2^{\mathbb{R}} - \varnothing \to \mathbb{R} \text{ with } \forall x, f(x) \in x$$

For the position operator. Now, the very fact that the departure set of f has cardinality $\aleph_2$, makes it out of reach of any Hilbert space whose cardinal is obviously $\aleph_1$. We are therefore comforted in the thought that this mysterious choice function in math or measurement function in quantum physics might have hidden variables, but that these variables are outside any Hilbert space and as such, could not be found by the orthodox theory which is, de facto, limited in its scope.

We shall conclude here that there is room for hidden variables which are out of the scope of traditional quantum physics and this potentially brings to weird properties in the theoretical approach of this discipline, bringing to such concepts as the abandonment of local realism which does not fit at all with any intuition, and being almost incompatible with relativity theory. So, in the following of this text, we shall take for granted that quantum physics is based on the axiom of choice and will draw some consequences of it.

Looking for the right geometry

We have just introduced in physics a set of cardinal $\aleph_2$. The fact is that whatever the sub discipline you tackle in physics today, you always deal with fields which are either, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ or $\mathbb{O}$ which are respectively the reals, the complex numbers, the quaternions and the octonions. All these fields have cardinal $\aleph_1$. One way out of this is to dive into what is called non-standard analysis, which consists in considering $\mathbb{R}^K = \mathbb{R}^{\mathbb{N}} / K$ where K is a non-trivial ultrafilter whose existence can only be guaranteed by the axiom of choice [5]. The problem with such a treatment is that the representation is not unique and heavily depends on K [5]. Therefore, let us propose an alternative approach. For this, Goblot’s approach is quite inspiring [5]. Indeed, he proposes a parallel approach through numbers and geometry. Let us therefore follow here the same kind of path and let us begin with geometry, intrinsic geometry should we say.

Can we introduce, through intrinsic geometry, a set of cardinal $\aleph_2$? There are obviously several ways to do this, but one is more obvious than the others. Indeed, since the $\aleph_\alpha$ values are circumscribed to the different kinds of infinite, this means that we must consider a geometry which naturally deals with infinite values. For this, there is a very axiom in “traditional” geometry which prevents this, and this axiom is the one of Archimedes. Let us remind the reader its formulation.

Archimedes axiom: Given any 3 distinct points, A, B, C, if we want to go from A to B along the line (A, B) with steps of length AC, then we shall reach B in a finite number of steps.

Clearly, this axiom, which is overwhelming in geometry in general and physics in particular, imposes to remain within the range of $\aleph_1$ which is the cardinal of the real numbers. If we want to go further, we need to cancel it, which means that we consider a universe in which there can exist 3 distinct points A, B, C such that to go from A to B with steps of length AC, we need an infinite number of steps! Even if the length of AB is finite! What this concretely means is that the length AC is an infinitesimal compared to the length of AB and the notion of infinite becomes a relative notion. Typically, viewed from the scale of AC, the length of AB is infinite, but viewed from the scale of AB, the length of AB is 1, a well- known real number.

Now, if we apply anew the contradiction of the Archimedes axiom to the length AC, then there can potentially be a length AD, which is infinitesimal with respect to AC and so on. We shall speak of scales; the scale of AB, the scale of AC, the one of AD, etc.

Now we need to make some remarks. We live at a scale and it is surprising that we considered, until now, the Archimedes axiom, because it seems obvious to us, since we do not necessarily have the intuition of the existence of any infinitesimal scale to ours. Surprisingly, we have the intuition of infinite, but very few people have the intuition that there might be an upper scale to ours, being infinite, in the sense of non-Archimedean geometry as we just saw it.

So, let us consider a non-Archimedean universe, having different scales of magnitudes. One very important thing is to be noticed:

since the Archimedes property is an axiom which is independent of the other axioms of geometry, if we drop it, we will not contradict anything. Another very important thing is that the Archimedes axiom is a constraint. Dropping it brings us to a much more general space which potentially brings additional properties since there are fewer restrictions. For example, at our scale, we can consider that our universe is Archimedean, but this constitutes then only a small part of the story.

But the very natural question which we can raise then, and which is absolutely paramount, is the one of the interaction between the scales. In other words, and in the physical world, because our goal is to describe our world, the question is:

Does what happens at our scale depend on what happens at the infinitesimal scale?

For sure, if it did not, we would just have different worlds which would be independent, put side by side, and such a model would be of no interest. The dependent case is much more interesting and potentially much more fruitful for us. The very laws of physics could come from an infinitesimal scale, out of reach to us. Really out of reach? We shall answer this question better later!

But, before, let us see until where we can be brought with such an approach. First of all, we need magnitudes to deal with the infinitesimals. As said above, the fields of hyperreal numbers could partly fit, but because of the non-uniqueness of the representation, we call for something else and this something else is the field of the surreal numbers built by John Conway in 1974 [6]. I shall not elaborate the construction of such a field, but I strongly advise the reader to read [6]. He will find there what I consider as a supremum of the mathematical science. What is important by the way, in this construction, is that the field of surreal numbers, called ***No***, is not a set. It is too big to be a set. It is a proper class. Everybody knows Russel's paradox: does the set of all sets exist? And the answer is no, but Gödel has figured out types [7] which bring to the notion of proper classes which do not face the problem raised by Russel's paradox. And this is why it is important in physics! Indeed, our universe must contain everything, even itself. It cannot be, therefore, any set. It must be a proper class! So, since ***No*** is a proper class, it seems to be a much better field than any other to describe our universe than either $\mathbb{R}$, $\mathbb{C}$ etc. To criticize a bit traditional physics, which aims at describing our universe with sets such as the reals, the philosophical question is:

Can a proper class be described by sets?

Let us keep on considering both physics and mathematics. In physics, we have punctual particles. Everybody thinks this is only a model, but that the elementary particles have constituents. But, for the specialists, this is not the case! The electron in modern quantum physics does not have any radius. It does not even have any trajectory around the nucleus in the atom, it does not have constituents. And this view perfectly fits with what we said earlier in this paper: quantum physics is non-causal. Out of a point, that is, nothing, would emerge a charge or a mass. With the non-Archimedean representation, we can overcome such a difficulty. Indeed, let us consider a sphere of radius $1/\omega$ where ω is the first transfinite ordinal of Cantor. Its volume obviously is $4\pi/(3\omega^3)$ which is an infinitesimal volume compared to any real number. So, from our scale (the real scale), we can only see this volume as a point. We can therefore, with this, justify the classical approach of physics, but see its limitations and we become again, causal. We shall see more about causality soon.

Now, the straightforward question is to know why or how we can have an influence from the infinitesimal scale into the real one. The point is pretty obvious and let us reason on an example only. Let us imagine infinitesimal particles of radius $1/\omega$ and let us consider ω such particles. Then, putting them side by side, we shall reach a global length of $\omega \text{x} 1/\omega=1 \in \mathbb{R}$. And we can imagine this can happen with any magnitude, not only the length, of the infinitesimal scale. This point of view is far from being exhaustive and will be the object of a further paper.

So, we are interested and have a model for explaining what happens at our scale by what happens at the transition between the finite and the infinite at the sub-scale. Let us see what this can bring.

A new model of particles

Let us begin considering particles at the real scale. Clearly, either we are part of the particle or we are outside. Without elaborating too much math, this means that a particle, as a subset of $\mathbb{R}^3$, must be a closed set. We have seen in the beginning of this text that the axiom of choice is part of our world, so this means that the Cantor-Bendixon theorem is not trivial. What this theorem says is that any closed set is made of a union of subsets which are "continuous" and a countable number of points. I refer the reader to [7] for more details. The interesting fact considering this through non-Archimedean geometry is that the points which are isolated in $\mathbb{R}$ and are in countable number, viewed from the sub-scale, are in their turn continuous, according to what we said above about the sphere of radius $1/\omega$.

This allows having particles which are extended in space with different parts, either locally continuous or points at the real scale and this would perfectly explain why, upon interaction, we cannot determine where the particle exactly is at the real scale; depending with which part of it we interact. It does not mean, however, that the particle itself is not very precisely positioned in space, but it can be a non-connected closed set of $\mathbb{R}^3$. So, we would have to integrate on a continuous volume of $\mathbb{R}^3$ to find the probability of presence of a particle, but not from $-\infty$ to $+\infty$ like in quantum physics, which seems much more reasonable and nearer from the reality even if we are going to see that things can be much weirder than this with entangled particles.

So, let us consider a couple of entangled particles. In fact, we consider it as a "big" particle with the same model as the one we described before thanks to the Cantor-Bendixon theorem. Now, since we can manipulate entangled particles, it suffices, to increase their spread into space, to keep the same structure with its constituents who are closed sets and stretch them into space so that we can have "the impression" that we have one particle in one place and the other one in another place. Now, the disentanglement process is going to consist in splitting the initial big particle into two sub particles. On a topological point of view, this just means making independent two closed sets by splitting an initial closed set. The process for splitting the initial closed set will be described in a further paper. This is a subtle point.

Causality

With our new model, we have all the ingredients for saving causality. Indeed, what happens at the real scale can very easily be explained though what happens at the infinitesimal scale. And what happens at this scale might perfectly be deterministic. So, we have full causality and can strictly verify the Kolmogorov axioms.

Can we justify the traditional point of view of quantum physics? For this, let us make a parallel with mathematics. Modern cryptography is born in the mid-1970s. It raised quite important questions about what randomness is. The culminating point has been reached by Yao [3]. In fact, given some data and some computation means, either we can extract some useful information out of the data (i.e. find some order) or not. If we can, the data are not random and if we cannot the data are random. Such a judgment is not absolute! It depends on our computation means. So, depending on them, we can have the impression of randomness or not. And this is what randomness is today, not more, but not less. Randomness for mathematicians today is an impression.

This should be the same for physicists. Indeed, we cannot see (yet!) what happens at the infinitesimal scale, therefore what happens at this scale looks like randomness. Only the fact of not having a non- Archimedean model prevents from concluding that this is an impression, otherwise, we have a potential source for such randomness, and we know that it cannot be scrutinized.

The question this raises is whether the concept of intrinsic randomness can be as much powerful as the Kolmogorov way through the infinitesimal scale. It seems pretty obvious that the second point of view is much more powerful, on the one hand, and much more promising on the other as I shall describe it next.

We also have solved the problem of the existence or not of the hidden variables. Indeed, in our model, they do exist, but they are out of reach of any Hilbert space, being at the infinitesimal scale. The Hilbert spaces are completely unable to describe them.

A first application

For the purpose of not revealing a patent to come, I am going to twist a bit the model of entangled particles I proposed before. This is an alternative, but it has the advantage to make the reasoning much easier in addition.

Let us assume that a couple of entangled particles is made of 2 particles linked through an infinitesimal physical channel, full of infinitesimal particles, allowing both particles to interact, like in the figure hereunder.

Figure 1: A couple of entangled particles

Let us imagine we send one of these particles towards a target. The channel is going to extend while they are entangled. Let us imagine that when hitting the target, this interaction disentangles both particles. If the particles are the same, then the channel will split, say, in the middle. The half of the channel which was attached to the particle we kept will retract and a part of it if not all, will reintegrate the particle. This can be considered as an echo, at the infinitesimal scale.

What do we have achieved? Simply a quantum radar for which we are not obliged to wait for the echo of the real particle in order to detect, but only the echo at the infinitesimal scale. This is a real breakthrough in the design of radars!

Fluid mechanics

There are two different scales to tackle fluid mechanics; either the atomic scale, through Boltzmann statistics or through the continuous approach and, say, the Navier-Stokes equations.

When comparing these two approaches, one thing is really puzzling: Why is there viscosity in the continuous case whereas at the atomic level the shocks are only elastic and there seems to be neither friction nor viscosity?

I asked the question and was answered that in the statistical approach, viscosity occurs when the number of molecules grows to the infinite. Right, but 6×10^{23}, for mathematicians, is a very small number. It has nothing to see with any infinite. So, once again, we have some discrepancy between the mathematical and the physical approaches.

The non-Archimedean model brings a big advantage. Of course, we naturally deal with infinite entities. As such, the viscosity naturally occurs even if it is the result of elastic shocks at some scale.

I shall not elaborate further on this, but I am now considering the design of an infinite impulse space propulsion engine based on this. It violates Newton's third law at the real scale, but not at the infinitesimal one. This is where all the difference is.

Mastering the infinitesimal scale

I wrote above that the infinitesimal scale cannot be scrutinized. This is not completely true. For this, I would like to refer to an address of Einstein to Heisenberg [8]:
Don't you see Heisenberg? This is the theory which says what can be observed.

And it is a fact that with non-causality and intrinsic randomness, we are not equipped to find anything else. The infinitesimal scale allows this, deeply. And there is a big cherry on the cake! Indeed, even if we cannot "see" the infinitesimal particles, the reader should realize that this is still the case in chemistry today. Nobody has ever seen any chemical reaction in real time. This did not prevent chemists from great successes. The exploration means in such a case is our intelligence. We have to tackle a system which we must understand with only a view from the outside and try to understand its way of functioning. And this is a way to introduce the necessity of intelligence in science, which was underlying until now, but never really proved to be compulsory. This is just done!

I shall end this short paragraph by noticing that with attosecond lasers and stroboscopy we are on the brink to have movies in real time of chemistry reactions. This should allow, in this 21st century, huge progress. There is no reason for not achieving such kind of same exploit and, say in some centuries, be able to "see" the infinitesimal world. That very day, we shall have to tackle the second order infinitesimal one, of course…!

Conclusion

I have proposed a breakthrough in the interpretation of quantum physics. A few years ago, at IHES, in Gif-sur-Yvette, one quarter has been dedicated to the 17 different interpretations of quantum physics. I just proposed here an 18th one. It has the merit to bring us back to well-known, mastered notions: determinism, with "true" laws of physics, and a vision of probability linked to our limitation and verifying the axioms of Kolmogorov. It also generalized greatly the universe as it is generally tackled, considering it as a proper class and no more a set, which is far more satisfactory on philosophical grounds.

Some applications have been evoked and depending on their experimental success will emerge or not the proposed point of view as the dominant one or not in the future.

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