

Two New Methods Based on $6x \pm 1$ equations To Break all Types of Evens in Sum of Two Primes

Bahbouhi Bouchaib*

Independent researcher. Nantes. Loire Atlantique. France

*Corresponding Author

Bouchaib Bahbouhi, Independent researcher. Nantes. Loire Atlantique, France

Submitted: 2025, Mar 25; Accepted: 2025, Apr 21; Published: 2025, Apr 28

Citation: Bahbouhi., B. (2025). Two New Methods Based on $6x \pm 1$ equations To Break all Types of Evens in Sum of Two Primes. *J Math Techniques Comput Math*, 4(4), 01-08.

Abstract

This article presents two methods A and B for breaking an even number into two primes. Both methods are inspired by the equations $6x \pm 1$ because all prime numbers and their multiples except 3 are $6x \pm 1$. Both methods can be very useful for even conversion in sums of two primes or to study the Goldbach's strong conjecture. Both methods can have applications in computer science.

Keywords: Method, Prime, Composite, Goldbach's strong conjecture, Congruence, Modulo, Sum of Two Primes.

Abbreviations: GSC : Goldbach's strong conjecture. P : prime. C : composite.

1. Introduction

I have recently reported in several papers that Goldbach's strong conjecture (GSC) depends on the presence of two equidistant primes p and q such that $p < S/2$ and $q > S/2$ and such that $S/2 - p = q - S/2$ therefore $S = p + q$ [1,6]. In addition I have shown that GSC depends closely on the gaps between primes especially gaps = 6 or 4 [1,5]. I have also shown that GSC might hold true to infinity [4].

In the present paper, I give a detailed description of two methods (A and B) to convert an even number into two primes according to GSC. Both of these methods are based on the equations $6x \pm 1$. Actually, there are three types of even numbers $6X$; $6X - 2$ and $6X + 2$ whose conversion into sum of two prime numbers depends on the equations $6x \pm 1$ given that $6X = (6x - 1) + (6x' + 1)$; $6X - 2 = (6x - 1) + (6x' - 1)$ and $6X + 2 = (6x + 1) + (6x' + 1)$.

2. Results

Method A
I. Even $S = 6X$ <ol style="list-style-type: none"> Be an even $S \geq 8$ (or ≥ 4 for any integer $n = S/2$). Determine whether $6X$; $6X + 2$ or $6X - 2$. $S = 6X \rightarrow S = \text{Odd1} + \text{Odd2} = (6x + 1) + (6x' - 1)$ or $S = (6x - 1) + (6x' + 1)$ Calculate all Odds $6x - 1 < S/2$ (denoted O6--) and $6x - 1 > S/2$ and $< S$ (O6--). Calculate all $6x + 1 < S/2$ (O6+-) and all $6x + 1 > S/2$ and $< S$ (O6++). Exclude any $3n$ or $5n$ or the prime factors of S. Either $S = 6X = (\text{O6--}) + (\text{O6++})$ or $S = 6X = (\text{O6+-}) + (\text{O6+-})$. In function of the Unit digit of $S \rightarrow$ select (O6--) and (O6++) with right partition unit digits. Do the same with (O6+-) and (O6+-) Determine $p < S/2$ or $\pi(S/2)$. $S = 6X \rightarrow$ Calculate $S - (6x - 1) = 6X + 1$; and $S - (6x + 1) = 6X - 1$. Both $6X + 1$ or $6X - 1$ can either be prime or composite (C) but $\neq 3n$. Calculate congruence of S and (O6++) or (O6+-) modulo (p). If $S \equiv (\text{O6+-}) \pmod{p}$ or $S \equiv (\text{O6++}) \pmod{p} \rightarrow$ Calculate $S = xp + r$; O6-- = $x'p + r$ and O6++ = $x''p + r$. If $x - x' = 1$ or $x - x'' = 1 \rightarrow S = p + q$ (p and q primes). If $x - x' > 1$ or $x - x'' > 1$ then $S = p + c$ or $S = p + c'$ such that $c = np$ or $c' = n'p$.

Example 180.

$180 = 6 \times 30$ therefore 180 is 6X. $S/2 = 90$.

Tables 1 show $6x - 1$ and $6x + 1$ numbers < 90 and > 90 but < 180 .

O6--	O6++	O6+-	O6+-
11	97	7	101
17	103	13	107
23	109	19	113
29	121	31	119
41	127	37	131
47	133	43	137
53	139	49	143
59	151	61	149
71	157	67	161
77	163	73	167
83	169	79	173
89			

Tables 1: Selection by the $6x \pm 1$ equation of odd numbers $< S = 180$ and either $< S/2 = 90$ or $> S/2$

Tables 2 show the next step which is selection of $6x - 1$ and $6x + 1$ numbers by their Unit digits. Only primes with right unit digits are involved in sum S. Prime numbers $p < S/2$ and $q > S/2$ such that $S = p + q$ are selected (Tables 3). Results are confirmed by

the calculation of congruence modulo (p) to explain why $S = p + q$ in some cases and why $S = p + c$ (c composite) in the other cases. Table 3 shows the final conversion of $S = 180$ in sums of two primes.

O6-- (1)	O6++	O6--	O6++	O6+-	O6+-	O6+-	O6+-
11	109	17	103	31	109	7	113
41	139	47	133	61	119	37	143
71	169	77	163	71	149	67	173
29	121	23	97	19	101	13	107
59	151	53	127	49	131	43	137
		83	157	79	161	73	167
		3	177				

Table 2: Selection by Unit digits of odd Numbers $< S = 180$ and either $< S/2 = 90$ or $> S/2$. Let us Note odd Numbers < 180 such XN with N their unit digit. Then $180 = XN1 + XN2$ Such that $XN1 < S/2$ and $XN2 > S/2$. We have $180 = X1 + X9$ or $180 = X9 + X1$; and $180 = X3 + X7$ or $180 = X7 + X3$. Indeed 0 (unit digit) = $1 + 9$ or 0 (unit digit) = $3 + 7$

P	q
3	177
7	173
13	167
17	163
19	161
23	157
29	151
31	149
41	139
43	137
71	269
53	127
67	113
71	109
73	107
79	101
83	97

Table 3: $S = p + q$ Such That p and q are Primes with $p < S/2$ and $q > S/2$ ($S = 180$ ad $S/2 = 90$). The Method Gives all Possible Sums

Calculation of Congruence

- $180 = c + q = 131 + 49 \rightarrow 180 \equiv 131 \pmod{7} \rightarrow 180 - 131 = 7n$ and $\rightarrow 180 - 131 = 7 \times 7 = 49$. $180 - 131 = X \rightarrow 180 = (7 \times 25) + 5$ and $131 = (7 \times 18) + 5 \rightarrow 25 - 18 = 7 > 1 \rightarrow X$ is composite = $7n = 49$ (composite).
- $180 = c + q = 77 + 103$. $180 \equiv 103 \pmod{7}$ and $180 \equiv 103 \pmod{11} \rightarrow 180 - 103 = (7 \times 11)n \rightarrow 180 - 103 = 7 \times 11 = 77$ (composite).
- $180 = p + q = 83 + 97 \rightarrow 180 \equiv 97 \pmod{83} \rightarrow 180 = (83 \times 2) + 14$ and $97 = (83 \times 1) + 14$.

$180 - 97 = X$ and so $X = (2 \times 83) + 14 - (1 \times 83) + 14 \rightarrow 2 - 1 = 1 \rightarrow X = p = 83$ (prime). Given that $180 \not\equiv 97 \pmod{p \neq 83}$, 83 is prime ($p \neq 83$ means any p different from 83).

- $180 = p + q = 19 + 161$. $180 \equiv 161 \pmod{19} \rightarrow 180 = (9 \times 19) + 9$ and $161 = (8 \times 19) + 9$.

$180 - 161 = X$ and so $X = (9 \times 19) + 9 - (8 \times 19) + 9 \rightarrow 9 - 9 = 1 \rightarrow X = p = 19$ (prime).

Given that $180 \not\equiv 161 \pmod{p \neq 19}$, 19 is prime ($p \neq 19$ means any p different from 19).

II. Even $S = 6X - 2$

1. Be $S = 6X - 2$.

$$(6X - 2) - (6x + 1) = 6X - 3 = 3N. (6X - 2) - (6x - 1) = 6X - 1.$$

Determine $\pi(S/2)$ with $p < S/2$. Calculate $S - p$. Either $S - p = 3N$ or $S - p = 6X - 1$. Then, remove all $3N$. Note $S - p = 6X - 1$ leads to either primes or composites $\neq 3N$. $S - p$ allows us to search for prime numbers p and q such that $p + q = S$.

$\pi(154/2)$	$154 - p = Xn$	Xn
3	151	X
5	149	X
7	147	3N
11	143	X
13	141	3N
17	137	X
19	135	3N
23	131	X
29	125	5N
31	123	3N
37	117	3N
41	113	X
43	111	3N
47	107	X
53	101	X
59	95	5N
61	93	3N
67	87	3N
71	83	X
73	81	3N

$S = p + q$	
$P < S/2$	$> S/2$
3	151
5	149
11	143
17	137
23	131
41	113
47	107
53	101
71	83

Table 4: Search for primes by $S - p$ ($p < S$) ($S = 154$; $S/2 = 77$). X in the right column means P or C

Calculation of congruence

- $154 = 3 + 151$; $154 \equiv 151 \pmod{3} \rightarrow 154 = (3 \times 51) + 1$ and $151 = (3 \times 50) + 11 \rightarrow 51 - 50 = 1 \rightarrow$ Given that $154 \not\equiv 151 \pmod{p > 3}$ 151 is prime.
- $154 = 11 + 143$. Give that 154 and 143 share two common factors 11 and 13, 143 is not prime.
- $154 = 17 + 137$. $154 \equiv 137 \pmod{17} \rightarrow 154 = (9 \times 17) + 1$ and

Example $S = 154$ (Table 4). Note 154 is $6X + 4$ and $6X + 4 = 6X - 2$.

Either $154 = 150(6X) + 4 = (6 \times 25) + 4$; or $154 = 156(6X) - 2 = (6 \times 26) - 2$.

$\pi(154/2) = \pi(77) = \{3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43; 47; 53; 59; 61; 67; 71; 73\}$.

$137 = (8 \times 17) + 1 \rightarrow 9 - 8 = 1 \rightarrow$ Given that $154 \not\equiv 151 \pmod{p \neq 17}$ 137 is prime.

• And so on.

Finally, $S = 154$ is broken 8 times in sum of two primes (Table 5). Note that this method gives all possible sums of $p + q$ for any even $S \geq 8$ with $q > p$ or $S \geq 4$ if $q \geq p$.

$S = p + q$	
$P < S/2$	$q > S/2$
3	151
5	149
17	137
23	131
41	113
47	107
53	101
71	83

Table 5: Final conversion $S = p + q$ for $S = 154$

• Note Units digits are ignored with $S = 6X - 2$ numbers because we calculate $S - p = X$ such that p and X have the right unit digits.

III. Even $S = 6X + 2$.

2. Be $S = 6X + 2$.

$$(6X + 2) - (6x - 1) = 6X + 3 = 3N. (6X + 2) - (6x + 1) = 6X + 1$$

Determine $\pi(S/2)$ with $p < S/2$. Calculate $S - p$. Either $S - p = 3N$ or $S - p = 6X + 1$. Then, remove all $3N$ (Table 6). $S - p = 6X + 1$ with $6X + 1$ prime or composite (C).

$\Pi(98/2)$	$98 - p$	X_n
3	95	5N
5	93	3N
7	91	X
11	87	3N
13	85	5N
17	81	3N
19	79	X
23	75	3N
29	69	3N
31	67	X
37	61	X
41	57	3N
43	55	5N
47	51	3N

Table 6: Example 98. Calculation of $98 - p$ with $p < 98/2 = 49$. Numbers 3N or 5N are shown. The Numbers 5N are Recognized by their unit Digit = 5

Calculation of Congruence

- $98 = 7 + 91$ to exclude because 91 and 98 sharing one common factor.
- Note prime factors of S can be excluded since the begining as

aforementioned.

- Conversion $S = p + q$ for $S = 98$ are therefore $98 = 19 + 79$; $98 = 31 + 67$ and $98 = 37 + 61$. The method gives all possible sums.

Method B

IV. Algorithm Based on a Postulate Analogous To Goldbach's Strong Conjecture (GSC)

1. Proposition A: « All primes p and q are equidistant and symmetrical at an integer $n = p + q/2$ except 2 and 3 ». Therefore $p + q = 2n$. This is true because any sum of two odds is even. If you add together the primes p and q , you always get an even number $S \geq 4$ and the two **primes are equidistant at $S/2$** .

2. Proposition B: « Any even number can be broken down into the sum of two primes ». Even if any even number can be the sum of any two odd numbers (Proposition A), we can't deduce from this that any even number ≥ 4 is the sum of two primes. Even if the addition of two known primes always produces an even number, we can't deduce from this that an even number can be broken into the sum of two primes. Hence breaking any even number ≥ 4 into the sum of two primes cannot be true until it has been demonstrated. Till now, this proposition known as GSC is not mathematically solved.

3. Proposition C: « A proposition is true until proven false by a counterexample or mathematical demonstration ». **We can then assume that for any natural number $n \geq 4$ there exists a value t such that $t < n$ and such that $n - t$ and $n + t$ are both primes.** Let $p = n - t$ and $q = n + t$ and thus $2n = p + q$. This proposition is analogous to the GSC. It can be applied to break an even number into the sum of two primes until a counterexample is found. Note here $q \geq p$. GSC means an even S can be broken down to two primes p and q such that $S = p + q$ and such that $p < S/2$ and $q > S/2$.

IVa. Even $S = 6X$.

To break an even $6X$ in two primes we calculate $n = 6X/2$ and then $n - t$ and $n + t$. Because $6X - (6x - 1) = 6X + 1$; and $6X - (6X + 1) = 6X - 1$; $t = 6x - 1$ or $t = 6x + 1$. Let us pose $n = S/2$ and then we calculate $S/2 - (6x - 1)$ and $S/2 + (6x - 1)$; or $S/2 - (6x + 1)$ and $S/2 + (6x + 1)$.

Example $S = 204$ and $S/2 = 102$. We first calculate $102 - p$ such that $p < 102$ (we add 1 although not prime). Then we calculate $102 - C$ with C a composite odd number $\neq 3n$ (Table 7). Both p and C are $6x \pm 1$.

$102 - p$	p (except 1)	$102 + p$	$102 - C$	$C \neq 3n$	$102 + C$
101	1	103	77	25	127
99	3	105	67	35	137
97	5	107	57	45	147
95	7	109	53	49	151
89	13	115	47	55	157
85	17	119	37	65	167
83	19	121	27	75	177
79	23	125	25	77	179
73	29	131	17	85	187
71	31	133	11	91	193
65	37	139	7	95	197
61	41	143			
59	43	145			

55	47	149
49	53	155
43	59	161
41	61	163
35	67	169
31	71	173
29	73	175
23	79	181
19	83	185
13	89	191
5	97	199
1	101	203

Table 7: Search for primes by $S/2 - p$ (p primes $< S/2$) and $S/2 - C$ (composites). Example $S = 204$ and $S/2 = 102$. Primes p and q are highlighted

Table 8 shows how 204 can be broken 14 times in two primes. This method gives all possible sums $p + q$.

204 = p + q
5+199
7+197
11+193
13+191
23+181
31+173
37+167
41+163
47+157
53+151
67+137
73+131
97+107

Tables 8: $S = p + q$. $S = 204$

IVb. Even $S = 6X + 2$ and $S/2 = 6X - 2$

$(6X - 2) - (6x + 1) = 6X - 3 = 3N$. And $(6X - 2) + (6x + 1) = 6X - 1$.

$(6X - 2) - (6x - 1) = 6X - 1$. And $(6X - 2) + (6x - 1) = 6X - 3 = 3N$.

For $n = S/2 = 6x - 2$, we can't use the $(n - t)$ and $(n + t)$ methods such as $(t = 6x - 1)$ or $(t = 6x + 1)$ because we'll always have a $3N$ and therefore never $S = p + q$.

As shown below (Tables 9 + 10 ; $n = S/2 = 124$) using numbers $t = 6x - 1$ or $t = 6x + 1$ we always get $3N$ numbers and therefore cannot break $S = 248$ in two primes. Numbers $6x - 1$ can be obtained by $5 + 6n$ equation while the $6x + 1$ ones by $7 + 6n$ ($n \geq 0$).

Example $S = 248$. $248 = (6 \times 41) + 2$; and $124 = (6 \times 20) + 4 \rightarrow 248$ is $6x + 2$ and $248/2 = 124$ is $6x + 4$ or $6x - 2$. We then calculate $n - t$ and $n + t$ such that $n = S/2 = 124$ and $t = 6x - 1$ numbers (Table 9).

124 - t	t = 6x - 1	124 + t
119	5	129
113	11	135
107	17	141
101	23	147
95	29	153
89	35	159
83	41	165
77	47	171
71	53	177
65	59	183
59	65	189
53	71	195
47	77	201
41	83	207
35	89	213
29	95	219
23	101	225

17	107	231
11	113	237
5	119	243

Table 9: $S = 248$. We calculate $n - t$ and $n + t$ such that $n = S/2 = 124$ and $t = 6x - 1$. The $6x - 1$ numbers are obtained by $5 + 6n$ equation. We have no case $S = p + q$. Note 3N numbers present on each line

We then calculate $n - t$ and $n + t$ such that $n = S/2 = 124$ and $t =$ number into two primes (see below). Note $6x + 1$ odd numbers are obtained with $7 + 6n$ equation (Table 10). We get the same results as with $t = 6x - 1$ due to constant presence of the 3Ns, which prevents us from breaking the

$124 - t$	$t = 6x + 1$	$124 + t$
117	7	131
111	13	137
105	19	143
99	25	149
93	31	155
87	37	161
81	43	167
75	49	173
69	55	179
63	61	185
57	67	191
51	73	197
45	79	203
39	85	209
33	91	215
27	97	221
21	103	227
15	109	233
9	115	239
3	121	241

Table 10 : Example $S = 248$. We calculate $n - t$ and $n + t$ such that $n = S/2 = 124$ and $t = 6x + 1$ numbers obtained by $7 + 6n$ equation. We have no case $S = p + q$. Note 3N numbers present on each line

While by contrast, we can use the odd-numbers which are multiples of 3 noted here as O3N, which are in order 3; 9; 15; 21; 27; 33. $n - O3N$ and $n + O3N$ for $S = 248$ and $S/2 = 124$. The O3N allows us to break 248 in two primes as shown below (Table 11). O3N. We then set $t = O3N$ and calculate

$124 - O3N$	O3N	$124 + O3N$
121	3	127
115	9	133
109	15	139
103	21	145
97	27	151
91	33	157
85	39	163
79	45	169
73	51	175
67	57	181
61	63	187
55	69	193
49	75	199
43	81	205
37	87	211
31	93	217
25	99	223
19	105	229
13	111	235
7	117	241

Table 11: Breaking an even number $S = 248$; $S/2 = 124$ which is $6X - 2$ in two primes by calculating $S/2 - t$ and $S/2 + t$ such that $t = O3N$ (note $n = S/2$). Primes p and q are highlighted

The number is therefore broken 6 times in two primes as below (Table 12). The method gives all possible sums.

$S = p + q$
7+241
19+229
37+211
67+181
97+151
109+139

Table 12: $S = p + q$ with $S = 248$

IVc. Even $S = 6X - 2$ and $S/2 = 6X + 2$

The even $S = 196$ and $S/2 = 98$.

$196 = (6 \times 32) + 4$ is $6X - 2$ whereas $98 = (6 \times 16) + 2$ therefore $6X + 2$.

$(6X + 2) - (6x - 1) = 6X + 3 = 3N$. And $(6X + 2) + (6x - 1) = 6X + 1$

$(6X + 2) - (6x + 1) = 6X + 1$. And $(6X + 2) + (6x + 1) = 6X + 3 = 3N$. Therefore we cannot use $n - t$ and $n + t$ such that $t = 6x - 1$ neither $t = 6x + 1$ because as seen with the previous case ; there will always be $3N$ numbers that prevents us from breaking the even S in sum of two primes. We only use $03N$ this time. We then break the numbers in two primes (Table 13).

$98 - t$	$t = 03N$	$98 + t$
95	3	101
89	9	97
83	15	113
77	21	119
71	27	125
65	33	131
59	39	137
53	45	143
47	51	149
41	57	155
35	63	161
29	69	167
23	75	173
17	81	179
11	87	185
5	93	191
3	95	193

Table13: Search For Primes By $n - t$ and $n + t$ with $n = S/2$ and $t = 03N$. Note 95 is not $03N$ (the last line of the table $95 = 5 \times 19$) But It Is Required To Check If The Number $S = 3 + q$. Note that $6X + 2$ and $6X - 2$ even Numbers Might have 3 as an Additive Prime On The Contrary To Even $6x$

$S = p + q$
3+193
5+191
17+179
23+173
29+167
47+149
59+137
83+113
89+107

Table 14: The number 196 ($S/2 = 98$) Can Therefore Be Broken 9 Times In Two Primes

Table 14 shows all possible conversions in sums of two primes of $S = 196$.

3. Conclusion

This article presents two methods for breaking an even number into two primes. Both methods are inspired by the equations $6x \pm 1$ because all prime numbers and their multiples except 2 and 3 are $6x \pm 1$. Method A finds prime numbers p and q with the correct unit digits to form the even number $S = p + q$. Congruence rules confirm or explain why $S - p = q$ or $S - p = c$ (composite).

Method B is based on proposition C which, explained differently, suggests that prime numbers are always formed in pairs from an integer $n \geq 4$. A prime number never appears alone but together with another one and both of them are symmetrical and equidistant to the integer n from which they come. In an interval $[0 - n - 2n]$ where we would then have at least one value t such that $t < n$ and such that $p = n - t$ and $q = n + t$ are primes and $2n = p + q$. The

value t differs depending on whether the even number S is $6X$; $6X - 2$; $6X + 2$. Method B shows that for $6X$; $t = 6x - 1$ or $t = 6x + 2$. Whereas it is necessary that $t = 03N$ (multiple of 3 that are odds) to obtain pairs of prime numbers (p ; q) such that $S = p + q$ from evens which are $6X - 2$, or $6X + 2$.

Both methods are not only useful to break evens in two primes but can help investigating the Goldbach's strong conjecture (GSC). There is a very common confusion to avoid, which is that even if prime numbers added together always give an even number; this does not mean at all that any even number can be broken into two prime numbers. To break an even number into two prime numbers, a method is required that can be simple or very complex. To prove the GSC, it is necessary to demonstrate by a theorem that an integer $n \geq 4$ always produces two prime numbers p and q such

that $p = n - t$ and $q = n + t$ ($t < n$). Such a theorem still does not exist. For the time being; it can only be achieved using simple or complex methods or algorithms. Both methods here are simple and accessible and could generate new algorithms of even conversion in sums of two primes.

Finally, this article defends the idea that prime numbers are formed in pairs from the same integer $n \geq 4$. For the moment, only methods and congruence rules can help us achieve this, but is this fact demonstrable? Can it obey a theorem that also solves the GSC? Time will tell.

References

1. Bahbouhi, B. (2025). Demonstrating Goldbach's Strong Conjecture by Deduction using $4x \pm 1$ Equations in Loops and Gaps of 4. *J Robot Auto Res*, 6(1), 01-10.
2. Bahbouhi, B. (2025). How to Pose the Mathematical Problem of the Goldbach's Strong Conjecture? A New Idea for a New Solution. *J Robot Auto Res*, 6(1), 01-03.
3. Bahbouhi, B. (2025). New Mathematical Rules and Methods for the Strong Conjecture of Goldbach to be Verified. *J Robot Auto Res*, 6(1), 01-36.
4. Bahbouhi, B. (2025). Verification of Goldbach's Strong and Weak Conjectures at Infinity Using Basic and Accessible Mathematics. *J Robot Auto Res*, 6(1), 01-11.
5. Bahbouhi, B. (2025). Proving Goldbach's Strong Conjecture by Analyzing Gaps Between Prime Numbers and their Digits. *J Math Techniques Comput Math*, 4(1), 01-18.
6. Bouchaib, B. (2025). Natural Equidistant Primes (NEEP) and Cryptographic Coding of the Goldbach's Strong Conjecture. *J Curr Trends Comp Sci Res*, 4(1), 01-09.

Copyright: ©2025 Bouchaib Bahbouhi. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.