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The Heliosheath Plasma Flow Downstream of the Solar Wind Termination Shock in a Consistent Two-Fluid View

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Abstract

In this article, we shall take it serious what has come out from recent theoretical studies on the plasma conditions in the heliosheath, namely downstream of the solar wind termination shock the total plasma pressure is dominated by the pressure of the electrons. We shall respect these results and try to develop the corresponding theoretical basis for an authentic multi-fluid description of the plasma flow downstream of the shock, starting with the dominant pressure action of the local electrons on the mass- and momentum flow of the multi-fluid plasma. It turns out that the usual claim made by Voyager-1 and-2 experimentalists that the bulk velocity of the upwind plasma flow is decreasing with distance from shock, would require that, in reaction to that bulkplasma momentum loss, the electron pressure even increases downstream of the shock. The alternative possibility would be that Voyager-1/-2 data up to now are misinterpreted, and that in fact solar wind bulk velocities in the upwind heliosheath downstream of the shock are increasing opposite to present understandings.

Keywords: Two-Fluid Plasmas, Incompressible Electrons and Protons, Shock-Processing

Introduction

With the Voyager-probes Voy-1 and Voy-2, the scientific community was hoping to get ideal observations in form of in-situ data on the real plasma scenario near the region of the solar wind termination shock and further downstream into the heliosheath region. This hope meanwhile has been cooled down a bit, since it became evident that the plasma near this shock region is of an unexpectedly complicated nature for which the Voyager detector packages were not well enough designed. This has mainly to do with the multifluid nature of the local plasma out there, where not only solar wind ions, but also even more pick-up ions and, surprisingly electrons do dictate the actual physics. Especially questions concerning the post-shock solar wind bulk velocity U and its direction have created big debates until now. In addition, the temperatures of the post-shock solar wind ions are under strong debates [1-4].

The latter points are due to the fact that bulk velocities and temperatures are velocity moments of the distribution function, and for their determination based on solar wind ion flux data one needs an assumption what kind of distribution function (e.g. Maxwell-, power-law-, or Kappa- distributions) needs to be imputed [5]. Things are even worse, since only Voyager-2 has low-energy proton data, while Voyager-1 has lost its low-energy proton detector and bulk velocities of post-shock protons have to be indirectly derived via use of the Compton-Getting anisotropy effects from GCR high-energy proton data [6].

Generally, electrons in plasma physics are the underestimated species. In most cases, they only have to guarantee electric quasi-neutrality, i.e. $n=n_i=n_e$, but they don´t count in terms of mass-, momentum-, and energy- flows. However, in many space plasmas,e.g. like the heliospheric plasma, especially downstream of the solar wind termination shock, the actual physical conditions there look different. More theories that are recent have shown that electrons in fact even dominate the plasma pressure and the plasma energy flow of the plasma bulk flow downstream of the termination shock [1,7-9]. Such conditions do require a genuine, authentic multi-fluid plasma theory for an adequate description of fields and flows, and we shall study here further down what this new theory will predict.

We first develop a pure two-fluid thermodynamics of such two-fluid plasmas and then study the actual situation for the given case that the total heliospheric plasma pressure downstream of the termination shock is dominated by the electron pressure, with the proton pressure being an inferior contribution near the shock. Under such auspices, the electron pressure determines the changes in the mass- and momentum- flows of the total plasma and in fact should increase with a decrease of the common bulk velocity U of the flow - or just vice versa.

The Multi-Fluid Aspect of the HeliosheathPlasma

Let us imagine a plasma consisting of heavy protons and light electrons, however, the protons are dominating the mass- and momentum- flows, while the electrons dominate the energy- flow, i.e. a "pressurized, hot- electron-fluid" in a massive background of cold protons! Under such auspices, it is evident that the change of the electron energy flow at the actual plasma motion will cause the changes both of the mass- and of the momentum- flows of the protons. Exactly such a twin-fluid dynamical situation in fact arises just downstream of the solar wind termination shock where electrons become shock-accelerated due to the action of the shock-electric field and there after are injected as a thermally randomized population into the shocked proton flow in form of a hot , strongly pressurized electron fluid [1,7-9]. The influence of this termination shock on electrons for instance is not taken into account in available multifluid approaches like those presented by Goedbloed and Usmanov, et al [10,11]. For instance, the situation downstream of the shock which we in this article feel confronted with is substantially different from what has been described with a four-fluid approach by Usmanov, Goldstein and Matthaeus, since in their approach the proton pressures downstream of the termination shock, different from present expectations, are two orders of magnitude higher than the corresponding electron pressures [11].

In a first view, and under standard plasma conditions, it may be self-suggestive that in the solar rest frame the change of the ion pressure P_i is responsible for the change of the plasma momentum flow, evidently connected with the bulk motion $\vec{U}$ of the ion mass-density $\rho_i=M_{ni}=M_n$. Consequently, this physical context should be formulated in the following form:

$$\frac{dP_i}{ds} = -\frac{d}{ds}[n\frac{M}{2}U^2]$$

Where $d\vec{s} = \vec{U}(s)dt$ denotes the line elementa long the streamline of the plasma flow.

In the unusual case studied here, however, that the electron pressure P_e would dominate over the ion pressure, this pressure P_{e}- would do an analogue job in influencing the momentum flow of the plasma, i.e. of the ions, according to:

$$\frac{dP_e}{ds} = -\frac{d}{ds}[n\frac{M}{2}U^2]$$

This for example would then evidently mean that a corresponding, additional term on the right side of Equ. (3) of Fahr and Dutta-Roy would be needed that takes into account the action of the electron pressure at influencing the momentum flow of the plasma [12]. In view of the upper equation and the expected decrease of the bulk velocity U along streamlines downstream from the termination shock (see dynamic models used by Nerney, et al, Fahr and Fichtner, Fahr, et al. or by Voyager observations presented by Decker or Boschini) would have the counter-intuitive effect of even increasing the dominant electron pressure as cause of a corresponding decrease of the plasma bulk velocity U with $dU/ds \leq 0$ [6,8,13-15]. Consequently, the question seems justified: What in fact does one know safely of the plasma bulk speed behavior downstream of the shock?

Modelling of the Coupled Ion-ElectronFlow

To model the plasma flow downstream of the termination shock usually the approximation of a potential flow has been used (see early work by Parker, 1963, or later by Fahr and Neutsch, Fahr and Scherer, Nerney et al., Fahr and Fichtner) [2,13,14,16]. This theoretical approach, namely to derive the flow field as a gradient of a flow potential according to $\vec{U} = -\nabla\Phi(\vec{r})$ is possible for a low Mach number flow $M \leq 0.1$ of a one-fluid plasma, since this kind of flows behave as nearly incompressible. Such a potential flow approximation thus is viable for an incompressible one-fluid plasma, i.e. with $dn/ds \simeq 0$. But when applied to the heliospheric two-fluid plasma system with even pressure-dominant electrons would then bring up a counter-intuitive result: Namely that in the heliosheath regions downstream from the upwind termination shock, where the expected bulk velocities according to such standard models decrease from about 130km/s at the termination shock to roughly 0 km/s at the heliopause (e.g. see Nerney et al, or Figure 1), the electron pressures should, in reaction to that, continue to increase even more in these regions towards the heliopause, since they had to reflect the loss of momentum flow [13].

This strange result, however, one may think, is due to the exaggerating assumption that exclusively the electron pressure dictates the plasma flow. Perhaps in addition one should also face the problem here, that under the discussed situation of a plasma determined in its dynamics by the pressures of two separate, individual plasma fluids, i.e. the electron and the ion fluid, single fluid solutions derived from one streaming potential $\Phi(\vec{r})$ may perhaps not be applicable anymore. What to do under these conditions?

What concerns the needed assumption of an incompressible fluid, the point that we want to face a two-fluid situation further on here in fact may not invalidate this. The only important point is that the effective multi fluid sound velocity $c_{s,eff}$ still has to be large in comparison to the plasma bulk velocity U that means (see Fahr and Heyl) [17].

$$c_{s,eff} = \sqrt{\frac{\gamma_{eff}}{\rho_i}}\sqrt{P_e + P_i} \gg U$$

Since this relation appears, however, well fulfilled in the heliosheath, one can tacitly make use of the "incompressibility assumption" also for the multifluid case here.

Since we in addition anyway have to treat isodensity conditions, i.e. $n = n_e = n_i$, and do not allow for electric currents by requiring co-convection of electrons and ions requiring, one can start the business here with a solution for $\vec{U}$ taken from a joint flow potential $\Phi = \Phi_{e,i}$ that only takes care of an acceptable fit of the resulting large-scale streaming configuration to the standard heliospheric boundary conditions: i.e. radial outflow at small solar distances

$r \leq L$ and a homogeneous downwind flow at large solar distances $r \gg L$ (with L denoting the heliopause distance). Thus, one can start from the following streaming potential (see e.g. Fahr and Fichtner) [14].

$$\Phi_{e,i}(\vec{r}) = -\sqrt{\rho_i}\, U_{IS} L \cdot [\bar{r} P_l(\cos\theta) - \sum_{l=1} \frac{A_l}{\bar{r}} P_{l-1}(\cos\theta)]$$

where $P_l(x)$ are Legendre polynomials of the order "l", " $\bar{r}$ =r/L is the normalized radial distance from the Sun and U_{IS} denotes bulk velocity of interstellar plasma relative to the Sun, i.e. the motion of the solar system relative to the local interstellar medium.

From this streaming potential, one derives, when hereby keeping only to the first order in *l, i.e. l =1*, the following streaming configuration shown in Figure 1.

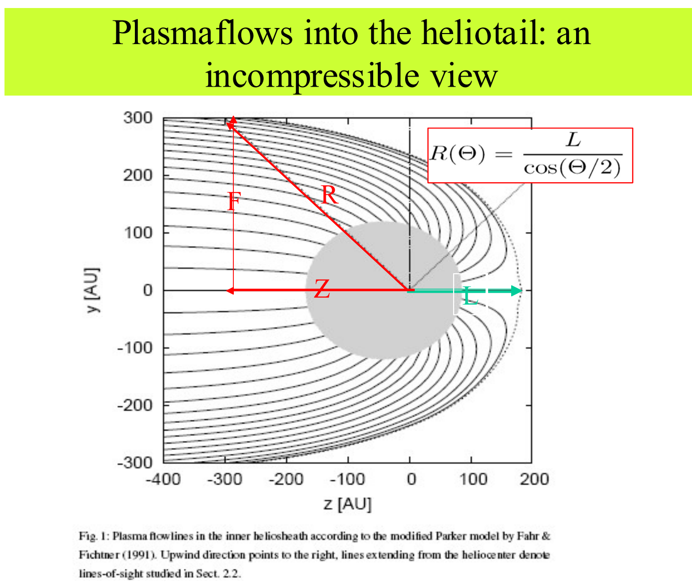


Figure 1: Heliospheric Streamline Configuration derived from a Joint Streaming Potential$\Phi_{e,i}(\vec{r})$

Keeping only to the first Legendre polynomial (*l*=1) one can derive from the streaming potential the following simpler solution (i.e. Parker's solution, Parker, 1963)

$$U^2(\vec{r}) = (\frac{\rho_{IS}}{\rho_i}) U_{IS}^2 [1 + 2\frac{\bar{z}}{\bar{r}^3} + \frac{1}{\bar{r}^4}]$$

where $r^2 = z^2 + y^2$ is the radial distance from the Sun, and the normalizations have been carried out with the heliopause stand-off distance L given by Fahr, Fichtner, Neutsch (1990) in the form [2].

$$L^2 = \frac{1}{2} \sqrt{\frac{\rho_{i0} U_0}{\rho_{IS} U_{IS}}} \sqrt{\frac{\rho_{i0} U_0^2}{K(M_{SH}) P_{IS}}}\, r_0^2$$

where ρ_{i0} and U_0 denote mass density and bulk velocity of the solar wind at a reference distance $r_0 = r_{SH}$(position of the termination shock), ρ_{IS}, U_{IS} and P_{IS} are the mass density, the bulk velocity and the total pressure of the interstellar medium, and $K(M_{SH})$ is the so-called pressure adaptation function which for general solar wind Mach numbers M_{SH} is given by the following expression [18].

$$K(M_{SH}) = \frac{\gamma(\gamma+1) M_{SH}^2}{2\gamma M_{SH}^2 - (\gamma - 1)} \cdot [\frac{4\gamma M_{SH}^2 - 2(\gamma - 1)}{(\gamma+1)^2 M_{SH}^2}]^{\gamma/(\gamma-1)}$$

Where $\gamma = 5/3$ denotes the polytrophic index. This above expression is more complicated than the one given by Axford, but the latter one is only valid for very high Mach numbers $M_{SH} \geq 10$, while here we want to apply this function to more realistic cases of the actual solar wind termination shock according to Voyager-2 observations with $3 \leq M_{SH} \leq 6$ with compression ratios of $s \simeq 2.8$ yielding adaptation values of $K(3) \simeq 1.08$ [3,18].

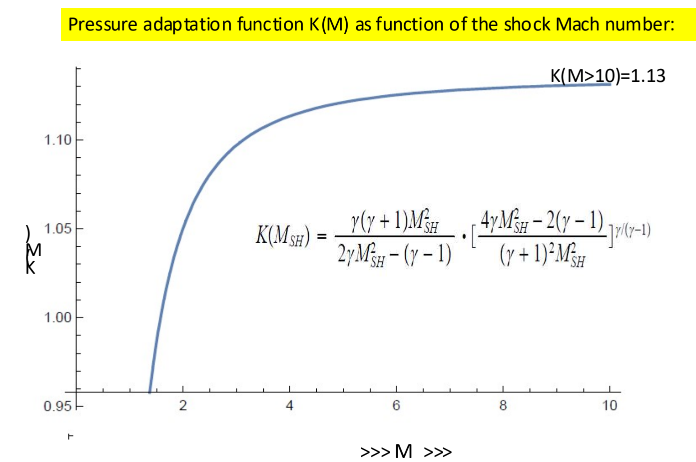


Figure 2: Pressure adaptation function $K(M_{SH})$as function of M_{SH}

A Consistent Two-FluidThermodynamics

The counterintuitive phenomenon suggested by equation (??) may for sure appear a bit controversial and embarrassing, hence for more clarification of this point, we now look at this situation from a slightly different view, following the standard thermodynamical procedure which states that the work done by the pressure at a change of the co-moving plasma volume ΔV is reflected by an associated change of the internal energy densities $\epsilon_{i,e,w}$ of that volume. This, when allowing for multi-fluid plasmas, requires that in the Solar Rest Frame (SRF) the following equation has to be fulfilled

$$-(P_e + P_i)\frac{d\Delta V}{ds} = \frac{d}{ds}[(\epsilon_i + \epsilon_e + \epsilon_w)\Delta V]$$

Where ΔV, as explained in Fahr and Dutta-Roy, denotes the co-moving plasma volume on the streamline, i.e. a fluid volume that locally moves with the plasma bulk velocity $\vec{U}$ [12].Hereby the indices "*i,e,w*"indicate ion-or electron-, or wawe-related quantities, e.g. as pressures $P_{i,e}$ or internal energy densities $\epsilon_{i,e}$ of ions or electrons, or energy densities of a self-sustained, co-convected turbulent wave field, respectively, like compressive or Alfvén turbulences with Alfvén velocities v_A small compared to bulk velocities U.

Hereby the Alfvénic turbulence spectrum $W(k)$ as function of the wave number k *is* normalized such that $\epsilon_w = \int_{-\infty}^{+\infty} W(k)dk$ represents the local energy density of the turbulence. In this respect we understand the thermodynamic action in its more completed form, namely also as that work done at a change of the fluid volume ΔV which partly reappears as equilibrium turbulence energy density $d_{\epsilon w}$. The latter as such also drives the energy diffusion of ions and electrons, i.e. the redistribution of energies between ions and electrons.

Somehow here with we follow a little bit the idea proposed by Fisk and Gloeckler for the inner heliosphere that energetic ions under resonant interaction with ambient compressive turbulences can enter into a quasi-equilibrium state with saturated power-law distributions (i.e. (-5) -power law tails!) and Kolmogorov-like turbulence spectra [20,21]. That means the thermodynamics of this wave-particle system also includes a consistent form of "frozen-in" wave fields. This means we take into account not only the facts that in the SRF (Solar Reference Frame) the ion energy density is given by $\epsilon_i = nMU^2/2+(3+2\pi)P_i$ while the electron energy density only is given by $\epsilon_e = (3+2\pi)P_e$ (n.b.: energy of the bulk motion of the electrons is negligible with respect to that of the protons according to the factor (m_e/M)), but also that an equilibrium energy density ϵ_w of turbulences is co-convected with the plasma bulk.

What concerns the change of turbulence energies ϵ_w per streamline element ds due to the action of velocity space diffusion of electrons and protons, assuming that the change in respective particle energies is due to a corresponding negative change in the wave energies, one can find from Equ. (3) in Fahr and Dutta-Roy [12].

$$\frac{d}{ds}(\epsilon_{wi}+\epsilon_{we}) = \frac{4\pi}{3U}[M\int v^2dv(\frac{1}{v^2}\frac{\partial}{\partial v}(v^2D_{i,vv}\frac{\partial}{\partial v}f_i)) + m\int v^2dv(\frac{1}{v^2}\frac{\partial}{\partial v}(v^2D_{e,vv}\frac{\partial}{\partial v}f_e))]$$

Where $D_{i,vv}$ and $D_{e,vv}$ denote velocity-space diffusion coefficients of ion and electrons, respectively. Assuming now that both these diffusion coefficients have the form $D_{i,vv} = D_{e,vv} = D_{0,i,e}\cdot(v/v_0)^2$ one obtains from the upper equation after solving the necessary integrals (see Equ. (11) of Fahr and Dutta-Roy) the following final result [12].

$$\frac{d}{ds}(\epsilon_{wi}+\epsilon_{we}) = \frac{1}{U}[10D_{i,0}P_i + 10D_{e,0}P_e]$$

Taking these things together with the earlier relations (i.e. Equ. (??)) one finally then obtains the following net equation:

$$-(P_e+P_i)\frac{d\Delta V}{ds} = \frac{d}{ds}[(\epsilon_i+\epsilon_e+\epsilon_{wi}+\epsilon_{we})\Delta V]$$

When furthermore being prepared for the situation that the electron pressure competes with the ion pressure, i.e. $P_e \simeq P_i$, will bring us to the following equation:

$$-(P_e+P_i)\frac{d\Delta V}{ds} = \frac{d}{ds}[(nMU^2/2 + \frac{3}{2\pi}(P_e+P_i) + \epsilon_{wi}+\epsilon_{we})\cdot\Delta V]$$

When additionally recognizing here that for an incompressible flow, as given here in the case of a strongly subsonic flow, the comoving plasma volume then is represented by the following relation $\Delta V = \Delta V_0 \cdot (U_0/U)$ and consequently the above equation displays into the form [12].

$$-(P_e+P_i)\frac{d\frac{1}{U}}{ds} = \frac{1}{U}[\frac{d}{ds}(\epsilon_i+\epsilon_e)+\frac{d}{ds}(\epsilon_{wi}+\epsilon_{we})] + [\epsilon_i+\epsilon_e+\epsilon_{wi}+\epsilon_{we}]\frac{d}{ds}\frac{1}{U}$$

which further develops into:

$$[(P_e+P_i) + [\epsilon_i+\epsilon_e+\epsilon_{wi}+\epsilon_{we}]]\frac{d}{ds}\frac{1}{U} = -\frac{1}{U}[\frac{d}{ds}(\epsilon_i+\epsilon_e)+\frac{d}{ds}(\epsilon_{wi}+\epsilon_{we})]$$

and furthermore yielding:

$$[P_e+P_i+\epsilon_i+\epsilon_e+\epsilon_{wi}+\epsilon_{we}]\frac{1}{U^2}\frac{d}{ds}U =$$

$$\frac{1}{U}\frac{d}{ds}[\frac{1}{2}\rho_i U^2] + \frac{3}{2\pi}\frac{1}{U}\frac{d}{ds}(P_e+P_i) + \frac{1}{U^2}[10D_{i,0}P_i + 10D_{e,0}P_e]]$$

Which finally for $U>0$ (i.e. excluding the stagnation point) leads to:

$$[P_e+P_i+\epsilon_i+\epsilon_e+\epsilon_{wi}+\epsilon_{we}-\rho_i U^2]\frac{dU}{ds} =$$

$$\frac{3}{2\pi}U\frac{d}{ds}(P_e+P_i) + [10D_{i,0}P_i + 10D_{e,0}P_e]$$

and after replacing ϵ_i and ϵ_e one finds:

$$\frac{2\pi+3}{2\pi}(P_e+P_i) - \frac{1}{2}\rho_i U^2 + \epsilon_{wi}+\epsilon_{we}]\frac{dU}{ds} =$$

$$\frac{3}{2\pi}U\frac{d}{ds}(P_e+P_i) + [10D_{i,0}P_i + 10D_{e,0}P_e]$$

Boundary conditions at the termination shock

When looking at the upper differential equation it may naturally be asked: What are the boundary conditions at the termination shock from which the integration of the upper differential equation must be started? Denoting the relevant quantities with suffixes "0", one would thus have to determine: $P_{e,0}$, $P_{i,0}$, $\epsilon_{wi,0}$, $\epsilon_{we,0}$ and U_0. Amongst those the easiest to define perhaps is the downstream bulk velocity U_0 which according toVoyager-2 data is found with U_0=130km/s or a dynamic pressure of $\epsilon_{dyn} = 1/2\rho_i U_0^2 = 28fPa$[6,15,22].

The total proton pressure, including pick-up ions, is given by these latter authors with a value of $P_{i,0}=160fPa$. Following Fahr, Siewert, Fahr and Verscharen the electron pressure $P_{e,0}$ compared to $P_{i,0}$ should be higher by about a factor of 2. 5, and according to the electron relaxation process by the Bunemann instability with a branching ratio of 1-to-1 on the downstream side should be about equal to the energy density of the whistler wave power $\epsilon_{we,}0$ [7,8]. Based on a similar branching ratio at the relaxation of the energetic protons (PUI's) we could most likely assume the Alfvenic wave energy density ϵwi,0 as being about equal to the total proton pressure, i.e. $P_{i,0} \simeq \epsilon_{wi,0}$. Therefore, since not equipped with any betterknowledge, we start with the following list of boundary values:

$$\epsilon_{dyn} = \frac{1}{2}\rho_i U_0^2 = 28fPa$$
$$P_{i,0} = 160fPa$$
$$\epsilon_{wi,0} = 160fPa$$
$$P_{e,0} = 400fPa$$
$$\epsilon_{we,0} = 400fPa$$

Not knowing anything better, we may also assume here that the velocity space diffusion coefficients $D_{e,0}$ and $D_{i,0}$ are related to each other, just as the energy densities of the responsible wave fields are. Hence, we assume:

$\epsilon_{we,0}/\epsilon_{wi,0}$=400/160 $\simeq D_{e,0}/D_{i,0}$

With these boundary values we now would have to start the integration of the above system of coupled differential equations.

The Total Pressure of the Turbulence-Free Plasma

When neglecting the involved turbulent wave energy densities, i.e. setting $\epsilon_{wi} = \epsilon_{we} = 0$, one would remain instead with Equ. (??) , with the following equation:

$$[\frac{2\pi+3}{2\pi}(P_e+P_i)-\frac{1}{2}\rho_i U^2]\frac{dU}{ds} = \frac{3}{2\pi}U\frac{d}{ds}(P_e+P_i)$$

which, introducing the total plasma pressure by $\Pi = P_e + P_i$, would lead to a differential equation like:

$$\frac{d}{ds}\ln\Pi = -\frac{2\pi}{3}[\frac{2\pi+3}{\pi}+\frac{\rho_i U^2}{\Pi}]\cdot\frac{d\ln U}{ds}$$

If one now keeps in mind, the assumption of incompressibility made for this derivation (See Equ. (AA), then this elucidates that the above equation with $\frac{2\pi+3}{\pi} \gg \frac{\rho_i U^2}{\Pi}$ simply states that the streamline derivative of the total pressure is given in the form:

$$\frac{d}{ds}\ln\Pi = -2\left(\frac{2\pi+3}{3}\right)\frac{d\ln U}{ds}$$

leading to:

$$\frac{d}{ds}[\ln\Pi + \ln U^{2\frac{2\pi+3}{3}}] = \frac{d}{ds}\ln(\Pi U^{\frac{4\pi+6}{3}}) = 0$$

and finally yielding as the "turbulence-free" solution for the total pressure:

$$\Pi(s) = C/U(s)^{\frac{4\pi+6}{3}}$$

where the constant C has to be evaluated at the line element $s = s_0$ at the termination shock and is given by the following expression:

$$C(s_0) = \Pi(s_0)U(s_0)^{\frac{4\pi+6}{3}} = [P_e(s_0)+P_i(s_0)]U(s_0)^{\frac{4\pi+6}{3}}$$

Hence the final solution of the total plasma pressure is given in the form:

$$\Pi(s) = \Pi(s_0)\cdot[\frac{U(s_0)}{U(s)}]^{\frac{4\pi+6}{3}}$$

So far, for the turbulence-free case, one finds the solution for the total pressure as the above function of the space coordinate s in the form Π = Π(U(s)).

If this equation would formulate the only physical requirement, then it would clearly state that a decrease of the bulk velocity *U(s)* with increase of the shock distance $s \geq s_0$ should necessarily invoke an increase of the total pressure Π(*s*), while not expressing clearly which of the pressures, P_i or P_e, does increase by how much. But to take this conclusion as the full truth, one should be sure that besides this upper condition for the total pressure Π there do not exist other independent conditions that have to be fulfilled for the two pressures P_e and P_i separately.

Conditions for the Single-Fluid Pressures

However, looking into the work published by Fahr and Dutta-Roy [12]. one may notice that there a pressure transport equation has been developed which separately describes how the electron pressure develops under multiple, energetically relevant physical processes, like magnetic moment conservation, whistler-wave driven velocity diffusion, and convection. The above considerations now have to be combined with their other terms in the pressure transport equation (see Equ. (14) in Fahr and Dutta-Roy) [12]. For the electron pressure as function of the streamline coordinate *s* one thus finds the following completed pressure transport equation for electrons:

$$\frac{d\ln P_e}{ds} = \frac{4}{3}\frac{d\ln B}{ds} + \frac{10D_{e0}}{U} - \frac{4\pi+6}{3}\cdot\frac{P_e}{\Pi}\frac{d\ln U}{ds}$$

where in the upper form of the pressure transport equation it has also now newly been taken into account that the change of the electron pressure is also influenced by the bulk velocity change due to the action of the partial pressure via the ratio $\Psi_e = P_e/\Pi$

As evident, a similar transport equation one has to expect for the protons in the analogous form:

$$\frac{d\ln P_i}{ds} = \frac{4}{3}\frac{d\ln B}{ds} + \frac{10D_{i0}}{U} - \frac{4\pi+6}{3}\cdot\frac{P_i}{\Pi}\frac{d\ln U}{ds}$$

Where, instead of electron-whistler wave diffusion, in case of ions the ion-Alfvén wave-driven velocity diffusion is taken into account.

In case of the actual heliosphere, when comparing heliospheric sound velocities c_S with Alfven velocities c_A yielding a ratio of $c_s/c_A \simeq 330/39$,one can tacitly get rid of the first terms on the right-hand side of the above differential equations (:::) and (:::), and thus the effects of the frozen-in magnetic fields connected with the conservation of magnetic particle moments can safely be neglected. If also the diffusion coefficients D_{e0} and D_{i0} could be assumed to vanish (i.e. a turbulence free plasma!), then the upper two differential equations can be simply added and lead to the solution of the total pressure Π which was already found earlier in the form $\Pi(s) = \Pi(s_0) \cdot [U(s_0)/U(s)]^{\frac{4\pi+6}{3}}$. If,however,turbulent wave-fields are becoming thermodynamically relevant, then this solution cannot be used, but the relevant consistent solution in that case must be found from the more complicated differential equation (???).

The Solution for the Bulk Plasma Flow

Taking Parker's solution for the postshock solar wind bulk velocity $U(s)$ (i.e. $l = 1$see Equ.(??)) we have:

$$U^2(\vec{r}) = (\frac{\rho_{IS}}{\rho_i})U_{IS}^2 \cdot [1 + 2\frac{\bar{z}}{\bar{r}^3} + \frac{1}{\bar{r}^4}]$$

where the positive z-axis, according to our convention here, is oriented into the tail direction. Evaluating, however, here U on the axial upwind streamline (i.e. anti-tail word) with $-z = r = s$, we thus obtain:

$$U^2(s) = (\frac{\rho_{IS}}{\rho_i})U_{IS}^2 \cdot [1 - \frac{2}{\bar{s}^2} + \frac{1}{\bar{s}^4}]$$

Where now the streamline element $\bar{s} = s/L$ normalized by the standoff distance L.

One can see,that exactly on the upwind axis at $s = L$ (i.e. $\bar{s} = 1$; at the heliopause!)from Parker's solution one finds $U(s = L)=0$ which inview of the relation found above $\Pi(s) = \Pi(s_0) \cdot [U(s_0)/U(s)]^{\frac{4\pi+6}{3}}$ wouldimply $\Pi(s = L) = \infty$.

At the stagnation point, however, the pressure of the heliospheric plasma should just compensate exactly the finite outer, interstellar pressure $\Pi(s = L) = \Pi_{IS}$. This would allow for an effective bulk velocity $U(s = L)$ near the stagnation point (i.e. triple point of a 3-d flow! see Baranov et al) [23]. according to,

$$U(s = L) = (\Pi(s_0)/\Pi_{IS})^{\frac{3}{4\pi+6}} \cdot U(s_0);$$

or to state it in Parker-conformal mathematics by a minimum distance given through [23].

$$U^2(s = L) = (\Pi(s_0)/\Pi_{IS})^{\frac{6}{4\pi+6}} \cdot U^2(s_0) = (\frac{\rho_{IS}}{\rho_i})U_{IS}^2 \cdot [\delta]$$

For the bulk velocity derivative with respect to s we find:

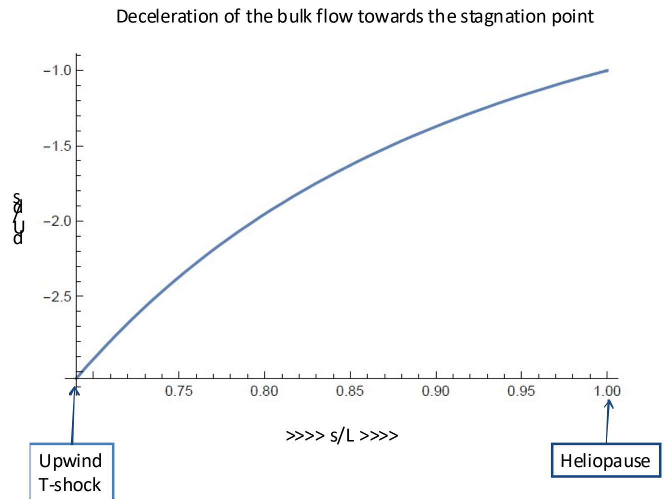
$$dU^2/d\bar{s} = 2UdU/ds = (\frac{\rho_{IS}}{\rho_i})U_{IS}^2 \cdot [4/\bar{s}^3 - 4/\bar{s}^5]$$

or

$$dU/d\bar{s} = \frac{2\sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2} \cdot [1/\bar{s}^3 - 1/\bar{s}^5]}{\sqrt{[1 - \frac{2}{\bar{s}^2} + \frac{1}{\bar{s}^4} + \delta]}}$$

and the term needed in the pressure transport equations:

$$d\ln U/d\bar{s} = \frac{1}{U}\frac{2\sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2} \cdot [1/\bar{s}^3 - 1/\bar{s}^5]}{\sqrt{[1 - \frac{2}{\bar{s}^2} + \frac{1}{\bar{s}^4} + \delta]}} = \frac{2[1/\bar{s}^3 - 1/\bar{s}^5]}{[1 - \frac{2}{\bar{s}^2} + \frac{1}{\bar{s}^4} + \delta]}$$



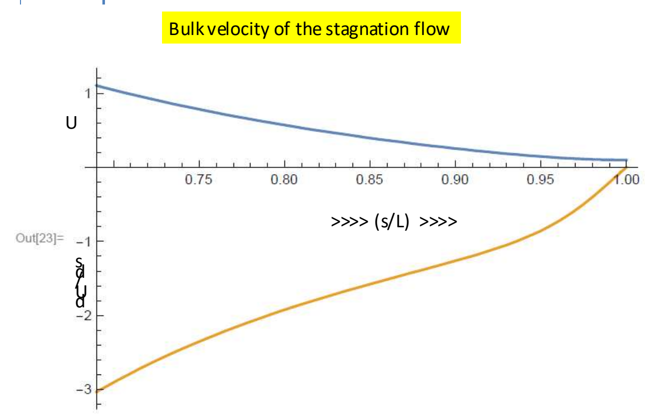


Figure 3: Plasma bulk velocity (blue) and its deceleration (yellow) along the stagnation line as function of the normalized distance s/L

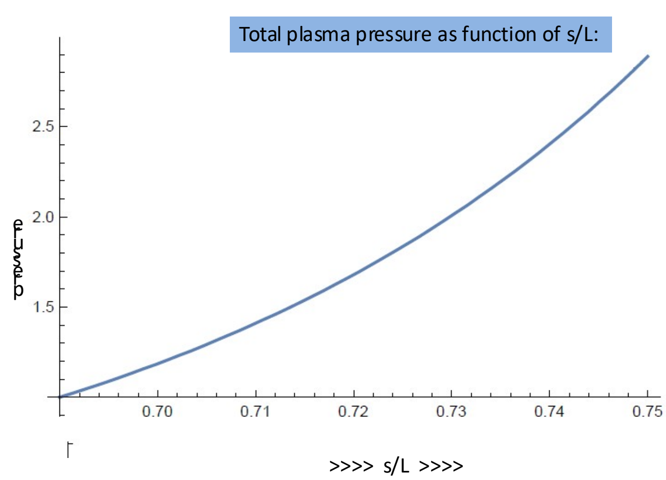


Figure 4: Total pressure Π(s)as function of distance s/L on the stagnation streamline for the turbulence-free plasma

Numerical Integration of the Completed Set of Differential Equations for the Turbulent Plasma Flow

Allowing for turbulences and the associated wave energy densities ϵ_{wi} and ϵ_{we} of the turbulence fields the completed thermodynamics of the two-fluid plasma flow does require (see Equ. (???)):

$$[\frac{2\pi+3}{2\pi}(P_e+P_i)-\frac{1}{2}\rho_i U^2+\epsilon_{wi}+\epsilon_{we}]\frac{dU}{ds}$$

$$=\frac{3}{2\pi}U\frac{d}{ds}(P_e+P_i)+[10D_{i,0}P_i+10D_{e,0}P_e]$$

Hereby now the change of the wave energy densities with the streamline element s is described by the following relation:

$$\frac{d}{ds}(\epsilon_{wi}+\epsilon_{we}) = \frac{1}{U}[10D_{i,0}P_i+10D_{e,0}P_e]$$

Furthermore, one has to respect and fulfill the two transport equations for the electron and the ion pressures separately given in the following forms:

$$\frac{d\ln P_e}{ds} = \frac{4}{3}\frac{d\ln B}{ds}+\frac{10D_{e0}}{U}-\frac{4\pi+6}{3}\cdot\frac{P_e}{\Pi}\frac{d\ln U}{ds}$$

and

$$\frac{d\ln P_i}{ds} = \frac{4}{3}\frac{d\ln B}{ds}+\frac{10D_{i0}}{U}-\frac{4\pi+6}{3}\cdot\frac{P_i}{\Pi}\frac{d\ln U}{ds}$$

where in the upper forms of the pressure transport equations it has been taken into account that the change of the electron/ion pressure is also influenced by the bulk velocity change according to the action of the partial pressures by means of the ratios $\Psi e,i = Pe,i/(P_e+P_i)$.

Neglecting further on the effect of the magnetic field change as already argued before (n.b.: $c_s \gg c_A$), we can arrange the two upper analogous equations for P_e and P_i in the following forms yielding the change of the partial pressures per step ds along the streamline in the form:

$$\frac{dP_e}{ds} = \frac{P_e}{U}[10D_{e0}-\frac{4\pi+6}{3}\frac{P_e}{P_e+P_i}\frac{dU}{ds}]$$

$$\frac{dP_i}{ds} = \frac{P_i}{U}[10D_{i0}-\frac{4\pi+6}{3}\frac{P_i}{P_e+P_i}\frac{dU}{ds}]$$

Hereby the functions U and dU/ds are given along the upwind streamline as known functions of s in the form:

$$U(\bar{s}) = \sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2}\cdot\sqrt{1-\frac{2}{\bar{s}^2}+\frac{1}{\bar{s}^4}}$$

$$dU/d\bar{s} = \frac{2\sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2}\cdot[1/\bar{s}^3-1/\bar{s}^5]}{\sqrt{[1-\frac{2}{\bar{s}^2}+\frac{1}{\bar{s}^4}]}}$$

Hence one obtains for the numerical integration the following four differential equations:

I:

$$[\frac{2\pi+3}{2\pi}(P_e+P_i)-\frac{1}{2}\rho_i U^2+\epsilon_{wi}+\epsilon_{we}]\frac{dU}{ds}$$

$$=\frac{3}{2\pi}U\frac{d}{ds}(P_e+P_i)+[10D_{i,0}P_i+10D_{e,0}P_e]$$

with the following additional relations to be simultaneously respected:

II:

$$\frac{d}{ds}(\epsilon_{wi}+\epsilon_{we}) = \frac{1}{U}[10D_{i,0}P_i+10D_{e,0}P_e]$$

III:

$$dP_e = \frac{P_e(s)}{\sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2}\cdot\sqrt{1-\frac{2}{\bar{s}^2}+\frac{1}{\bar{s}^4}}}[10D_{e0}-\frac{4\pi+6}{3}\frac{P_e(s)}{P_e(s)+P_i(s)}\frac{2\sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2}\cdot[1/\bar{s}^3-1/\bar{s}^5]}{\sqrt{[1-\frac{2}{\bar{s}^2}+\frac{1}{\bar{s}^4}]}}]ds$$

and

IV:

$$dP_i = \frac{P_i(s)}{\sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2}\cdot\sqrt{1-\frac{2}{\bar{s}^2}+\frac{1}{\bar{s}^4}}}[10D_{e0}-\frac{4\pi+6}{3}\frac{P_i(s)}{P_e(s)+P_i(s)}\frac{2\sqrt{(\frac{\rho_{IS}}{\rho_i})U_{IS}^2}\cdot[1/\bar{s}^3-1/\bar{s}^5]}{\sqrt{[1-\frac{2}{\bar{s}^2}+\frac{1}{\bar{s}^4}]}}]ds$$

Solutions of this system of coupled interdependent differential equations and the discussion of these solutions shall be presented in a forthcoming paper.

Conclusions

In earlier papers we have shown that classical monofluid MHD theory delivers straightforward and consistent MHD solutions for the magnetic field configuration and the plasma flow in the heliosheath, both for the upwind case, i.e. for streamlines approaching the region near the heliopause stagnation point, and for the downwind case, i.e. for streamlines leading into the heliospheric tail region (see Figure 2, taken from Nickeler, Goedbloed, Fahr) [23,24]. Here in this article we, however, do now demonstrate, those monofluid solutions infact cannot be accepted as valid solutions of the actually given problem in the heliosheath region, because it turns out that electrons beyond the solar wind termination shock develop their own independent pressures which are comparable with or even dominant over the proton pressures. This requires an authentic two-fluid representation of the plasma flow system in the heliosheath. Under these conditions namely the electron pressures become a dynamically relevant quantity which strongly co-influences the resulting plasma dynamics, i.e. an authentic two-fluid treatment of the plasma flow is definitely required here.

In order to be able to describe electrons and protons as independent, but dynamically coupled fluids, one, however, has to pay a look on the kinetic level of the underlying plasma system and has to derive separate kinetic transport equations for electrons and protons describing the evolution of their kinetic distribution functions along streamlines. When converting these equations into pressure transport equations anlogous to the method in Fahr and Dutta-Roy,[12]. one can arrive at independent solutions for the pressures of the electrons and the protons as functions of the streamline coordinate s[12]. In this paper here, we give solutions for the MHD plasma flow under the special condition that the electron pressure either dominates over or competes with the proton pressure. This in fact is shown to be the case immediately downstream of the solar wind termination shock, but at decreasing plasma bulk velocities downstream from the shock the electron pressures fall down and finally are comparable or even lower than proton pressures. As we can show in this paper, this phenomenon can be controlled with an additional differential equation describing the total pressure $\Pi = P_e + P_i$ and the actual and consistent turbulence energy densities ϵ_{wi} and ϵ_{we} [25-49].

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