Review Article

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# Pulsar Identification of Background Spectra of LIGO Data

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## Abstract

*By choosing the metric (called physical metric) in general relativity as the exact solution to the Einstein equation that fits the time delay data, one can determine the size and gravitational redshift on the surface of compact objects (neutron stars and black holes). The author shows that the physical metric is invariant by rotation. As a result, the frequencies of gravitational waves from pulsars are represented as n * f /$\sqrt{3}$ for pulsar frequency f and harmonics n. Based on this result, the author has identified potential pulsar candidates with gravitational wave spectra. This result will be critical in the study of gravitational redshift of compact objects.*

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## I Introduction

LIGO has observed many line spectra as gravitational waves from pulsar rotations, but have yet to declare their identity. The reason is the existence of gravitational redshift due to the copactness of neutron stars. The author shows that the gravitational redshift of the observed pulsars is $\sqrt{3}$, irrespective of its rotation frequency. The theoretical basis for this claim is presented in Section II and Section III provides the discussion on the effect of rotation. As a result, gravitational wave (gw hereafter) frequency, $f_{gw}$, becomes in terms of pulsar rotation frequency, f,

$$f_{gwn} = \frac{nf}{\sqrt{3}} \qquad n = 1,2,3, ...... \tag{1}$$

where n is the harmonics number. The formulae which include the effect of the pulsar rotation in Section III are intended for future use as more accurate data will become available. Using the table of publicly reported gw spectra[1], the identification of observed pulsars is presented. For the lower frequency part of the observed gw, the identification with n = 1, 2 is presented as Table 1 in Section IV, while for the higher frequencies, higher harmonics, n ≥4, will be used as Table 2 in Section V. Table 1b and Table 2b are for the alternative choices within 1 % accuracy for pulsar identification. A summary is given in Section VI.

## II Time delay experiment of Shapiro and the physical metric in general relativity

The spherically symmetric and static (SSS) metric is expressed as

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - e^{\mu(r)}r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

for a mass point $M$. Let us start with what Schwarzschild has discovered. Stating it in the form of S-lemma [2] (Schwarzschild lemma),

**Lemma 1** By coordinate transformation, $r' = r\,e^{\mu(r)/2}$, $g_{00}(r') = -g_{11}(r')^{-1} = 1 - r_s/r'$ is the exact solution of the Einstein equation, where $r_s = 2GM/c^2$ is the Schwarzschild radius.

In other words, the Schwarzschild metric and all its coordinate transformations are the exact solutions of the Einstein equation. Which one is the correct metric? It will be decided by comparing its prediction with observational data.

In order to discuss the time delay experiment of the solar system by Shapiro et al.[3,4], one can derive the integrals of the geodesic equation[6] for the light path,

$$\frac{dt}{dr} = \pm e^{(\lambda(r)-\nu(r))/2} / \sqrt{1 - J^2 e^{\nu(r)-\mu(r)}/r^2}, \tag{3}$$

in exact form, where J is angular momentum constsnt, an integration constant of the geodesic equation. Writing $J = r_0$ ($r_0$ being the impact parameter of the light path), the condition,

$$e^{\nu(r)} = e^{\mu(r)}, \tag{4}$$

Will yield the integral of Eq. (3) to give the formula for time delay by the gravity of the sun to be

$$\triangle t = 2r_s \ln(\frac{2r}{r_0}) \tag{5}$$

for the half of the trip. This formula is consistent with the most recent data with the accuracy of 1 in $10^5$ [4]. On the other hand, the prediction of the Schwarzschild metric gives [5].

$$\triangle t = 2r_s(\ln(\frac{2r}{r_0}) + \frac{1}{2}\sqrt{\frac{r-r_0}{r+r_0}}), \tag{6}$$

which spoils the agreement between the data and the prediction in the accuracy of 1 in $10^5$.
Writing

$$\omega = e^{\nu(r)} = e^{\mu(r)} \tag{7}$$

and

$$r' = r\ \omega^{\frac{1}{2}}, \tag{8}$$

where $r'$ is the Schwarzschild coordinate, the correct metric yields

$$\omega = 1 - \frac{r_s}{r\omega^{\frac{1}{2}}}. \tag{9}$$

Solving for $r_s$=$r$, one gets

$$\frac{r_s}{r} = \omega^{\frac{1}{2}}(1-\omega). \tag{10}$$

This function, $\omega = \omega(r)$, terminates from outside at

$$r = R_1 = \frac{3\sqrt{3}}{2}\ r_s = 2.60\ r_s \tag{11}$$

and

$$\omega = \omega_1 = \frac{1}{3}. \tag{12}$$

Incidentally, the other metric function becomes

$$e^{\lambda(r)} = (\frac{d}{dr}(r\omega^{\frac{1}{2}}))^2/\omega = (\frac{2\omega}{3\omega-1})^2. \tag{13}$$

This new metric is called the physical metric [6]. The physical metric is the exact solution of the Einstein equation due to S-lemma and ts the the data of time delay experiment accurately. It is the correct metric. In this metric, the speed of light in the direction perpendicular to the gravity that is the radial direction is unchanged from that of vacuum. It is remarkable that such a natural condition is required to fit the experimental test of general relativity. The radius, $R_1$, of Eq. (11) is called the extended horizon and represents the size of compact objects, neutron stars and black holes [7]. The gravitational redshift on the surface of compact objects is given by

$$1 + z = \frac{1}{\omega_1^{\frac{1}{2}}} = \sqrt{3}. \tag{14}$$

Eq. (11) gives the radius size of a neutron star of 1.4 solar mass to be 10.7 km, which is consistent with the observation. It suggests a linear proportionality between mass and radius of neutron stars. This should be tested by observation in the future. The observation of the radius size of the black hole at the center of the Milky Way by the Event Horizon Telescope [9] has been reported as (2.64 ± 0.18) * $r_s$, consistent with Eq. (11).

Eq. (5) has been derived by Parametrized Post Neutonian approximation (PPN) [10]. However, such an approach would lose the opportunity to find a correct metric in general relativity, as is done here.

## III Rotation with the Physical Metric

The rotation expressed by the Kerr metric[11] indicates an expansion of the radius as a result of rotation. However, for the physical metric one can derive from Eq. (10)

$$\frac{d\omega}{dr} = \frac{2\omega^{\frac{1}{2}}}{3\omega - 1}\frac{r_s}{r^2}.$$

This implies that

$$\frac{d\omega}{dr} = \infty$$

on the surface of compact objects (neutron stars and black holes), for which Eq (12) is valid. In other words, the gravity on the surface is infinitely large attractive force. As a result, the Kerr metric is not an appropriate metric for rotation of compact objects. The author cocludes that the physical metric is invariant. by rotation due to Eq. (4). Therefore, Eq. (11) for the size and Eq. (14) for the gravitational redshift on the surface are unchanged by rotation for the compact objects.

## IV Identification of the gw spectra of ligo with pulsar fre-Quencies

Based on the table of observed spectra of LIGO, one can find the numerical details of spectral frequencies accurately. By making the average of the measurements of the both instruments at Hanford and Livingston[1], the observed gw frequencies below 600 Hz are listed in the first column of Table 1a. The identi cation of pulsars with n =2 and 1 are listed in the other columns. A suggestion is to make a pulsar catalogue, ATNF, in the order of frequency[12]. The other choices of pulsar candidates for each spectrum within 1 % accuracy are shown in Table 1b, supplemented in the reference. The * signs in column 3 in Table 1b indicate the items shown in column 2 and 3 in Table 1a.

**Table 1a: The pulsar identi cation of obswerved gw frequency, fgw, below 600 Hz. f and fgwn stand for pulsar frequency and the calculated gw frequency of the n-th harmonics in Eq. (1). n is restricted to less than 2 here**

| $f_{gw}$ (Hz) | pulsar | f (Hz) | $f_{gw2}$ (Hz) | $f_{gw1}$ (Hz) |
|---|---|---|---|---|
| 0.25 | J1414-6802 | 0.216 | 0.249 | 0.25 |
| | J1939+10 | 0.433 | | |
| 3.40 | J1929+19 | 2.948 | 3.40 | 3.38 |
| | J1548-5607 | 5.85 | | |
| 6.03 | J1848+0351 | 5.223 | 6.03 | 6.00 |
| | J2043+2740 | 10.40 | | |
| 10.2 | J1844-0346 | 8.861 | 10.2 | 10.3 |
| | B2127+11B | 17.81 | | |
| 14.2 | J1856+0245 | 12.36 | 14.3 | 14.1 |
| | J1518+4904 | 24.43 | | |
| 35.6 | J2222-0137 | 30.47 | 35.2 | 35.5 |
| | J1748-2021E | 61.49 | | |
| 60.0 | J1745-0952 | 51.61 | 59.6 | 59.6 |
| | J0922-52 | 103.3 | | |
| 120 | J0922-52 | 103.3 | 119.3 | 119.3 |
| | J1207-5050 | 206.6 | | |
| 180 | J1824-2452B | 152.7 | 176.3 | 179.3 |
| | J1710+49 | 310.5 | | |

| | | | | |
|---|---|---|---|---|
| <330, 332> | J1721-2457 | 286.0 | 330 | 331 |
| | J1902-5105 | 573.9 | | |
| <501, 503> | J1701-3006F | 435.8 | 503 | |
| <508, 509> | J2256-1024 | 436.7 | 504 | |

**Table 1b: The option for pulsar candidates for fgw below 600 Hz**

| fgw (Hz) | n pulsar [f (Hz)] |
|---|---|
| 0.25 | 2 J2007+20 [0.2158]; J1414-6802 [0.2160]; J1901+0511 [0.217]*<br>1 J1749+16 [0.433]; J1939+10 [0.433]*; J1303-6305 [0.4335];<br>1 J1941+2525 [0.4336]; J2105+6223 [0.4339]; J2015+2524 [0.434] |
| 3.40 | 2 J1638-5226 [2.94]; J0519+54 [2.94]; J1929+19 [2.95]*; and 20 others[13]<br>1 J1548-5607 [5.85]*; J1907+05 [5.928]; J1624-4041 [5.957] |
| 6.03 | 2 B1557-50 [5.19]; J1843-0702 [5.22]; J1848+0351 [5.22]*; and 8 others[14]<br>1 J1837-0604 [10.385]; J2043+2740 [10.40]*; J1753-2240 [10.51] |
| 10.2 | 2 J1739-3023 [8.74]; J1844-0346 [8.86]*; J0821-4300 [8.87]; J0537-69 [8.88]<br>1 B2127+11B [17.81]*; J0609+2130 [17.95] |
| 14.2 | 2 J1856+0245 [12.36]*<br>1 J1829+2456 [24.384];<br>J1518+4904 [24.43]* |
| 35.6 | 2 J2222-0137 [30.47]*<br>1 J1748-2021E [61.49]*; J0537-6910 [62.03]; J2145-0750 [62.30] |
| 60.0 | 2 J1745-0952 [51.61]*<br>1 J1748-3009 [103.3]; J0922-52 [103.3]*; J1748-2446I [104.5] |
| 120. | 2 J1748-3009 [103.3]; J0922-52 [103.3]*; J1748-2446I [104.5]<br>1 J1823-3021F [206.2]; J1207-5050 [206.6]*; J0024-7204V [207.9];<br>1 J2322+2057 [207.97]; J1725-3853 [208.69];J0024-7204X [209.58] |
| 180 | 2 J1824-2452B [152.74]* |
| | 1 J1701-3006E [309.2]; J1836-2354B [309.4]; J1710+49 [310.5]*; |
| | 1 B0021-72B [311.5]; J1628-3205 [311.5] |
| <330, 332> | 2 B1639+36B [283.4]; J1721-2457 [286.]*; B0021-72I [286.9]; |
| | 2 J0024-7204R [287.3]; J0751+1807] |
| | 1 J1902-5105 [573.9]*; J1748-2446P [578.5] |
| <501, 503> | 2 J0218+4232 [430.5]; J1701-3006F [435.8]*; J2256-1024 [436.7] |
| <508, 509> | 2 J2256-1024 [436.7]*; J1748-2446L [445.5] |

The gw spectra at 60, 120 and 180 Hz could be higher harmonics of a single pulsar rotation, if not the local electricity origin. Using Eq. (1), J1745-0952 with pulsar frequency, 51.61 Hz, will give gw frequencies, 59.6, 119.2 and 178.8 Hz for n = 2, 4 and 6 respectively.

## V High frequency gw by higher harmonics of pulsar rotation

Above 800 Hz for gw frequency, one has to use higher harmonics of pulsar rotation with n > 2. These are listed in Table 2a, and Table 2b is for multiple candidates for pulsars within 1 % accuraccy.

**Table 2a: Higher gw frequency above 800 Hz are identi ed with higher harmonics of pulsar frequency**

| $f_{gw}$ (Hz), observed | pulsar | f, pulsar freq. (Hz) | n | fgwn (Hz), calculated |
|---|---|---|---|---|
| <992, 999> | J0218+4232 | 430.5 | 4 | 994 |
| 1,010 | J2256-1024 | 436.7 | 4 | 1,009 |
| 1,457 | B1957+20 | 622.1 | 4 | 1,437 |
| | J1946+3417 | 315.4 | 8 | 1,457 |
| <1,469, 1,472> | B1937+21 | 641.9 | 4 | 1,482 |
| | J0740+41 | 318.6 | 8 | 1,472 |
| 1,923 | J1124-3653 | 414.9 | 8 | 1,916 |
| <1,941, 1955> | J1858-2216 | 420.2 | 8 | 1,941 |

| | | | | |
|---|---|---|---|---|
| 2,385 | J0610-2100 | 259.0 | 16 | 2,393 |
| <2,422, 2,424> | J1748-2021F | 263.6 | 16 | 2,435 |
| <2,878, 2,881, 2,884> | B1957+20 | 622.1 | 8 | 2,873 |
| | B0021-72H | 311.5 | 16 | 2,878 |
| 3,617 | J0557+1550 | 391.2 | 16 | 3,614 |
| 3,724 | J1518+0204C | 402.6 | 16 | 3,719 |
| 5,165 | J1545-4550 | 279.7 | 32 | 5,168 |
| <5,959, 5,963.> | B1937+21 | 641.9 | 16 | 5,930 |
| | J1012-4235 | 322.5 | 32 | 5,958 |
| <6,049, 6,052> | B0021-72N | 327.4 | 32 | 6,049 |

**Table 2b: The option for pulsar canditates for fgw above 800 Hz**

| $f_{gw}$ | n pulsar [f (Hz)] |
|---|---|
| <992,999> | 4 J1731-1847 [426.5]; J1017-7156 [427.6]; J0218+4232 [430.5]*; |
| | 4 J1701-3006F [430.8]; J2256-1024 [436.7] |
| 1,010 | 4 J1701-3006F [435.8]; J2256-1024 [436.7]* |
| 1,457 | 4 J1957+20 [622.1]* |
| | 8 J1518+0204E [314.3]; J1946+3417 [315.4]*; and 7 others[15] |
| <1,469, 1472> | 4 B1937+21 [641.9]* |
| | 8 J1946+3417 [315]; J1640+2224 [316]; J0308+74 [316]; J1614-2230 [317]; |
| | 8 J0614-3329 [318]; J0740+41 [319]*; J2214+3000 [321]; J1641+3627D [321] |
| 1,923 | 8 J1124-3653 [415]; J1342+2822B [419]; J1858-2216 [420]; J2043+1711 [420] |
| <1941, 1955> | 8 J1342+2822B [419]; J1858-2216 [420]*; J2043+1711 [420]; J0024-7204W [425] |
| 2,385 | 16 J1835-3259A [257.1]; J0610-2100 [259.0]* |
| <2,422, 2,424> | 16 J1748-2021F [264]*; J1923+2515 [264]; J1652-48 [264]; J1905+0400 [264] |
| <2,878, 2881, 2884> | 8 B1957+20 [622.1]* |
| | 16 J1701-3006E [309]; J1836-2354B [309]; J1710+49 [311]; B0021-72H [311]*; |
| | 16 J1628-3205 [312]; J1816+4510 [313]; J1905+0154 [313] |
| 3,617 | 16 J0101-6422 [388.6]; J1311-3430 [390.6]; J1909+21 [390.6]; |
| | 16 J0557+1550 [391.2]*; J1342+2822A [392.9]; J0251+26 [393.7] |
| 3,724 | 16 J1641+3627E [402.1]; J1518+0204C [402.6]*; J1748-2446Z [406.1] |
| 5,165 | 32 J1514-4946 [279]; J1545-4550 [280]*; J1748-2446M [280]; and 7 others[16] |
| <5.959, 5,963> | 16 B1937+21 [641.9]* |
| | 32 J2214+3000 [320.6]; J1641+3627D [320.7]; J1125-5825 [322.4]; |
| | 32 J1012-4235 [322.4]*; J1400-1438 [324.2] |
| <6,049, 6,052> | 32 J1400-1438 [324]; J1536-4948 [325]; J0619-0200 [327]; |
| | 32 J1807-2459A [327]; B1821-24A [327]; B0021-72N [327]*; J0023+0923[328] |

The LIGO should determine the frequencies of line spectra of gw, fgw, and publish the result. Then, the accurate identication of gw frequency with Eq. (1) result. The doublets and triplets in Table 1 and 2 might indicate the detailed mechanism of gw emission from rotating pulsars. The directional orientation of gw relative to the rotational axis of the neutron star may be the cause of the multiplets.

## VI Summary

By choosing the correct metric among the possible exact solutions of the Einstein equation that  ts the time delay experiment of the solar system, the physical metric, one can determine the size and the gravitational redshift of compact objects, black holes and neutron stars. The effects of the rotation for neutron stars are shown to be within 1 % accuracy. As a result, all observed background gw frequencies are identi ed with the observed pulsar rotation, with the harmonics parameter, $n \leq 2$ below 600 Hz and $n \geq 4$ above 800 Hz. The detailed determination of the gw frequencies in the future will be crucial for clarifying the unknown parameters, neutron star masses and mass distribution parameter, as well as the unique identi cation of the pulsar. Once you have determined the original location of a gw peak, one can study the effect of the Earth on gw by measuring the day and night difference, and study the peak time variation. The physical metric provides the correct basis to systematically study gw from pulsars.

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14. ATNF, 8 others are: J1954+2407 [5.171]; B1338-62 [5.172]; J1743-3153 [5.179]; J1612-49 [5.190]; // J1752-2410 [5.235]; J1911+10 [5.239]; J1755-0903 [5.244]; J1913+1050 [5.261]
15. ATNF, 7 others are: J1816+4510 [313.2]; J1905+0154A [313.2];// J1640+2224 [316.1]; J0308+74 [316.5]; J1614-2230 [317.4]; J0614-3329 [317.6]; J0740+41 [318.6]
16. ATNF, 7 others are: J1902-70 [277.8]; J2234+06 [277.8]; J1600-3053 [277.9]; J1903-7051 [277.9]; J1701-3006B [278.3]; // J1850+0124 [280.9]; J1933-6211 [282.2]