Research Article

Current Research in Statistics & Mathematics

Proof That the Real Part of All Nontrivial Zeros of Riemann Zeta Functions is 1/2

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Submitted: 2023, Aug 02; **Accepted**: 2023, Sep 01; **Published**: 2023, Oct 10

Citations: Xiaohui, L. (2023). Proof that the Real Part of All Nontrivial Zeros of Riemann Zeta Functions is ½. *Curr Res Stat Math, 2*(1), 38-41.

Abstract
By analytically extending the Euler Zeta function, the Riemann Zeta function is obtained. The Riemann Zeta function has zero points, which are trivial and non-trivial, respectively. By analyzing the internal structure of the Riemann Zeta function, it was found that the key to the value of 0 in the complex plane of the Riemann Zeta function is sin(sπ)=0, thus proving the validity of the Riemann hypothesis. That is, the real parts of all non trivial zeros of the Riemannian Zeta function are on the complex plane 1/2.

Keywords: Trivial Zero, Nontrivial Zero, Analytic Continuation

Riemann first starts from a prime number relationship proposed by Euler

$$\prod \frac{1}{1-\frac{1}{\mathrm{p}^{s}}} = \sum \frac{1}{\mathrm{n}^{s}}$$

In the above equation, p runs through all prime numbers ,And n runs through all natural numbers,Then the following Riemann functions $\zeta(s)$ are constructed through Analytic continuation:

$$2\mathrm{sins}\pi \cdot \Gamma(s-1) \cdot \zeta\ (\mathrm{s}) = i\int_{-\infty}^{+\infty} \frac{(-x)^{s-1}dx}{e^{x}-1} \qquad (\ s_{\mathrm{Re}} \neq 1\ ) \qquad (1)$$

This integral expression is analyzed on the whole Complex plane except that it is meaningless at

$s=1$.According to the integral expression (1) above, it can be proven that $\zeta(s)$ atisfy algebraic relationship:

$$\zeta\ (\mathrm{s}) = 2^{s}\pi^{s-1}\sin\frac{s\pi}{2}\Gamma(1-s)\zeta\ (1\text{-}\mathrm{s}) \qquad (2)$$

Note: In $\sin\frac{s\pi}{2}$ of the above equation,When the value of s is an even number such as 2, 4, 6, etc., the value of $\zeta(s)$ is zero,At this point, s is the trivial zero of the $\zeta(s)$.

Furthermore, it was found that in equation (2), the factors that cause the $\zeta(s)$ to be zero are more than $\sin\frac{s\pi}{2}$,in fact in $\zeta(1-s)$, if s in $\sin\frac{(1-s)\pi}{2}$ is equal to an odd number such as 3, 5, etc., it can also make the value of the $\zeta(s)$ equal to zero,This is a new discovery about the trivial zero point.

Next, let's analyze the situation when s is a complex number.

On the right side of equation (2),it can be seen that the product of $\sin\frac{s\pi}{2}$ in $\zeta(s)$ and $\sin\frac{(1-s)\pi}{2}$ in $\zeta(1-s)$ determines whether the value of the $\zeta(s)$ is zero.if the product of $\sin\frac{s\pi}{2}\times\sin\frac{(1-s)\pi}{2}$ is zero, then the value of $\zeta(s)$ will also be zero accordingly ,the product values given below:

$$(\sin\frac{s\pi}{2}) \times (\sin\frac{(1-s)\ \pi}{2}) = \frac{1}{2}\sin(s\pi) \qquad (3)$$

Then,starting from this $(\sin\frac{s\pi}{2})\times(\sin\frac{(1-s)\ \pi}{2}) = \frac{1}{2}\sin(s\pi)$,we will search for non trivial zeros.

The method is as follows:

Let $s=c+ib$ on the Complex plane be the nontrivial zero point of $\zeta(s)$, s_{Re} is the real part of s ,and $s_{\mathrm{Re}}=c$ (It has been proven that 0< c < 1), ib is the imaginary part of s.

In equation (3), as long as $\sin(s\pi)=0$,then $\zeta(s)=0$

Now expand $\sin(s\pi)$:

$$\sin(s\pi)=\sin(c\pi+ib\pi)=\sin c\pi\cdot\cos ib\pi+\cos c\pi\cdot\sin ib\pi \qquad (4)$$

Because $\sin c\pi\neq 0$,So equation (4) can be transformed as follows:

$$\sin c\pi\cdot\cos ib\pi+\cos c\pi\cdot\sin ib\pi=\frac{\cos ib\pi+\cot c\pi\cdot\sin ib\pi}{\frac{1}{\sin c\pi}} \qquad (5)$$

Because: the value of $\zeta(s)$ at $s=c+ib$ is zero.

So: the molecules in equation (5) must be zero,that is:

$$\cos ib\pi+\cot c\pi\cdot\sin ib\pi=0 \qquad (6)$$

Next, analyze equation (6) as follows:

If $\sin ib = 0$,then $\cos ib \neq 0$

then $\cos ib\pi + \cot c\pi \cdot \sin ib\pi \neq 0$,then $\zeta(s) \neq 0$,obviously contradictory.

So it can be inferred that: $\sin ib \neq 0$.

So equation (6) can be transformed as follows:

$$\cot c\pi = -\frac{\cos ib\pi}{\sin ib\pi} = -\cot ib\pi$$

It can be seen that the symbols on the left and right sides of equation (7) are opposite, and the left Trigonometric functions has no imaginary part, and the right Trigonometric functions has no

real part, then immediately we can get:

$$\cot c\pi = -\cot ib\pi = 0$$

So:

At this point: $(\sin\frac{s\pi}{2})\times(\sin\frac{(1-s)\ \pi}{2}) = \frac{1}{2}\sin(s\pi) = 0$, which is: $\zeta(s) = 0$

Below is an analysis of the value of c:

When $\cot c\pi = 0$, then $c\pi = \left(\frac{1}{2} + n\right)\pi, (n \in N)$

We know that the range of values for the real part c is $0 < c < 1$,We can directly obtain: $c = \frac{1}{2}$

That is, the real part of the non trivial zero of $\zeta(s)$ is: $s_{\text{Re}} = c = \frac{1}{2}$

So the non trivial zero of $\zeta(s)$ is:

$$s = \frac{1}{2} + ib$$

Now,we can immediately conclude that the Real Part of All Non trivial Zeros of Riemann Zeta Functions is 1/2.

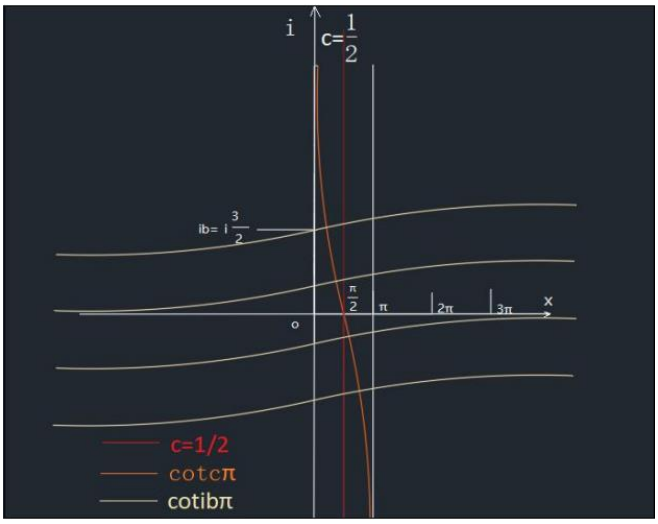
When the value of $\zeta(s)$ is zero,So the value of the imaginary part ib should be $\cot ib\pi = 0$,

When $\cot ib\pi = 0$, then:

$$ib\pi = i\left(\frac{1}{2} + m\right)\pi, (m \in N)$$

Example:

When $m=1$, $ib\pi = i\frac{3}{2}\pi$,at this time, the function image of $\cot c\pi = -\cot ib\pi = 0$ on the Complex plane is:



(a)

Below is a comparison table for non trivial zero values.

Modern Computed Values of Non trivial Zeros (c+ib)	m	c+ib=1/2+i(1/2+m)	error value
1/2+14.1347251i	14	1/2+14.5i	-0.3652749i
1/2+21.0220396i	21	1/2+21.5i	-0.4779604i
1/2+25.0108575i	25	1/2+25.5i	-0.4891425i
1/2+30.4248761i	30	1/2+30.5i	-0.0751239i
1/2+32.9350615i	32	1/2+32.5i	0.4350615i
1/2+37.5861781i	37	1/2+37.5i	0.0861781i
1/2+40.9187190i	40	1/2+40.5i	0.4187190i
1/2+43.3270732i	43	1/2+43.5i	-0.1729268i
1/2+48.0051508i	48	1/2+48.5i	-0.4948492i
1/2+49.7738324i	49	1/2+49.5i	0.2738324i
1/2+52.9703214i	52	1/2+52.5i	0.4703214i
1/2+56.4462476i	56	1/2+56.5i	-0.0537524i
1/2+60.8317785i	60	1/2+60.5i	0.3317785i
Note: π≈3.1415926			

(b)

From Figure (b), it can be seen that the error between the non trivial zero values calculated in this paper and the 13 non trivial zero values currently calculated is relatively small.the reason for the error is that modern calculations estimate the results of Π (x) and Li (x) functions, while in this paper, non trivial zero values are calculated using the exact value of ib=i(1/2+m).

Conclusion

The Real Part of All Non trivial Zeros of Riemann Zeta Functions is 1/2,the Riemann hypothesis holds.

References

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