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Research Article

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Nonthermal Messages from Thermal Planetary Atmospheric Systems

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Abstract
Light atmospheric gas constituents tend to evaporate from the planetary gravitational fields. The point is that not only the uppermost atmospheric layer contributes to this gas escape, but the lower layers contribute their share as well and give the outcoming particle flow a nonthermal, non-Maxwellian character. In this article we do study the outflow of hydrogen atoms from a planetary oxygen atmosphere assumed to be one-dimensionally stratified by the action of the planet's gravitational field. This outflow is modified by local elastic collisions of upwards flying H-atoms with the heavy major atmospheric background constituent, as in case of the terrestrial atmosphere, the monoatomic oxygen atoms. This shock-modulation of the upwards particle flow produces nonthermal kinetic features of the particle distribution which we want to decribe. Since angle-integrated elastic collision cross sections are velocity-dependent, falling off with the velocity v like (1/v), the occuring collision-modulation of the H-atom flow does change the kinetic velocity profile of the escaping H-atoms. Deeply down in the lower atmophere the local H-atoms, like the O-atoms as well, are in thermodynamical equilibrium characterized by Maxwell Boltzmann distributions with a common temperature $T_H = T_O$*. Nevertheless, at the upper exobase border of the atmosphere the resulting H-atom escape flow is shown to be a non-Maxwellian, non-equilibrium flow with non-thermal escape-relevant properties. We describe this collisional modification of the H-escape flow and can quantify the upcome of kinetic non-equilibrium features like power laws in the H-distribution function. Thus, as we demonstrate in this article, this collisional modulation effect via velocity-dependent collision cross-sections acts as a typical process to convert equilibrium distributions into non-equilibrium distribution functions. On the basis of this new kinetic theoretical approach we then calculate the effective escape flux of H- atoms to open space, demonstrate its nonthermal character, and quantify its difference with respect to the classical Jeans escape value. We find that with the classical Jeans formula one slightly overestimates the actual escape flux by an amount that varies between 30 - 80 percent, depending on the temperature of the lower atmosphere of the planet which varies with the solar 10.7-cm- radioflux.*

Keywords: Nonthermal Distributions, Kinetic Theory, planetary Escape, Hydrogen Loss, Planetary Atmospheres

Introductory Motivation: Superpositions of Gaussians with Statistically Weighted Widths

In an earlier paper [1] we have shown that a series of nonthermal distribution functions, like especially Kappa-functions [2, 3], can be understood as originating from superpositions of thermal distributions with different stochastic weights. Especially we could show that the superposition of a series of stochastic processes will give rise to power-law distribution functions, a kind of degenerated Kappa functions with small kappa indices of $k \geq 1.5$. Thereby we could show that the v^{-5} - power law distributions, prominently appearing in heliospheric and heliosheath physics [4] can arise from a collection of Poisson processes, from a collection of Gaussian distributions, or from a collection of different shock-accelerated distributions. Though this proof indicated how nonthermal distributions in plasma physics easily can arise from superpositions of thermal distributions, the argumentation given there was, however, strongly based on theoretical physics and stochastic principles, without a deeper look on the physical scenarios that have to arrange in fact such stochastic superpositions. Here in this paper we shall, however, start from a physically well-posed problem connected with a planetary atmosphere stratified by the action of the planet´s gravitational field and connected with locally thermal equilibrium distribution functions, and will demonstrate that the collision-modulated escape flow from below, arriving at the top level of the atmosphere, i.e. the exobase, does show clearly pronounced signatures of a non-thermal kinetic distribution function.

The Thermosphere-Modified Escape Flow at The Planetary Exobase

We are interested in describing the particle outflow from the top of an atmospheric layer, one-dimensionally stratified by the earth´s gravity between coordinates $x = x^0$ (base of thermosphere) and $x = x^1$(exobase). Below the top layer at x^1 the gas, especially the heavy component, the O-atom gas, is kept more or less under thermal equilibrium conditions with a common local temperature T (x) as function of x, and the particle distribution functions of the atomic gases can be assumed to be local Maxwellians f(v, x) = Max (v, T(x)). From below x_1 particles like H- atoms, distributed according to such a local Maxwellian, are emitted towards the top layer, but before arriving there, they will undergo elastic collisions with other gas particles, like, in case of the earth, especially the dominant O- atoms. What comes out under such conditions is a "sheath-modified" H - atom outflow with a collisionally modified, transformed, non-Maxwellian distribution function. This latter function finally determines the effective H-escape from the atmosphere, and it is just the kinetics of this function which we want to describe in the forthcoming paper.

Taking a 1d-structured atmosphere with an H/O - gas density distribution given by $m_{H,O}(x) = m_{H,O}(x) \exp[-(x-x_0)/S_{H,O}]$, with a height-coordinate x, and with the atmospheric scale heights $S_{H,O} = kT(x)/m_{H,O}g$ specific for *H*- and *O*- atoms, where g denotes the gravitational acceleration within this layer, one obtains a local particle emissivity (j_H) into the direction ϑ of

$$\vec{j}_H(v,\vartheta)dvd\vartheta = n_H(x)[\vec{v}\cdot(\frac{m_H}{kT(x)})^{3/2}\exp[-\frac{m_H v^2}{kT(x)}]v^2 dv\cos\vartheta d\vartheta]$$

It could appear as suggested to introduce here as an appropriate velocity normalization $v^2_{H,x}=kT(x)/m_H$ and use the normalized velocity $w = v/v_{H,x}$ furtheron. This then would lead to

$$\vec{j}_H(v,\vartheta)dwd\vartheta = n_H(x)[\vec{w}\cdot\exp[-w^2]w^2 dw\cos\vartheta d\vartheta$$

where, however, this normalization of *v* unfortunately implies a mixing of coordinate dependencies on *v* and on *x*, since $T = T(x)$ must be taken as an x-dependent sheath temperature. This, however, is a hindrance for the integration procedure to come, and is the reason why we better stay with the first expression.

Now one has to pay attention to the fact that the upward flux (v, ∂) ($\vec{j}_H$) $(v, \vartheta \simeq 0°)$ of H-atoms originating at a coordinate x, before it reaches the top layer at x = x1, is reduced on its way up due to elastic collisions with the dominant gas constituent, in case of the Earth's atmosphere, the O-atoms. This reduction can be described by a transmission function Tr(x, v), assuming that the colliding, low-mass H-atoms by $H - O$ - collisions are completely redistributed to other direction s $\vartheta' \gtrless \vartheta$, i.e. representing in essence a loss for $\vec{H}$- atoms of the flow (j_H) (v,∂) which come along the upward direction , and hence one obtains a transmission given by:

$$Tr(x,v,\vartheta) = \exp[-\sigma_O(v)\int_x^{x_1} n_O(z)\frac{dz}{\cos\vartheta}]$$

Where $\sigma_0(v)$ denotes the angle-averaged elastic collision cross section between O and H- atoms at a relative velocity $v_{rel} \simeq v_H = v$. This cross section must be described as a type of a polarization cross section with a central interaction potential $V_{H,O}(r)$~r-4 (i.e. Maxwell model!) between the collision partners (polarized atomic shell), effectively leading to the following velocity-dependence $\sigma_0(v) = \sigma_0(v_0).(v/v_0)^{-1}$. Here in case of H-O-atom collisions a reference cross section of σ_0 ($v_0 = \sqrt{(kT/_0/m_H)} = 3.10^{-17}cm^2$ or $\sigma_0=8.10^{-17}cm^2$ can be used [5, 6], or, when averaged over the scattering angle, of $\sigma_0 = 10^{-15}cm^2$ [7, 8]. Thus alltogether we obtain for velocities $v \geq v_0$ the following transmission function:

$$Tr(x,v) = \exp[-\sigma_O\cdot(\frac{v_0}{v})^1 n_O(x)\int_x^{x_1}\exp[-\frac{(z-z_0)}{S_O}]\frac{dz}{\cos\vartheta}] =$$
$$\exp[-\sigma_O\cdot(\frac{v_0}{v})^1 n_O(x) S_O[\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]$$

One may keep in mind that the general validity with respect of the zenith inclination angle ∂ only is serious within a small range of inclination values $\vartheta \leq \vartheta_c \simeq 30^0$, since our one-dimensional atmospheric approach requires limitations due to the sphericity of the real planetary atmosphere.

This then leads to the following total H-atom outflow $J_H(v,\vartheta)$ upwards from the top layer at $x = x_1$, say the exobase :

$$\vec{J}_H(v,\vartheta)dvd\vartheta = \frac{1}{x_1 - x_0}\int_{x_0}^{x_1}\frac{dx}{\cos\vartheta}n_H(x)\cdot[\vec{v}(\frac{m_H}{kT(x)})^{3/2}\exp[-\frac{m_H v^2}{kT(x)}]\cdot$$
$$\exp[-\sigma_O(\frac{v_0}{v})^1 n_O(x) S_O[\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]v^2 dv\cos\vartheta d\vartheta$$

When simplified with $v^2_x = kT(x)/m_H = v_0^2 \cdot (T_0/T(x))$, where $v_0 = \sqrt{(kT_0/m_H)} = 5.8$km/s≃,0.5$v_{esc,H}$. the exobasic value of the escape velocity, denotes the mean thermal velocity of H –atoms at the lower most layer at $x=x_0$, then leads to the following expression:

$$\vec{J}_H(v,\vartheta)dvd\vartheta = \frac{1}{x_1 - x_0}\int_{x_0}^{x_1}dx\cdot n_H(x)[\vec{v}\cdot(\frac{1}{v_0})^3(\frac{T(x)}{T_0})^{3/2}\exp[-\frac{v^2}{v_0^2}\frac{T(x)}{T_0}]\exp[-\sigma_O(\frac{v_0}{v})^1 n_O(x) S_O$$
$$[\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]v^2 dvd\vartheta$$

or finally with w = v/v0 given in the form:

$$\vec{J}_H(w,\vartheta)dwd\vartheta = v_0\frac{1}{x_1-x_0}\int_{x_0}^{x_1} dx \cdot n_H(x)[w \cdot (\frac{T(x)}{T_0})^{3/2}\exp[-w^2\frac{T(x)}{T_0}]\exp[-\sigma_O\frac{n_O(x)}{w}S_O \cdot \quad \# \\ [\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]w^2dwd\vartheta$$

Atmospheric conditions for a standard terrestrial atmosphere between heights of x_0 = 200km and x_1 = 500km:

We take as a standard atmosphere the one at 14. ^{00}h day time for medium solar irradiance conditions $F_{10.7}$ = 150. According to CIRA [9], we then have to use the following input numbers:

$$n_H(x) \simeq n_{H0} = 10^4 cm^{-3}$$

$$n_O(x) = n_{0,o}\exp[-(x-x_0)/S_0] = 10^{10} \cdot \exp[-(x-x_0)/50]cm^{-3}$$

Oxygen Scale height: S_O=50km

$$T(x) = T_0 + \frac{T_1-T_0}{x_1-x_0}\cdot(x-x_0) = T_0 + \frac{\Delta T}{\Delta x}\cdot(x-x_0)$$

with

T_0 = 700K

and

T_1 = 700K

and:

ΔT = 700K

Δx = 300km

To study at this, occasion the influence of the given temperature profile in the atmospheric layer, we also shall compare the upcoming results with results for an alternative atmosphere which, in contrast to the upper CIRA-typical one, has an inverted temperature profile with the highest temperature T = T_1 at x = x_0 and the lowest temperature T = T_0 at the top layer x= x_1. The temperature profile in this alternative atmosphere (a corona-like profile) would thus be given in the following form:

$$T^*(x) = T_1 - \frac{T_1-T_0}{x_1-x_0}\cdot(x-x_0) = T_1 - \left|\frac{\Delta T}{\Delta x}\right|\cdot(x-x_0) \quad \#$$

Finally with these above standard input numbers we obtain the following expression for the regular atmosphere:

$$\vec{J}_H(w,\vartheta)dwd\vartheta = v_0 n_{H0}\frac{1}{x_1-x_0}\int_{x_0}^{x_1} dx \cdot [w \cdot (\frac{T_0+(\Delta T/\Delta x)(x-x_0)}{T_0})^{3/2}\exp[-w^2\frac{T_0+(\Delta T/\Delta x)(x-x_0)}{T_0}] \quad \# \\ \exp[-\sigma_O\frac{n_O(x)}{w}5\exp[-(z-4)][\exp(-\frac{x}{50\cos\vartheta}) - \exp(-\frac{500}{50\cos\vartheta})]]w^2dwd\vartheta$$

or numerically using the value given by Massey (1968) with σ_O =8 • $10^{-17}cm^2$ and introducing the quantity $\Delta = \Delta T/T_0\Delta x$ resulting in:

$$\vec{J}_H(w,\vartheta)dwd\vartheta = v_0 n_{H0}w^3dwd\vartheta\frac{1}{x_1-x_0}\int_{x_0}^{x_1} dx \cdot (1+\Delta(x-x_0))^{3/2}\exp[-w^2(1+\Delta(x-x_0)) \quad \# \\ \exp[-\frac{4}{w}\exp[-(x-x_0)/S_O] \cdot (\exp(-\frac{x}{50\cos\vartheta}) - \exp(-\frac{10}{\cos\vartheta}))]$$

When introducing the normalized integration variable by z = x/S_O, one finally obtains:

$$\vec{J}_H(w,\vartheta) = v_0 n_{H0}\frac{S_O}{x_1-x_0}w^3\int_4^{10} dz \cdot (1+\Delta(z-4))^{3/2}\exp[-w^2(1+\Delta(z-4)) \cdot \quad \# \\ \exp[-\frac{4}{w}\exp[-(z-4)] \cdot (\exp(-\frac{z}{\cos\vartheta}) - \exp(-\frac{10}{\cos\vartheta}))]$$

where the quantity Δ evaluates to:

$$\Delta = \Delta T/T_0\Delta x = \frac{(1400-700)}{700\cdot(300/50)} = \frac{1}{6} = \frac{S_O}{x_1-x_0} \quad \#$$

To study the above considered, collision-caused modulation effect under a bit wider perspective, we also treat here quantitatively the modulation effect in a "non-standard" atmosphere with the inverted "corona-like" temperature profile given by Equ.(A), already discussed further above and, instead of the flux $\vec{J}_H(w,\vartheta)$, now we find an alternative flux $\vec{J}^*_H(w,\vartheta)$

This flux $\vec{J}^*_H(w,\vartheta)$, resulting for the alternative atmosphere (inverted temperature profile) T*(x)!) is given by:

$$\vec{J}^*_H(w,\vartheta) = v_1 n_{H0}\Delta \cdot w^{*3}\int_4^{10} dz \cdot (1-\Delta(z-4))^{3/2}\exp[-w^{*2}(1-\Delta(z-4)) \cdot \qquad \#$$
$$\exp[-\frac{4}{w^*}\exp[-(z-4)] \cdot (\exp(-\frac{z}{\cos\vartheta}) - \exp(-\frac{10}{\cos\vartheta}))]$$

pronouncing the phenomena of a non-equilibrium distribution function even stronger as in case of the standard atmosphere, where, however, in this latter case the normalized velocity now is given by $w^{*2} = 2kT_1/m_H$.

Main Results

First here we look at results obtained for the standard terrestrial atmosphere.

According to the expression (??) derived above for the upward hydrogen flux at $x = x_1 = 500km$ one obtains the flux values $J_H(w,\vartheta = 0)$ shown in **Figures 1** and **2**. While in **Figure 1** the flux values themselves are plotted versus the normalized velocity w = v/v0, in Figure 2 we have plotted the logarithm of these fluxes, because this type of plot more clearly manifests the nonthermal characteristic of the function $J_H(w,\vartheta = 0)$, since *w*- power law characteristics then show up in straight-lines, i.e. linear dependences on w.

As one can see, beyond the velocity w = 2. 5 this kind of power law spectrum seems to come up in the expression (B) for $J_H(w,\vartheta = 0)$.

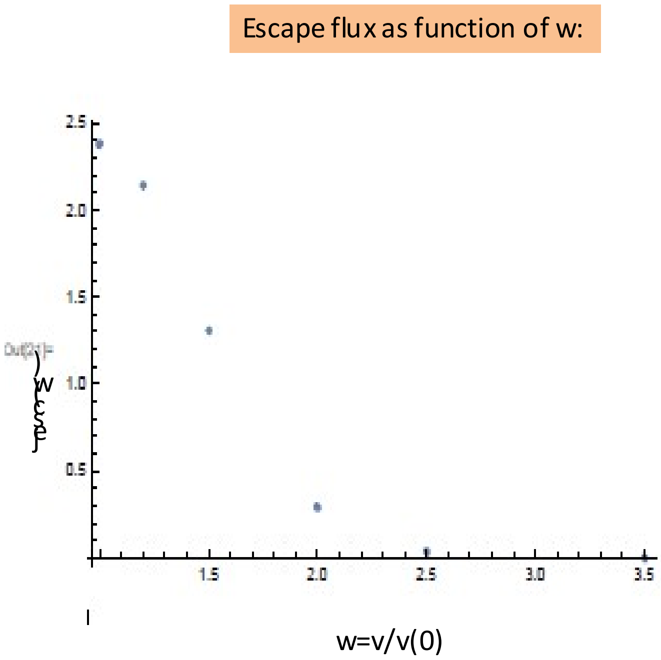

Figure 1: The upward hydrogen flux at $x = x_1$ (exobase) as function of the normalized velocity $w = v/v_0$ for the standard atmosphere

More impressive this power law feature turns up in a logarithmic display of the flux function $J_H(w,\vartheta)$ which is given in Figure 2.

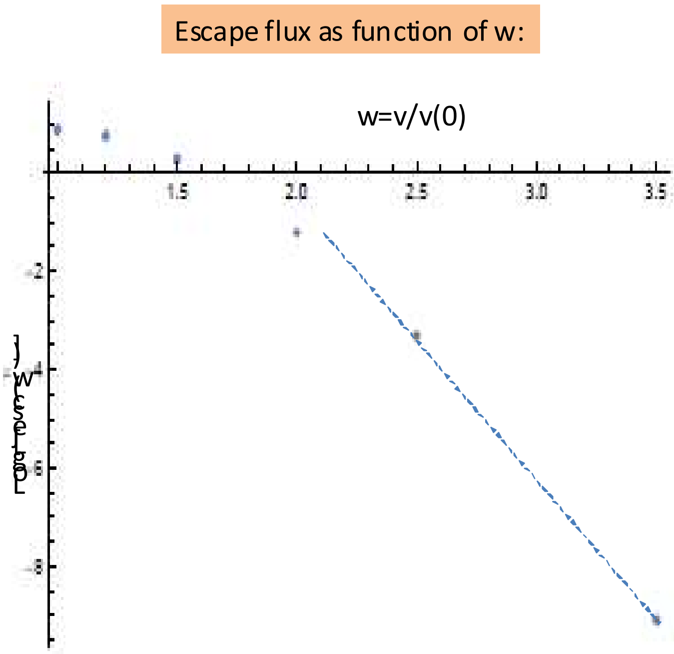

Figure 2: The logarithm of upward hydrogen flux at $x = x_1$ (exobase) as function of the normalized velocity $w = v/v_0$ for the standard atmosphere

One can see that beyond velocities $w = v/v_0 = 2.5$ the function $J_H(w,\vartheta)$ turns into a power-law behavior.

Doing the same calculations for the case of the "alternative atmosphere" (inverted temperature profile Equ. (A)!) we can see substantial differences and a much stronger emphasis of the conversion into a non-thermal character of the resulting upward H-flux, as is shown in the next Figures 3 and 4. In the region of small velocities w ≤ 1. 5 one can recognize the depletion with respect to a Maxwellian of low velocity particles, and beyond velocities of w = 2. 5 a much milder decrease as compared to a Maxwellian decrease.

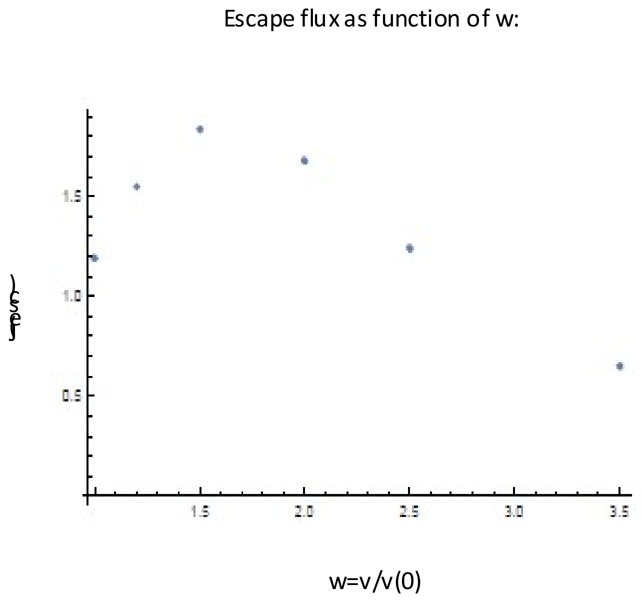

Figure 3: The upward hydrogen flux $J^*_H(w)$ at $x = x_I$ as function of the normalized velocity $w = v/v_0$ for the non-standard atmosphere with the inverted temperature profile (see Equ. (A)

This phenomenon of a systematic conversion from an equilibrium towards a non-equilibrium distribution function is even more evident in Figure 4 where we have shown the logarithm of the function $J^*_H(w)$ showing that upwards from velocities $w \geq 1.5$ the function turns to a power law.

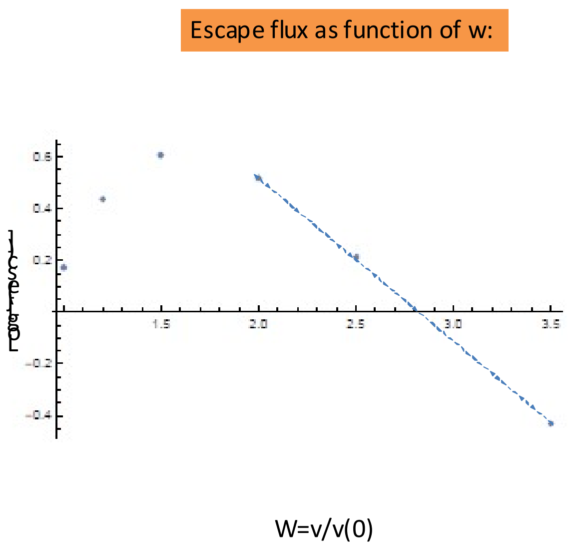

Figure 4: The logarithm of the upward hydrogen flux $J^*_H(w)$ at $x = x_I$ as function of the normalized velocity $w = v/v_1$ for the non-standard atmosphere with the inverted temperature profile (see Equ. (A)

In the case of Figure 4, i.e. for the inverted temperature profile, one can clearly recognize the flux depletions at low velocities $w \leq 1.5$ and non-thermal flux increases beyond $w = 2.0$, where the clear tendency of a power law spectrum becomes visible. This effect is due to the collision-determined expression for the H-atom transmissivity function $Tr_H(w, x)$ which describes the effect of elastic *H - O* - collisions with collision cross sections $\sigma(v)=\sigma_1.(v_1/v)$ that fall off with the reciprocal of the velocity $w = v/v_I$ yielding, in case of the inverted T- profile, higher transmissivities at higher velocities and a stronger dominance of flux contributions from deeper and hotter regions of the atmosphere. In case of the temperature-inverted atmosphere Figure 4 higher transmissions from the lower, but hotter layers of the atmosphere thus are guaranteed. Taking all these results together it shows that collision-modified thermal hydrogen flows from atmospheric layers with different temperatures may lead to nonthermal characteristics of the particle outflows from the uppermost exobasic layer.

There exist some differences in the recommended values of the relevant collision cross sections, expressed in the quantity of the reference cross-section $\sigma_0=\sigma_0.(v_0)$ given by the quantity $\sigma_0(v_0) = \sigma^{(1)}=10^{-17}\ cm^2$ or by the quantity $\sigma_0(v_0)=\sigma^{(2)}=10^{-15}\ cm^2$ [5]. In Figure 5 we have shown the differences that are solely due to these cross-section differences concerning the upcoming results for *Log[j(esc, w]* as function of $w = v/v(o)$.

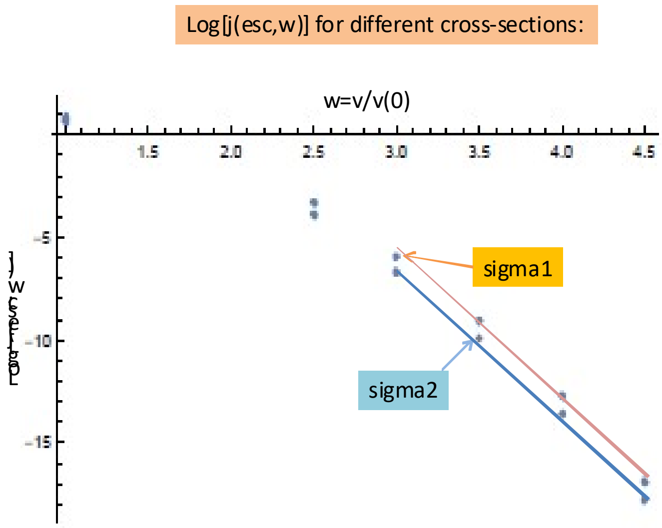

Figure 5: Log [jesc (w)] is shown as function of $w = v/v(o)$ for two different cross section values: sigma1= $10.^{(-17)}\ cm^2$ and sigma2= $10.^{(-15)}\ cm^2$.

The Collision-Modulated Planetary Escape Flux

At the end of this article, the important question must be put how these effects of a collision-induced transformation of the H-atom distribution function into a non-equilibrium function do influence the final hydrogen escape flow from the earth´s exobase to space. This escape flow namely determines the total hydrogen loss of the Earth´s atmosphere per time to space, and now this must be quantified under the new auspices treated in the sections above. Usually, for the purpose to determine this escape flux, the classical Jeans approach is used [10-12] leading to the result that this flux J_{jeans} is given by;

$$J_{jeans} = 2\pi \int_{v_{esc}}^{\infty} \int_{\vartheta=0}^{\vartheta=\pi/2} f_{H,MAX}(v) v \cos\vartheta d(\cos\vartheta) v^2 dv \qquad \#$$

with the thermal exobase hydrogen distribution function given at a central exobase distance $r_{exo}=r_E+r_{ex}$ with $r_{ex} = 500km$ by

$$f_{H,MAX}(v) = n_H \cdot (\frac{m_H}{2kT_H})^{3/2} \exp[-\frac{m_H v^2}{2kT_H}] \qquad \#$$

and leading to the well-known result [13]:

$$J_{jeans} = \frac{n_H}{2\sqrt{\pi}} (\frac{2kT_H}{m_H})^{1/2} (1+\lambda_H) \exp[-\lambda_H] \qquad \#$$

where λ_H is the escape energy parameter given by:

$$\lambda_H = \frac{G m_H M}{k T_H r_{exo}} \qquad \#$$

where M and $r_{exo} = r_e + 500km$ here denote the mass of the earth and the central exobase radius.
Now, in comparison to that classical result from Jeans (1923), we here in this article find the following result for the collision-modified escape flux:

$$\begin{aligned} J_{esc} &= v_0 n_{H0} \Delta \int_{w_{esc}}^{\infty} w^3 \int_4^{10} dz \cdot (1+\Delta(z-4))^{3/2} \exp[-w^2(1+\Delta(z-4)) \cdot \\ &\quad \exp[-\frac{4}{w}\exp[-(z-4)] \cdot (\exp(-\frac{z}{\cos\vartheta}) - \exp(-\frac{10}{\cos\vartheta}))] \\ &= v_0 n_{H0} \Delta \cdot \frac{1}{4} \int_{w_{esc}}^{\infty} J_H(w, \vartheta = 0) dw \end{aligned} \qquad \#$$

Let us remember that we have used as normalization of the H-atom velocity: $kT(x)/m_H = v_0^2 \cdot (T_0/T(x))$, with the normalizing velocity $v_0 = \sqrt{(kT_0/m_H)} = 5.8km/s\simeq$,$0.5 v_{esc}$,H. This means that the lower border in the upper w-integral delivering Jesc is given by $w_{esc} = v_{esc},H / v_0 = 2$. Thus, we obtain the modulated Jeans escape flow by:

$$J_{esc} = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} J_H(w, \vartheta = 0) dw \qquad \#$$

The numerically determined dependence of $J_H(w, \partial = 0)$ beyond values of $w = 2$ as can be seen in Figure 2 appears power-law-like, i.e. yielding in the range $w \geq 2$ a straight line when plotting $Log[J_H(W)]$ against w with the following algebra:

$$Log[J_H(w)] = A + B \cdot w \qquad \#$$

and when fitting this algebraic expression to the plot shown in Figure 2 one obtains the following result:

$$Log[J_H(w)] = Log[v_0 n_{H0} \Delta] + 12.87 - 6.05 \cdot w \qquad \#$$

Since the escape flux is an integral over the differential flux, and not over its logarithm, we have to use the following expression:

$$J_H(w) = v_0 n_{H0} \Delta \cdot 10.^{(12.87-6.05w)} \qquad \#$$

yielding the total escape flux according to Equ.(B) in the following form:

$$J_{esc} = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} J_H(w, \vartheta = 0) dw = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} 10.^{(12.87-6.05w)} dw \qquad \#$$

The remaining integral evaluates to $\int_2^{\infty} 10.^{(12.87-6.05w)} dw = 0.163$ and hence one finally finds

$$J_{esc} = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} J_H(w, \vartheta = 0) dw = 2.8 \cdot 10\hat{\ }5 \cdot 0.163/12 \cdot n_{H0} = 3.81 \cdot 10^3 \cdot n_{H0} \qquad \#$$

Comparing this above result with earlier results obtained by using the classical Jeans expression (Jeans, 1923, see also Fahr and Shizgal (1983, especially their Figure 3 for concretes) we find as a relative surprise that the present value $J_{esc}=3.8.10^3 n_{H0}$ obtained for a thermally structured atmosphere with a lower temperature $T_0(x_0) = 700K$ and an upper temperature of $T_1(x_1) = 1400K$ not only, as expectable, is larger than the Jeans flux for the lower temperature, i.e. $J_{jeans}(T_0= 700K)=80n_{H0}$, but, less expectable, is slightly smaller than the Jeans flux for the higher temperature, namely $J_{jeans}(T_1= 1400K)=7000n_{H0}$. Therefore, one can say that the classical Jeans formula does lead to a slight overestimation of the actual hydrogen escape flow from the earth.

This result also came already out from several earlier studies following different aspects of the escape problem like those considered by Brinkmann (1970), Fahr (1976), Fahr and Weidner (1977), Lindenfeld and Shizgal (1979), Shizgal and Blackmore (1986) or Pierrard (2003) [14-19]. In Fahr (1976) it was considered that the H-population at exobase heights in its upward velocity branch contains particles that escape from the earth's gravity field, i.e. particles that do not return to the exobase from above, meaning that this part of the population in the downward velocity branch is permanently missing at exobase heights, i.e. it thus does not appear in the downward branch of the distribution function and somehow needs to be replaced via collisions. This loss of escaping particles can be expressed as a permanent loss of thermal energy from the exobasic *H*-population cooling down the exobasic hydrogen gas by about *80K* relative to oxygen (Fahr, 1976) and thereby reducing the Jeans escape rate by about a factor of 0. 5. A similar reduction of the Jeans flux values is elaborated in a study by Fahr and Weidner (1977) determining the influence on the *H*-escape rate in the sub-exobasic atmospheric layers due to collisions with O-atoms, however, treated in this case as hard-core elastic collisions with velocity-independent cross sections. For the atmospheric exobasic temperature of 1400 K the authors find a similar reduction of the Jeans escape value by a factor of 0. 35.

Putting things together, it turns out that this present study is not the first one demonstrating that classical Jeans escape rates are undermined by present day more realistic results, if collisional effects in the thermosphere of the Earth are taken into account. However, this study shows for the first time that the effect of elastic collisions of escaping H-atoms with O-atoms leads to a transformation of the original thermal Maxwell distribution into a non-thermal kappa-like distribution with power-law characteristics at larger velocities of $v \geq 2\ v_0 = 2\sqrt{(kT_0 / m_H)}$. This should demonstrate that when detecting exobasic H-atoms with kappa-like or power-law distributions, this does not allow to conclude that these H-atoms originate from non-thermal lower atmospheric regions [20].

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