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Research Article

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Harmony and Precision of the Research Method by Fundaments: More Commutative Algebra and Consistence Theorems

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Abstract
As has been mentioned in many expositions and international publications, the research method by fundaments is determined and established as a derived categories scheme on a commutative ring, whose morphisms between categories have inverses, conforming quasi-isomorphisms among the categories of the research method such as are the theory ThΣ, a true propositions set Φ_α, the sub-theory subThΣ, prototypes $\{\pi_j\}$, etcetera, which are nodes of the flow diagram of the research method by fundaments. The direct morphisms, as well as the inverses demonstrate isomorphisms in commutative diagrams to consistence theorems which define the harmoniousness and precision in the functioning of the research method by foundations, even in the order of the experimentation and design of applied theories in scientific research.

Keywords: Derived Category, Derived Functors, Method by Foundations, Research, Sub-Theory, Theory, True Propositions, Mathematics and Physics Knowledge

1. Introduction

The research method by fundaments arises considering the "research" as an object of study both in its process, as in its resolution to obtain results in all areas of the knowledge. The research method by fundaments is a most important result obtained from the mathematical theory of research realized and obtained considering the identification of certain invariants that define it and its realization in scientific research on applied sciences where the observation directly is impossible and requires an extension of the observation, the creation of a valid hypothesis to a field deduction and analysis on true propositions (already demonstrated and evident facts), the creation of a theory $Th\Sigma$, as entity fundamental and generator of true propositions and new true propositions further knowledge [1-5]. But also, in the experiments development comes enforced with the true propositions as results of a research operator $\mathfrak{T}$, on the set of true propositions which we denote as Φ_α, in the class α, or knowledge type α (for example a knowledge class can be the belonging to a sub-branch of a study branch). The proof, will be on the theorems and experiments supported by these theorems (almost always are the true propositions obtained by $\mathfrak{T}$). Then the final law obtained is a theorem, which has more weight than a law. The theorems are eternal. The laws are changing depending on the context or environment where they occur. Likewise, one of the most important results obtained under this research theory is the flow diagram of the research methods by fundaments. The spaces or nodes are derived categories of objects in goal of study that is the scientific research. Likewise, the spaces I, E_{FISMAT}, $Th\Sigma$, $\mathfrak{T}\Phi_\alpha$, $\{\pi_j\}$, π *, P, are set objects with morphisms (arrows) between them and whose inverses generate quasi-isomorphism between these spaces. The laws of identity and associativity to their compositions of these objects are satisfied. Likewise, we have the following considerations.

Let be the categories (whose points are sets of propositions) $\Phi_1, \Phi_2, \ldots, \Phi_n, \ldots$, of an Abelian category $\mathcal{R}$. These sets can be set of propositions of certain class α, then these can to define the following category $\mathfrak{T}\Phi_\alpha$, whose objects are sets with arrows and therefore sequences as:

[1] This can be a modules category on a ring, or category of sheaves of Abelan groups on a topological space, for example topological groups.

$$\Phi_0 \overset{d_0}{\to} \Phi_1 \overset{d_1}{\to} \Phi_2 \overset{d_2}{\to} \Phi_3 \to \ldots$$

which defines the research process starting of the category of true propositions belonging to a class of the knowledge in mathematics and physics E_{FISMAT}. Also here each Φ_i, is an object of $\mathcal{R}$, and each compositions $d^i \circ d^{i+1}$, is zero. The ith −cohomology group of the complex is $H^i(\Phi_i) = \ker d^i / \operatorname{im} d^{i-1}$.

If in particular we have the categories of objects in the research context; I, E_{FISMAT}, $Th\Sigma$, $\mathfrak{T}\Phi_i$, $\{\pi_j\}$, π *, P, and 0, defined in [4, 6, 7] we have:

$$\mathcal{P}$$

$$\downarrow$$

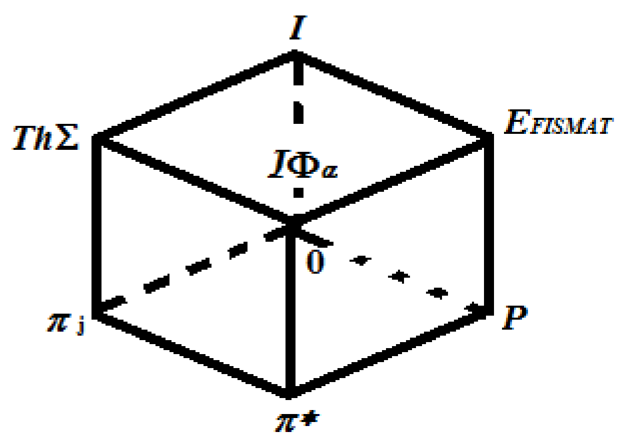
$$I \to E_{FISMAT} \to \Phi_\alpha \to \mathfrak{T}\Phi_\alpha \to Th\Sigma \to \{\pi_j\} \to \pi * \to \mathrm{P} \to 0, \quad (1)$$

$$\downarrow \qquad\qquad \downarrow$$

$$subTh\Sigma \qquad\qquad Pat$$

which is totally reversible generating isomorphism between categories [8]. The diagram (1) is an exact sequence in cohomology [9, 10].

We consider the cubic arrangement of (1), whose weight vertices are I, E_{FISMAT}, $Th\Sigma$, $\mathfrak{T}\Phi_i$, $\{\pi_j\}$, π *, P, and 0, only. Then we have:



(2)

which is commutative [11, 12, 13]. The vertex of major weight is the category $Th\Sigma$.

Theorem (F. Bulnes) 1. 1. The flow diagram of the research method by fundaments is isomorphic to the commutative scheme (2).

Proof. [8].

In the work on implement additional steps due the research extension we can to have one cube scheme with commutative triangles [8].

Lemma 1. 1. $\mathfrak{T}\Phi_\alpha$,is a derived category.

Proof. [8].

2. Some Consistence Theorems and Results of the Mathematical Theory of the Research

In the development of the mathematical theory of the research through two theorem explained in Cuba in different postgraduate courses on algorithms development and published in we have the following theorem [3,14].

Theorem 2. 1. Let be $\Phi_\alpha(E)$, the category of true propositions coming directly of the universe of mathematics and physics knowledge $E = E_{FISMAT}$. Let be Π, [2] the category of prototypes (or technologies in research process). Then is commutative the following diagram:

[2] **Def.** We call to a technology t_σ, a prototype (is tos ay an element of the category Π) if $\forall\, \phi \in Hom_K(\kappa_\alpha, t_\sigma)$, and $x \in E_{FISMAT}$, we have $\phi(x) = t_\sigma$. This says that "*a prototype is a technology under research. Then is only product of research*"

$$
\begin{array}{ccc}
\Phi_\alpha(E) & \rightarrow & \Pi \\
\upharpoonleft\!\downharpoonright & & \upharpoonleft\!\downharpoonright \\
\Pi & \rightarrow & \mathrm{Hom}_K(L, t_\sigma)
\end{array} \qquad (3)
$$

Proof. We prove the isomorphism $\mathrm{Hom}_K(L, t_\sigma) \cong \Pi$. Let $\phi \in \mathrm{Hom}_K(L, t_\sigma)$. Then $\phi(L) = t_\sigma$. But t_σ, can be a prototype if $\forall\, x \in E_{FISMAT}$, we have $t_\sigma = \phi\,(x)$. Indeed, we consider (by homomorphism[3] property of ϕ):

$$\phi\big(L(x)\big) = \phi(L)\phi(x) = \phi(L)t_\sigma$$

$$\phi(x) = t_\sigma,$$

which is in Π. Now suppose that if $t_\sigma \in \Pi$, then $\forall\, x \in E_{FISMAT}$, we have $t_\sigma = \phi(x)$. Indeed, we consider now $\phi(x) \in \mathrm{Hom}_K(L(x), t_\sigma)$. Then $\phi(L)\phi(x) = t_\sigma\, \phi(x)$. Thus $\phi(L) = t_\sigma$, is a homomorphism of $\mathrm{Hom}_K(L, t_\sigma)$. The inferior row is proved with last implication. The another isomorphism $\Phi_\alpha(E) \cong \Pi$, is trivial, because $\forall\, x \in E$, we have $\phi(x) = t_\sigma$, which is an element of Π. The reciprocal is trivial. The upper row is this first implication proved to isomorphism in the left side. Therefore the diagram is commutative.

Another theorem is the following.

Theorem 2. 2. The following diagram of categories is commutative:

$$
\begin{array}{ccccc}
\Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta) & & \leftrightarrow & & \mathrm{Hom}_K(E_{FISMAT},\, t_\sigma) \\
& \phi_\beta \nwarrow & & \swarrow\ \phi_\delta & \\
\phi_\alpha \downarrow & & \mathrm{Hom}_K(\kappa_\alpha,\, t_\delta) & & \downarrow \phi_\gamma \\
& \swarrow & & \nwarrow\ \phi_\sigma & \\
\mathrm{Hom}_K(ThMod\Phi_\alpha(E_{FISMAT}), Cn\Phi_\alpha) & & \cong & & \phi_\zeta \in \mathrm{Hom}_K(\phi_\sigma(t_\gamma),\, t_\eta) \\
& \lfloor\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \uparrow & & & \\
& & \mathrm{FET} & &
\end{array} \qquad (4)
$$

Proof. The implication $\leftrightarrow$, in the top line of the diagram (4) is an equivalence given by an isomorphism. Indeed, let be $\tau \in \Lambda_{SYSTEM}$, where Λ_{SYSTEM}, is given by (A. I) in the Appendix. Then $\exists\, \tau \in \Lambda_{SYSTEM}$, such that $\phi(t_\tau) = t_\delta$. However, for another side $\phi \in \mathrm{Hom}_K(E_{FISMAT}, t_\sigma)$, such that $\phi(x) = t_\sigma$. Then $\exists\, \varphi \in \Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta)$, such that

$$\varphi\big(\phi(t_\tau)\big) = \varphi(t_\delta) = t_\sigma = \phi(x), \quad (5)$$

Therefore are isomorphic. The central space $\mathrm{Hom}_K(K_\alpha, t_\delta)$, is isomorphic to both spaces of top line. Also is isomorphic to the space in right side in the bottom line. However the isomorphism between two spaces conforms the commutativity of all diagram. Further, due to the before theorem 2. 1, $\mathrm{Hom}_K(K_\alpha, t_\delta) \cong \Pi$, and Π, is an important product of the engineering (proving the FET isomorphism).

Now we consider all sequences of spaces (that include $\Phi_\alpha(E_{FISMAT})$, $\Phi_\alpha(E)$, E_{FISMAT}, $\phi_\sigma(t_\gamma)$, $Mod\Phi_\alpha(E_{FISMAT})$, and K_α

$$0 \rightarrow Th\Sigma \rightarrow \Phi_0 \rightarrow \Phi_1 \rightarrow \Phi_2 \rightarrow \Phi_3 \rightarrow \Phi_4 \rightarrow \Phi_5, \quad (6)$$

Then we consider the derived functor Hom_K, under the commutative ring K, (from $ThMod\Phi_\alpha(E_{FISMAT})$ which is a particular case of $Th\Sigma$to models Mod applied to the true propositions set) and we apply it to the sequence:

[3] $\phi: G(\circ) \rightarrow \widetilde{G(\cdot)}$, with rule of correspondence $t_\alpha \rightarrow t_\beta$, is homomorphism since that $\forall\, t_\gamma,\, t_\delta \epsilon G$, is had that

$$\phi\big(t_\gamma \circ t_\delta\big) = \phi(t_\gamma) \cdot \phi(t_\delta)$$

$$0 \to \mathrm{Hom}_K(Th\Sigma, \Phi_0) \to \mathrm{Hom}_K(Th\Sigma, \Phi_1) \to \mathrm{Hom}_K(Th\Sigma, \Phi_2) \to \ldots \to \mathrm{Hom}_K(Th\Sigma, \Phi_5), \quad (7)$$

All these spaces are the spaces of the diagram (4), which are derived categories. Then the diagram (4) is commutative. Likewise the commutativity and strong equivalences determine the commutative prism (see the Figure 1).

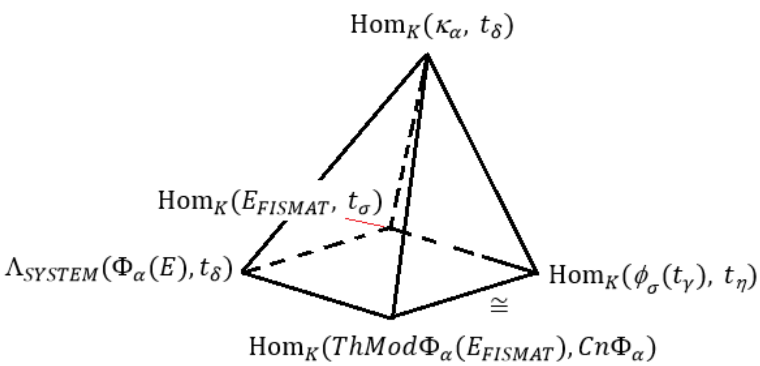


Figure 1: Commutative prism (tetragonal prism) of consistence theorem to research in engineering. Of the theorem 2. 2, the bottom line represents the FET (formal engineering theory).

By definition, "engineering" is the creation of technologies on bases of the mathematics and physics (axiom 2. 1). However, by the axiom 2. 2, also these technologies are designed and constructed to that function on energy signals. Finally, the perfection in technology is risked with the improvement given by $\phi_\sigma(t_\gamma)$. All this defines the engineering, as knowledge product (axiom 2. 3) [8]. The space definition of $\Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta)$, can be seen in the appendix.

Example 2. 1. In all research diagram used the applied research is given by

$$\begin{array}{ccccc} Th_{app} & \to & \{\pi_j\} & \to & \pi * \\ \downarrow & & \downarrow & & \downarrow \\ \{\pi_j\} & \to & \pi * & \to & P, \\ \downarrow & & \downarrow & & \downarrow \\ \pi * & \to & P & \to & 0 \end{array}$$

The consistence of this diagram stays established in five cycles identified in the diagram, which satisfy the diagram (3) of the theorem 2 .1, for example

$$\begin{array}{ccc} Th_{app} & \to & \{\pi_j\} \\ \downarrow & & \downarrow \\ \{\pi_j\} & \to & \pi * . \end{array}$$

Example 2. 2. Let be $\boldsymbol{\mu}_{EFISMAT}$, a measure of Lebesgue type. Then the value of a research in engineering is a prototype. Likewise for example if we consider the space $\mathrm{Hom}_K(\kappa_\alpha, t_\delta)$, which establish the research process to create technologies, then we can evaluate the integral $\int_{E_{FISMAT}} f\, \mu_{\boldsymbol{E_{FISMAT}}}$ on the upper triangle of the diagram (4) and according to the example 3. 3 in [8] we have:

$$\int_{E_{FISMAT}} \phi_\alpha(x)\,\mathrm{Isom}\,\phi(x)\mu_{E_{FISMAT}} = \frac{1}{c}\int_{E_{FISMAT}} \phi^2(x)\boldsymbol{\mu}_{\boldsymbol{E_{FISMAT}}} = \lambda\|\phi\|_2,$$

which evaluates the functioning of the technologies through of research[4]. Then we have a prototype. Likewise, we consider

$$\begin{array}{ccc} \Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta) & \leftrightarrow & \mathrm{Hom}_K(E_{FISMAT},\ t_\sigma) \\ \phi_\beta \nwarrow & & \swarrow\ \phi_\delta \\ & \mathrm{Hom}_K(\kappa_\alpha,\ t_\delta) & \end{array}$$

The homomorphisms ϕ_β , and ϕ_δ , are inside the integrand as identity element, is to say,

$$\phi_\alpha(x)\mathrm{Isom}\,\phi(x)\phi_\delta\phi_\beta^{-1} = t_\beta t_\beta^{-1}\phi_\alpha(x)\mathrm{Isom}\,\phi(x) = \phi_\alpha(x)\mathrm{Isom}\,\phi(x) = \frac{1}{c}\phi(\mathrm{x})\mathrm{Isom}\,\phi(x).$$

3. Design and Determination of Experiments In Research

According with the $TFI \subset Mathematical\ Theory\ of\ the\ Research$,[5] and accord with the research method by fundaments, exists Th_{app}, applied theory of a $Th\Sigma$, such that

$$Th_{app} \cong subTh\Sigma, \quad (8)$$

which has been demonstrated in a evident way [1, 8, 11, 14, 15]. Likewise the experiments in a scientific research determine the category Th_{app}, considering the commutative diagram

$$\begin{array}{ccccc} Th_{app} & \rightarrow & \{\pi_j\} & \rightarrow & \pi* \\ \downarrow & & \downarrow & & \downarrow \\ \{\pi_j\} & \rightarrow & \pi* & \rightarrow & P, \\ \downarrow & & \downarrow & & \downarrow \\ \pi* & \rightarrow & P & \rightarrow & 0 \end{array} \qquad (9)$$

Likewise, we can observe that the diagram (9) in each square complies with the first consistent theorem of the mathematical theory of the research, which establishes the equivalence, for example with a square:

$$\begin{array}{ccc} \Phi_\alpha(E) & \rightarrow & \Pi \\ \upharpoonleft\downharpoonright & & \upharpoonleft\downharpoonright \\ \Pi & \rightarrow & \mathrm{Hom}_K(L, t_\sigma) \end{array} \cong \begin{array}{ccc} \{\pi_j\} & \rightarrow & \pi* \\ \upharpoonleft\downharpoonright & & \upharpoonleft\downharpoonright \\ \pi* & \rightarrow & P \end{array} \qquad (10)$$

where $\Phi_\alpha(E)$, is the category of the true propositions set on the universe of mathematics and physics knowledge $E = E_{FISMAT}$. Π, is the category of prototypes. Also is isomorphic to the others squares of the diagram (9). The compositions $Th_{app} \rightarrow \{\pi_j\} \rightarrow \pi*$, and $\pi* \rightarrow P \rightarrow 0$, are valid.

Likewise the commutative diagram (9), establish the quality of experiments (through prototypes, test technologies, tools etcetera) to risk the optimal prototype $\pi*$, to obtain the final technological product that will give a solution to a necessity of the society. The qualities are:

a). Mental scheme with clear goals, is to say the implication $\mathfrak{T}\Phi_\alpha \rightarrow Th\Sigma$.

b). Manufacturing of high quality, then we have the implication $Th\Sigma \rightarrow Th_{app}$.

c). Harmoniousness and precise ensembles and constructions, is equivalent to the implication $Th_{app} \rightarrow \{\pi_j\}$.

d) Stability in the units and system, we have finally the implication $\{\pi_j\} \rightarrow \pi*$.

The a), is obvious, since come from of the proper research method

[4] Remember that the definition of engineering due the mathematical theory of research is the creation of technology on the energy [1, 4, 5, 8]. And a prototype is a technology under research (no product yet).

by fundaments. Then to have $\pi *$, is totally trivial (the potential of a research is consigned in a theory, which is a big potential to create applied research, which is implicated a) in b). After the potential of a theory is implicated in its realization, which is an applied theory which serve to obtain a set of prototypes, then b) implies c). Finally the realization of the set of prototypes serve to realize a final prototype (optimal prototype). Then c) implies d).

Likewise we can obtain the following extensive commutative diagram:

$$\begin{array}{ccccc}
I & \rightarrow & E_{FISMAT} & \rightarrow & \Phi_{\boldsymbol{\alpha}} \\
\downarrow & & \downarrow & & \downarrow \\
E_{FISMAT} & \rightarrow & \boldsymbol{\mathfrak{T}}\Phi_{\boldsymbol{\alpha}} & \rightarrow & \Phi_{\boldsymbol{\alpha}} \\
\downarrow & & \downarrow & & \downarrow \\
\Phi_{\boldsymbol{\alpha}} & \rightarrow & \mathfrak{T}\Phi_{\alpha} & \rightarrow & Th\Sigma \\
\downarrow & & \downarrow & & \downarrow \\
\mathfrak{T}\Phi_{\alpha} & \rightarrow & Th\Sigma & \rightarrow & Th_{app} \\
\downarrow & & \downarrow & & \downarrow \\
Th\Sigma & \rightarrow & Th_{app} & \rightarrow & \{\pi_j\} \\
\downarrow & & \downarrow & & \downarrow \\
Th_{app} & \rightarrow & \{\pi_j\} & \rightarrow & \pi * \\
\downarrow & & \downarrow & & \downarrow \\
\{\pi_j\} & \rightarrow & \pi * & \rightarrow & P \\
\downarrow & & \downarrow & & \downarrow \\
\pi * & \rightarrow & P & \rightarrow & 0
\end{array} \qquad (11)$$

with the valid compositions $Th\Sigma \rightarrow Th_{app} \rightarrow \{\pi_j\}$, $I \rightarrow E_{FISMAT} \rightarrow \mathfrak{T}\Phi_{\alpha}$, $\mathfrak{T}\Phi_{\alpha} \rightarrow \Phi_{\alpha} \rightarrow Th\Sigma$,

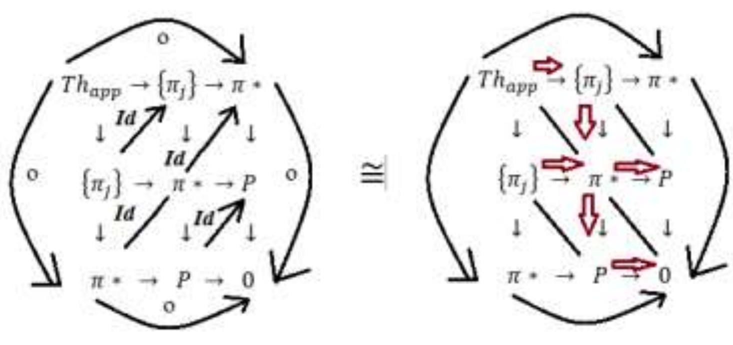


Figure 2: Commutative compositions and movements

5. Harmony, Tessellations and Games

We can establish the following game having the horizontal and vertical (←,↑,→,↓,↔) movements with a box 0, as space unfilled [16]. Considering the arbitrary arrangement

[5] Formal Theory of Engineering. Mathematical Theory whose study object is the research in engineering. This theory consider how fundamental derived category Th_{app}, which consists of a theory created in the applications context, which can be prototypes $\{\pi_i\}$.

Th_{app}	P	$Th\Sigma$	$\{\pi_j\}$
$\{\pi_j\}$	0	P	P
$\{\pi_j\}$	π^*	π^*	π^*

Board 1

Then we want to obtain the correct arrangement given by the research method by fundaments:

$Th\Sigma$	Th_{app}	$\{\pi_j\}$	π^*
Th_{app}	$\{\pi_j\}$	π^*	P
$\{\pi_j\}$	π^*	P	0

Board 2

How much movements we require realize to obtain the solution given in the board 2 only with (←, ↑, →, ↓, ↔)-movements? We let this question to the reader.

This game can be extended for all the commutative diagram (11) that defines the complete research method by fundaments, also considering the box 0, unfilled.

Another games can be established considering the commutative compositions and movements given in the figure 2. For example we can consider the loops or closed cycles in a commutative diagram

$$\begin{array}{ccccc} Th\Sigma & \rightarrow & Th_{app} & \rightarrow & \{\pi_j\} \\ \downarrow & & \downarrow & & \downarrow \\ Th_{app} & \rightarrow & \{\pi_j\} & \rightarrow & \pi *, \\ \downarrow & & \downarrow & & \downarrow \\ \{\pi_j\} & \rightarrow & \pi * & \rightarrow & P \end{array} \qquad (12)$$

Then we have the figure 3 A and 3B. These closed cycles or loops satisfy the consistence theorem 2. 1.

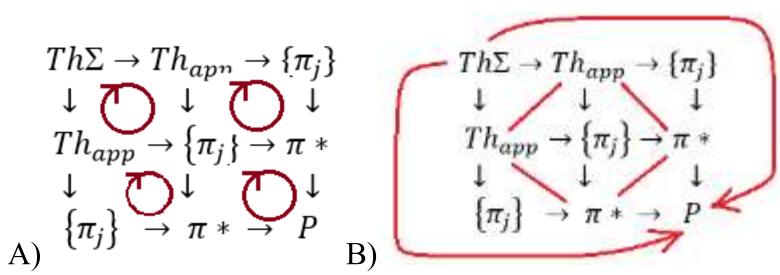
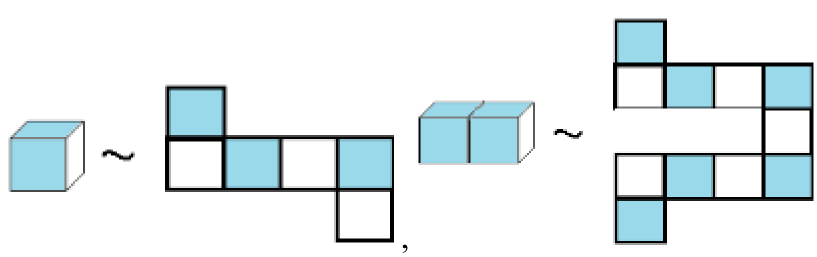


Figure 3: A). Closed cycles or loops in the diagram (12). B). Central cycle or loop and total composition of the research method of fundaments.

We consider the theorem 1. 1. Then the commutative cube isomorphic to the research method by fundaments can be extended in a plane as:



However there are eleven distinct nets of a cube[6]. Then the tessellations can be more complex, and depends on the object of research considered by the research method by fundaments.

If we consider colors assigned to each categorical element based in their context, we have six colors in a matrix of 12 entries having best way six colors arrangement:

$Th\Sigma$	Th_{app}	$\{\pi_j\}$	π^*
Th_{app}	$\{\pi_j\}$	π^*	P
$\{\pi_j\}$	π^*	P	0

Board 3

In the top corner is the category whose node is the heaviest, and in the bottom corner has the lower weight. What difficult is included to the Rubik's cube, if this consideration is taken into account? For a matrix designed to the diagram (11), can be obtained arrangements of colors in the same way. When is the best minimal selection of colors to this matrix such that we have nine colors to give the solution ... →0? This is left to the reader.

6. Conclusions

The research method by fundaments is beyond that a precise and effective research method in all areas of knowledge whose vertebral base or rector axis are the mathematics, the fundamentals science. The research method by fundaments is really that, construct a theory $Th\Sigma$, that will give whole the base required in a research and more. Having a theory, is had all perspectives of a research, including the technological applications that born from a Th_{app}, passing for a sequence of prototypes until to obtain the optimal prototype (technology under research yet) and finally give the technology or product P, that gives the solution to a social necessity. However, in the research technological part, which concerns to the engineering, because this is the purpose of engineering, the creation of useful technologies to solve diverse processes, we need an ordered way to create technologies through of the application of certain morphism called scientific technologiscisms, which belong to a derived functor Hom_K, on the commutative ring K, and whose derived categories satisfy isomorphisms in different levels, even the consistence fundamental theorems developed in the mathematical theory of the research. Likewise the engineering is the cognitive process that through the investigation of $\Phi_\alpha \subset E_{FISMAT}$, and with the concrete application of a $Th\Sigma$, to obtain a Th_{app}, are obtained the prototype set Π, where by the due research $\mathrm{Hom}_K(ThMod\Phi_\alpha(E_{FISMAT}), Cn\Phi_\alpha) \cong \mathrm{Hom}_K(\phi_\sigma(t_\gamma), t_\eta)$, are obtained the "technologies" or technological products such as are described in (9). The commutativity of all these diagrams establish a harmoniousness form of thought, which contributes to the clarity and to a logical development of a research due to that each step is an equivalence justified (quasi-isomorphisms) between its objects that are derived categories. From the harmony can be created and developed many games based in find an order that establish the solution in the research method of fundaments [17].

Appendix A. Basic Definitions and Relations

Def. A. 1. A technologiscism is a neologism that obtains a technology from a technology given. Likewise, $\phi_\sigma(t_\gamma) = t_\delta$.

Def. A. 2. A prototype π, is a technology under research (non-finished product). The theory that generates is a sub-theory or the category $subth\Sigma$. An optimal prototype $\pi*$, is a technology ready to be product P.

Proposition A. 1. [1, 2]. An applied theory is a sub-theory, therefore $SubTh\Sigma \cong Th_{app}$.

All research unit $Cn\Phi_\alpha$, is defined as the place (space) where only exist true propositions. Then a theory based on models of true propositions sets is the category of morphisms given by $\mathrm{Hom}(Th\mathrm{Mod}\Phi_\alpha(E_{FISMAT}), Cn\Phi_\alpha)$ is isomorphic to the set of morphisms (creation of technologies) $Hom(\phi_\sigma(t_\gamma), t_\eta)$.

Def. A. 3. The space $\Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta)$, defines the set of technology transferences of the class τ, to the class σ, formally:

$$\Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta) = \{\tau | \tau \Longrightarrow \sigma\}, \qquad (A.1)$$

Def. A. 4. The category $Cn\Phi$, is the set of tautological applications of the true proposition category Φ, is to saya

$$Cn\Phi = \{\sigma | \Phi \vDash \sigma\}, \qquad (A.2)$$

This category is called research unit.

Technical Notation.

E_{FISMAT} − Derived category of all knowledge in mathematics and physics. Locally is a Banach space [15], for example in the technology applications.

Π − Set of prototypes. This set is a derived category whose

[6] We have twenty nets of a cube [17].

morphisms are $\{\pi_j\} \to \pi*$.

ϕ_α −Scientifical technologiscism or simply technologiscism. This is a homomorphism on the the group of technologies, that is to say, $\forall\ t_\gamma$, and $t_\delta \in G(\circ)$, $\phi_\alpha(t_\gamma \circ t_\delta) = \phi_\alpha(t_\gamma) \cdot \phi_\alpha(t_\delta)$, where the image $\phi_\alpha(t_\gamma \circ t_\delta) \in \tilde{G}$.

α −Technology. Are the points belonging to the group G.

$\Phi_\alpha(E)$ −Set of true propositions (useful propositions) in the creation of a theory. This is a sub-category such that $\Phi^\alpha(E) \subset E_{FISMAT}$. Locally also can be a Banach space [15].

κ_α −The class α. This sub-category belongs to a partition of the groups G.

Λ_{SYSTEM} −Derived category of systems of utilities, manufactures and tools. Locally also is a Banach space.

$\mathrm{Hom}(A, B)$ −Space of homomorphism from the set A. until set B. In a commutative or noncommutative ring is a derived category of functors Hom.

$Th\mathrm{Mod}\Phi_\alpha$ −Derived category of all the statements that are true in all models of $\Phi_\alpha(E)$.

$\to$ −Morphism.

$Th\Sigma$ − Derived category of all theories on the set of demonstrated and verified proposition in the specific research. This category is the most important in the research method by fundaments

$Cn\Phi$ − Derived sub-category of all consequences by the research realized as final products.

$\cong$ −Isomorphism (structural equivalence between categories, rings, groups, any sets of elements). Another similar notation in the diagrams and schemes is $\leftrightarrows$. Also Isom, in the context of a derived category of functors Hom.

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