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Review Article

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Embedding the Photon with Its Relativistic Mass As a Particle into the Electromagnetic Wave Explains the Gouy Phase Shift as an Energetic Effect

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Abstract

In the paper "Embedding the photon with its relativistic mass as a particle into the electromagnetic wave" a new aspect concerning the relationship between photon and electromagnetic wave has been developed by considering the question why the energy and the mass density of an electromagnetic wave are propagating in the same direction [1]. For instance, in optical resonators the energy density usually propagates along curved lines. However, according to Newton's first law the mass density should propagate along a straight line, if no force is exerted it. To solve this problem, the assumption has been made that a transverse force is exerted on the mass density and in consequence on the mass of the photons which forces them to follow the propagating energy density. This leads to the result that the photon is moving within a transverse potential which allows describing the transverse quantum mechanical motion of the photon by a Schrödinger equation. These results are used to show that in case of a Gaussian wave the effective axial propagation constant $\bar{k}_{z,\,nm}(z)$ can be expressed as $\bar{k}_{z,\,nm}(z) = [E_{ph} - E_{nm}(z)]/\hbar c$ where E_{ph} is the total energy of the photon, and the $E_{nm}(z)$ are the energy eigenvalues of the transverse quantum mechanical motion of the photon. Since according to this result $\hbar c\bar{k}_{z,\,nm}(z)]$ represents a real energy, it has been concluded that also the effective axial propagation constant represents a real propagation constant. This leads to the conclusion that $\lambda_{nm}(z) = 2\pi/\bar{k}_{z,\,nm}(z) = hc/\big((E_{ph} - E_{nm}(z)\big)$ represents the real local wave length of the electromagnetic wave at the position z. According to this conclusion, $\lambda_{nm}(z)$ increases inversely proportional to the energy difference E_{ph}-$E_{nm}(z)$, which decreases with decreasing z, and therefore describes the Gouy phase shift in agreement with wave optics. This shows that the deeper physical reason for Gouy phase shift consists in the fact that the energy of the photon is increasingly converted into its transverse quantum mechanical motion when the photon approaches the focus. This explains the Gouy phase shift as an energetic effect.

Keywords: Quantum Optics, Electromagnetic Wave, Paraxial Wave Optics, Physical Optics, Laser Theory, Laser Resonators

Introduction

In [1] the question has been considered, why the energy density and the mass density of an electromagnetic wave propagate in the same direction, even if the energy density, whose propagation direction is described by the Poynting vector, is propagating along a curved line as shown by Figure 1 which displays a resonant Gaussian wave between two spherical mirrors of an optical resonator [1]. Therefore, if a particle of relativistic mass propagating with the electromagnetic wave is assumed to represent a quantum of energy, its propagation should be described by the Poynting vector. On the other hand, if the particle is considered to represent a real particle of mass, its propagation should be described by Newton's first law, if no force is exerted on it. Therefore, the question arises, why does the propagating relativistic mass density follow the propagating energy density?

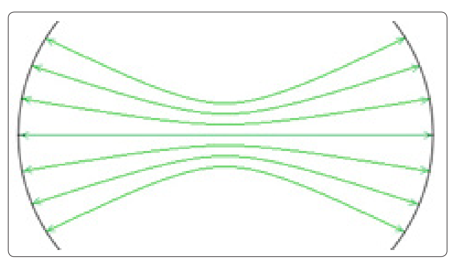

Figure 1: Resonant Gaussian mode between two spherical mirrors. The green lines visualize the propagating energy density

This consideration leads to a contradiction between fundamental physical laws:

- Theory of relativity ($E=mc^2$),
- Newton's first law,
- Maxwell's equations.

I this context it shall be stressed that this contradiction not only concerns the modes in an optical resonator but furthermore concerns any electromagnetic wave. To solve this problem, the assumption has been made that a transverse force is exerted on the mass density which forces it to follow the propagating energy density [1]. Based on a consideration of the directional change of the Poynting vector during an infinitesimal propagation step of the wave, in the following expression has been derived for the force exerted on a small particle with relativistic mass $\tilde{M}$ which propagates with the electromagnetic wave [1].

$$\vec{K}(r,z) = -\tilde{M}c * \lim_{z_1\to z_2, r_1\to r_2, \Delta t\to 0} \frac{1}{\Delta t}\left[\vec{S}_N(r_2,z_2) - \vec{S}_N(r_1,z_1)\right] \qquad (1)$$

Here, the $\mathbf{S}_N$ (r_i, z_i) are normalized Poynting vectors erected in the points r_1 and r_2 on the wave fronts Φ_1 and Φ_2 as shown by Figure 2. The infinitesimal time interval, which the phase front Φ_1 takes to propagate into Φ_2, is designated by Δt. $\tilde{M}$c is the momentum of the particle.

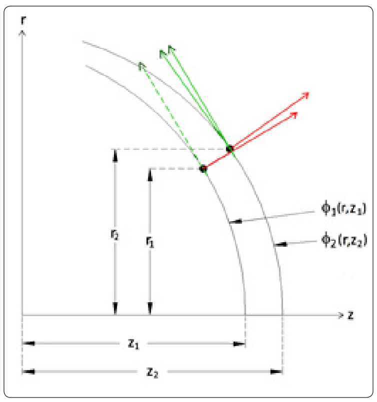

Figure 2: Visualization of two phase fronts Φ_1 and Φ_2 intersecting the z axis at z_1 and z_2

z_1 and z_2 are the points where the wave fronts Φ_1 and Φ_2 intersect the propagation axis of the wave. In the infinitesimal limit the points r_i as well as z_i are assumed to move into each other. Since the right hand side represents an infinitesimal change of a momentum versus an infinitesimal time step, it represents according to Newton's second law a force. And since furthermore the above consideration does not require to define the mass of the small particle exactly, it can be assumed that its mass is identical with the relativistic mass M=h/(cλ) of the photon. This delivers for the force exerted on the photons

$$\vec{K}(r,z) = -E_{ph} * \lim_{r_1\to r_2, z_1\to z_2} \frac{1}{\Delta z}\left[\vec{S}_N(r_2,z_2) - \vec{S}_N(r_1,z_1)\right] \qquad (2)$$

Here E_{ph} is the energy of the photon given by

$$E_{ph} = Mc^2 = \frac{hc}{\lambda} \qquad (3)$$

where λ and c are the wavelength and the speed of light in a vacuum, respectively. For the sake of simplicity in Equation (2) the consideration has been restricted to a rotational symmetric wave. Since under this condition the normalized Poynting vectors $\mathbf{S}_N$ (0, z_1) and $\mathbf{S}_N$ (0, z_2) are pointing in the same direction, Δt can be replaced by $\Delta z = z_2 - z_1 = c\Delta t$ as shown in [1].

From the above results it has been concluded in that the photon is moving within a transverse potential V(r,z) which is obtained by integrating the negative value of the force **K**(r,z) given by Equation (2) over r along the curvature of the phase front Φ(r,z). Based on this conclusion in [1] the assumption has been made that the transverse motion of the photon is described by the following Schrödinger equation

$$\left[\frac{\hbar^2}{2M}\Delta_\perp + E(z) - V(\mathrm{r},z)\right]\chi(\mathrm{r},z) = 0 \ , \qquad (4)$$

like the motion of the electron except for the difference that the mass of the electron is replaced by the relativistic mass M of the photon. In Equation (4) r and z are not Cartesian coordinates in the usual sense, since z describes the point where the phase front Φ(r,z) intersects the optical axis.

In case of a Gaussian wave the force described by Equation (2) can be explicitly computed as shown in [1]. This delivers

$$\vec{\mathrm{K}}(\mathrm{r},\mathrm{z}) = -E_{ph} r\left(\frac{z_R}{z^2 + z_R^2}\right)^2 . \qquad (5)$$

Here $z_R = \pi w_0^2/\lambda$ is the Rayleigh range, and w_0 is the spot size at the beam waist. To compute the potential, -K(r,z) has to be integrated along the curvature of the phase front. However, in paraxial approximation this integration can be carried through directly along r. This delivers

$$V(r,z) = \frac{1}{2} M\omega_\perp^2(z) r^2 \qquad (6)$$

Here ω⊥ is the frequency of the transverse quantum mechanical oscillation of the photon, and is according to Equation (5) given by

$$\omega_\perp(z) = \frac{cz_R}{z^2 + z_R^2} \qquad (7)$$

If the potential V(r,z), as given by Equation (6), is inserted into the Schrödinger equation given by Equation (4) the following expression

$$\left[\frac{\hbar}{2M}\Delta_\perp + E_{nm}(z) - \frac{1}{2}M\omega_\perp^2(\mathrm{z})(x^2 + y^2)\right]\chi_{nm}(x,y,z) = 0 \qquad (8)$$

is obtained, where the coordinates x, y and z are used in the same way as r and z in Equation (5). Equation (8) is identical with the Schrödinger equation of the 2-dimensional harmonic oscillator.

In [1] the Equations (7) and (8) have been verified by comparing the probability density of the photon, which is represented by the squared absolute values $|\chi_{nm}(x,y,z)|^2$ of the eigenfunctions χ_{nm} of the Schrödinger equation, with the result obtained by the use of wave optics for the normalized local intensity of a Gaussian wave of order n,m as described by Equation (16.60) in [2]. This comparison provided full agreement between both quantities.

as well known, the energy levels E_{nm} of the Schrödinger equation of the 2-dimensional harmonic oscillator are given by

$$E_{nm}(z) = \hbar\omega_\perp(\mathrm{z})(n + m + 1) \qquad (9)$$

In the next section it will be shown that a direct relation exists between the energy levels E_{nm} (z) and the Guoy phase shift which seems to allow for explaining the Guoy phase shift as an energetic effect [3].

Is the Gouy phase shift an energetic effect?

To answer this question we consider the effective axial propagation constant for a finite beam which according to the Equations (3) and (19) in [4] can be expressed as

$$\bar{k}_{z,nm}(z)=\frac{<k_{z,nm}^2>}{k}=k-\frac{<k_{x,nm}^2>+<k_{y,nm}^2>}{k} \tag{10}$$

and establish a relationship between the expectation value $<k^2_{xnm}>$ + $<k^2_{ynm}>$ of the transverse part of the square of the propagation constant k, and the expectation value of the square of the photon's transverse quantum mechanical momentum given by

$$<\chi_{nm}|\hat{p}_\perp^2(z)|\chi_{nm}>=\hbar M\omega_\perp(z)(n+m+1) \tag{11}$$

which, by the use of Equation (9), can be transformed into

$$<\chi_{nm}|\hat{p}_\perp^2(z)|\chi_{nm}>=ME_{nm}(z) \tag{12}$$

In order to establish for the case of a Gaussian wave a relationship between $<k^2_{xnm}> + <k^2_{ynm}>$ and $<\chi_{nm}|\hat{p}_\perp^2(z)|\chi_{nm}>$ we take into account that the momentum of a freely propagating photon is given by p = ħk. This shows that the expression for the wave number k is obtained, if p is divided by ħ. It can therefore be concluded that division of the expectation value $<\chi_{nm}|\hat{p}_\perp^2(z)|\chi_{nm}>$ by ħ2 delivers $<k^2_{xnm}> + <k^2_{ynm}>$. In this way, we obtain

$$<k_{x,nm}^2>+<k_{y,nm}^2>=\frac{<\chi_{nm}|\hat{p}_\perp^2(z)|\chi_{nm}>}{\hbar^2}=\frac{ME_{nm}(z)}{\hbar^2} \tag{13}$$

This delivers after insertion into Eq. (10)

$$\bar{k}_{z,nm}(z)=k\left(1-\frac{ME_{nm}(z)}{k^2\hbar^2}\right) \tag{14}$$

If we replace in this equation the relativistic mass M of the photon according to Equation (3) by $M=E_{ph}/c^2$, we obtain

$$\bar{k}_{z,nm}(z)=k\left(1-\frac{E_{ph}E_{nm}(z)}{c^2k^2\hbar^2}\right) \tag{15}$$

If we take into account now that E_{ph} can be expressed as

$$E_{ph}=ck\hbar \tag{16}$$

we obtain

$$\bar{k}_{z,nm}(z)=k\left(1-\frac{E_{nm}(z)}{E_{ph}}\right)=\frac{k}{E_{ph}}\left[E_{ph}-E_{nm}(z)\right]=\frac{1}{\hbar c}\left[E_{ph}-E_{nm}(z)\right] \tag{17}$$

This equation can be rewritten as

$$\hbar c\bar{k}_{z,nm}(z)=E_{ph}-E_{nm}(z) \tag{18}$$

This shows that $\hbar c\bar{k}_{z,nm}$ (z) is equal to the difference between the total energy E_{ph} of the photon and the energy eigenvalues E_{mn} (z) of the Schrödinger equation which increase with decreasing z, since ω⊥ increases according to Equation (7). It can therefore be concluded that part of the total energy of the photon is transformed into the energy of the transverse quantum mechanical motion of the photon, and another part is transformed into $\hbar c\bar{k}_{z,nm}$ (z). Since according to this result $\hbar c\bar{k}_{z,nm}$ (z) represents a real energy, it must be assumed that also the effective axial propagation constant $\bar{k}_{z,nm}$ (z) represents a real propagation constant. This leads to the conclusion that

$$\lambda_{nm}(z)=\frac{2\pi}{\bar{k}_{z,nm}(z)}=\frac{hc}{\hbar c\bar{k}_{z,nm}(z)}=\frac{hc}{E_{ph}-E_{nm}(z)} \tag{19}$$

represents the real local wave length of the electromagnetic wave at the position z. According to this conclusion λ_{nm} (z) increases inversely proportional to $\bar{k}_{z,nm}$ (z), and therefore, describes the Gouy phase shift in agreement with wave optics. But this conclusion also shows that λ_{nm} (z) increases inversely proportional to energy difference E_{ph} – E_{nm} (z) which decreases with decreasing z. This seems to show that the deeper physical reason for Gouy phase shift consists in the fact that the energy of the photon is increasingly converted into its transverse quantum mechanical motion when the photon approaches the focus. This allows concluding that the Gouy phase shift is an energetic effect. Therefore, it seems to be reasonable to designate the energy $\hbar c\bar{k}_{z,nm}$ (z) as Gouy energy, and to introduce for this energy the term $E_{G,nm}(z)$. Based on this result Equation (18) can be rewritten as

$$E_{ph}=E_{G,nm}(z)+E_{nm}(z) \tag{20}$$

which shows that the total energy E_{ph} is the sum of an axial part described by $E_{G,nm}(z)$, and the energy eigenvalues $E_{mn}(z)$ of the Schrödinger equation. There is a more intuitive releationship between $E_{G,nm}(z)$. and the local wave length defined by Equation (19). Since due to the increase of the wave length the frequency of the oscillation of the wave is slowing down, the energy, which is transported by the oscillation of the wave, decreases, when the wave approaches the focus. This decrease of the energy transported by the oscillation of the wave is described by $E_{G,nm}(z)$.

However, the particle picture not only explains the Gouy phase shift as an energetic effect, it also allows to compute the mathematical expression describing the Gouy phase shift in agreement with wave optics as already shown in [1].

The upper part of Figure 3 shows for λ=1[μm] and w_0=1.12 λ/(π√2) [μm] how the energies $E_{G,00}(z)$ and $E_{00}(z)$ change with the propagating wave. The lower part shows how the spot size changes. Under the same conditions, the upper part of Fig. 4 shows how the local wave length λ_{00} (z) changes as a function of z [μm]. The lower part shows how the Gouy phase shifts changes.

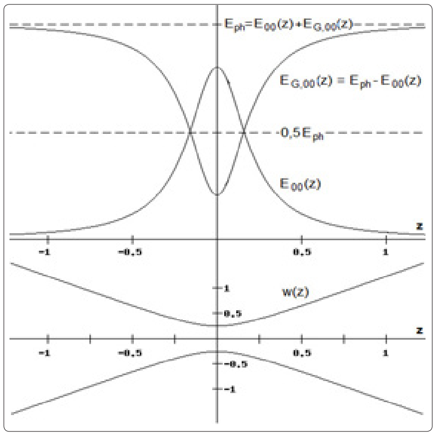

Figure 3: The upper part of this figure shows for λ=1[µm] and w_0=1.12 λ/ (π√2) [µm] how the energies $E_{G,00}(z)$ and E_{00} (z) change with the propagating wave. The lower part shows how the spot size changes as a function of z [µm].

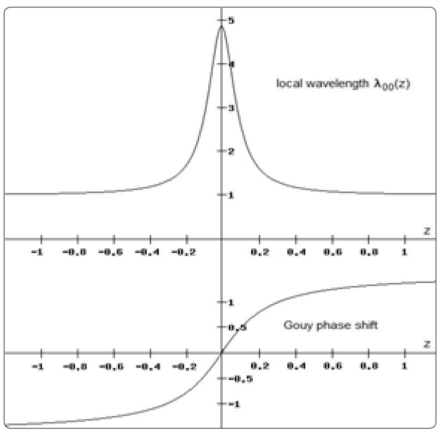

Figure 4: Under the same conditions as used for Figure 3. The upper part of this figure shows how the local wave Length λ_{00} (z) changes as a function of z. The lower part shows how the Gouy phase shifts changes.

Since $E_{G,nm}(z)$ vanishes for $E_{nm}(z) = E_{ph}$, $\lambda_{nm}(z)$ goes to infinity under this condition. Therefore, $E_{nm}(z) < E_{ph}$ must be considered to be a limiting condition for E_{nm}. The condition $E_{nm}(z) < E_{ph}$ transforms, after replacing E_{nm} (z) according to Equation (9) and E_{ph} according to Equation (3), into

$$\hbar\omega_{\perp}(z)(n+m+1) < \frac{hc}{\lambda} \tag{21}$$

which after replacing ω⊥(z) according to Equation (7) transforms into

$$\frac{z_R}{z^2+z_R^2}(n+m+1) < \frac{2\pi}{\lambda} \tag{22}$$

This equation transforms for z=0 into

$$n+m+1 < \frac{2\pi}{\lambda} z_R = 2\left(\frac{\pi w_0}{\lambda}\right)^2 \tag{23}$$

This delivers for the spot size w_0 at the beam waist the condition

$$w_0 > \frac{\lambda}{\pi}\sqrt{\frac{n+m+1}{2}} \tag{24}$$

If we replace in this equation the ">" sign by the "=" sign we obtain for n=m=0

$$w_0 = \frac{\lambda}{\sqrt{2}\pi} = \frac{\sqrt{2}}{k} \tag{25}$$

This delivers the relation

$$\omega_{\perp}(0) = \frac{c}{z_R} = \frac{2\pi c}{\lambda} = \omega \tag{26}$$

This result shows that under the condition $E_{nm}(z) = E_{ph}$ the frequency ω⊥ of transverse quantum mechanical oscillation of the photon is becoming equal to the frequency ω of the initially incoming electromagnetic wave. Therefore, in this case the oscillation of the wave is in the focus totally transformed into the transverse quantum mechanical oscillation of the photon. Simultaneously, the effective axial propagation constant $\bar{k}_{z,00}$ vanishes according to Equation (17), and the local wave length λ_{nm} (z) goes to infinity according to Equation (19).

Concerning the Equations (23-25) it shall be mentioned that there is mathematical agreement between Equation (25) and Equation (30) given in [5] which according to the considerations given in [5] describes the fundamental mode radius. Equation (24) can therefore be considered as a generalization of Equation (30) in [5] for higher order Gaussian modes. This shows once more that the particle picture provides results in agreement with wave optics as has been used in [5].

The fact that effective axial propagation constant $k_{z,00}$ (0) vanishes under the condition w_0=λ/ (π√2) also can be derived from the wave optics description of the Gouy effect by differentiating the phase of the fundamental mode which is given by Φ (z) =kz+Φ_G (z) with Φ_G (z) =arctan (z/z_R). Differentiation of Φ (z) versus z delivers for z=0

$$\bar{k}_{z,00}(0) = \frac{\partial\Phi(z=0)}{\partial z} = -k + \frac{\partial\Phi_G(z=0)}{\partial z} = -k + \frac{1}{z_R} = 0 \tag{27}$$

This result shows that the phase Φ (z) of the fundamental mode stops to change at the beam waist under the condition w_0=λ/ (π√2). Therefore, the question arises what is happening with a wave whose phase stops to oscillate with the propagating wave. Can it still be considered to be a wave which transports energy? The presented particle picture says that the total energy of the photon is totally transformed into its transverse quantum mechanical motion under the above mentioned condition and the wave itself does not transport energy anymore.

This problem finds a solution, if one takes into account that the far-field angel where the intensity of the fundamental beam drops to 1/e² is according to Equation (17.17) in [2] given by θ_{e^2}=λ/(πw_0). This delivers after insertion of w_0 according to Equation (25) θ_{e^2}=√2 which is approximately equal to π/2. Moreover, in Chapt.17 in [2] the statement is given that 99% of the energy of a beam with waist spot size w_0 is distributed over a cone of full angle spread λ/w_0.

From this statement it can be concluded that the energy of a beam with $w_0=\lambda/(\pi\sqrt{2})$ is distributed over a cone of full angle spread $\sqrt{2}\pi$. This shows that the propagating energy of a beam with $w_0=\lambda/(\pi\sqrt{2})$ fills in the far field an angle larger than the full solid angle. Therefore, $w_0=\lambda/(\pi\sqrt{2})$ seems to represent a physical limit for gaussian beams. This conclusion opens an interesting relationship with the above derived result of the particle picture which says that in case of a gaussian beam with $w_0=\lambda/(\pi\sqrt{2})$ the energy of the photons is in the focus totally converted into the energy of the transverse quantum mechanical motion, and that therefore the wave does not transport energy anymore.

Is the force exerted on the photons a real force?

To answer this question we take into account that the expression given by Equation (2) for the force **K**(r,z) exerted on the photon has been derived from Equation (1). Therefore, since the right hand side of Equation (1) represents an infinitesimal momentum change versus an infinitesimal time step, and in consequence physically represents a force, it can be concluded that also the right side of Equation (2) physically represents a force, though it may not be possible to assume that a photon, which is found at a certain position r,z, changes the direction of its momentum at this position. However, it seems to be necessary to assume that an overall continuous directional change of the momentum of the photons propagating with an electromagnetic wave takes place, since otherwise the photons would not follow the propagating wave, and would spread out from the wave due to their relativistic mass. Therefore, it seems to be possible to conclude that the **K**(r,z) represents a force which causes a directional change of the momentum of the photons, and in this way, forces the photons to follow the electromagnetic wave.

Another interesting aspect of the force exerted on the photon consists in the fact that, in case of a Gaussian wave, a ray bouncing between two virtual phase fronts with infinitesimal distance carries through a sinusoidal motion which according to Equation (48) in [1] is described by $r=A*\cos(\omega_\perp t)$, and therefore oscillates with a frequency $\omega_\perp$ identical with the frequency of the quantum mechanical transverse oscillation of the photon given by Equation (7). Therefore, since a ray of infinitesimal length can be considered to represent the classical equivalent of the photon, it follows that the transverse motion of this classical equivalent particle is described by the same potential which describes the motion of the photon with the difference that the photon obeys the laws of quantum mechanics. It can be therefore concluded that the same force, which is exerted on the classical equivalent particle, is also exerted on the photon. This shows that the force derived above by considering of the momentum change of the photon, also can be derived from a ray optics consideration.

Equation (1) also shows that this force does not change due to the splitting of E_{ph} into $E_{G,nm}+E_{nm}$, since the total momentum Mc of the photon remains unchanged. The latter follows from the fact that the expectation value of the momentum of the harmonic oscillator vanishes with the consequence that the total momentum Mc of the photon remains unchanged. Therefore, also the radiation pressure in the focal region remains unchanged. The latter is important, since both quantities are used to describe the optical tweezers [6-8].

Summary and conclusions

Based on a consideration of the question, why the energy density and the relativistic mass density of an electromagnetic wave are propagating in the same direction, in [1] the assumption has been made that a transverse force is exerted on the mass density and in consequence on the mass of the photons which forces them to follow the propagating energy density. This assumption leads to the result that the photon is moving within a transverse potential which allows to describe the transverse quantum mechanical motion of the photon by a Schrödinger equation which is identical with the Schrödinger equation describing the motion of the electron, except that the mass of the electron is replaced by the relativistic mass of the photon [1]. In case of Gaussian waves this Schrödinger equation is identical with the Schrödinger equation of the 2-dimensional harmonic oscillator [1]. This Schrödinger equation is used to compute the expectation value of the square of the photon's transverse momentum which, after some transformations, allows to express the effective axial propagation constant $\bar{k}_{z,nm}(z)$ of the wave as given by Eq. (3) in [4] by $\bar{k}_{z,nm}(z) = [E_{ph} - E_{nm}(z)]/\hbar c$ where E_{pn} is the total energy of the photon, and the $E_{nm}(z)$ are the energy eigenvalues of the transverse quantum mechanical motion of the photon. Since according to this result $\hbar c\bar{k}_{z,nm}(z)$ represents a real energy, also the effective axial propagation constant $\bar{k}_{z,nm}(z)$ must be assumed to represent a real propagation constant. This leads to the conclusion that $\lambda_{nm}(z) = 2\pi/\bar{k}_{z,nm}(z) = hc/(E_{ph}-E_{nm}(z))$ represents the real local wave length of the electromagnetic wave at the position z. According to this conclusion, $\lambda_{nm}(z)$ increases inversely proportional to $\bar{k}_{z,nm}(z)$ with decreasing z, and therefore, describes the z dependence of the Gouy phase shift in agreement with wave optics. But this conclusion also shows that $\lambda_{nm}(z)$ increases inversely proportional to the energy difference E_{ph}-$E_{nm}(z)$ which decreases with decreasing z. This shows that the deeper physical reason for Gouy phase shift seems to consist in the fact that the energy of the photon is increasingly converted into its transverse quantum mechanical motion when the photon approaches the focus. This explains the Gouy phase shift as an energetic effect. Based on this result the proposal has been made to designate the energy $\hbar c\bar{k}_{z,nm}$ as Gouy energy, and to introduce for this energy the term $E_{G,nm}(z)$. This delivers for the total energy of the photon the expression $E_{ph}= E_{G,nm}(z) + E_{nm}(z)$. However the particle picture not only explains the Gouy phase shift as an energetic effect, it also allows to compute the mathematical expression describing the Gouy phase shift in agreement with wave optics as already shown in [1].

Since according to these results only $E_{G,nm}(z)$ changes when photon approaches the focus but not its total energy E_{ph}, it follows that the wave length of the photon as measured by a spectroscopic device is $\lambda=hc/E_{ph}$ and not $\lambda_{nm}(z)$, because the measurement process changes the structure of the propagating wave with the consequence that the change of the wave length disappears together with the Gouy phase shift.

Since $E_{G,nm}(z)$ decreases, when the photon approaches the focus, and is transformed into $E_{nm}(z)$, which simultaneously increases by the same amount, it can be concluded that the wave structure of the photon is becoming less important compared with its particle property near the focus. This result may provide new aspects concerning the theoretical description of the optical tweezers [7-9].

The question, if the force exerted on the photons represents a real force, has been considered in Sect. 3. Equation (1) shows that the force described by **K** (r.z) is equal to the momentum of the photon multiplied with the infinitesimal directional change of the normalized Poynting vector versus a time step Δt describing an infinitesimal propagation step of the wave. Therefore, since an infinitesimal momentum change versus an infinitesimal time step represents a force, the right hand side of Equation (1) and in consequence of Equation (2) physically represents a force, though it may not be possible to conclude that a photon, which is found at a certain position r,z, changes the direction of its momentum at this position. Since however nevertheless an overall continuous directional change of the momentum of the photons must take place, which forces them to follow the electromagnetic wave, it seems that **K** (r.z) represents the force which causes this momentum change.

Since a close relationship exists between the Gouy phase shift and the reduced effective axial propagation constant in a waveguide as described in [2] Sect. 17.4, it can be expected that the reduction of the propagation constant in a waveguide can be explained in the same way as the Gouy phase shift as an energetic effect induced by a partial transfer of the photon's energy to the transverse quantum mechanical motion of the photon, as will be described in more detail in a subsequent paper.

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