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Distinguish Between The Average Magnetic Field on A Circular Coil and The Original Definition of Magnetic Field, and Correct The Serious Loopholes In Classical Electromagnetic Theory

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Abstract

The author found that magnetic fields can be measured using the ampere force on a small current element, or using a small loop current coil. Using a small loop coil current measurement actually measures the average magnetic field on the loop. Under quasi-static or magnetic quasi-static conditions, the average magnetic field measured on the loop and the magnetic field measured using a linear current element are the same. Two definitions are equivalent. There are also two methods for measuring magnetic fields for changing currents or alternating electromagnetic fields using a small coil and a linear current element. The magnetic field obtained from the measurement of alternating magnetic field by a small coil on the loop under quasi-static or magnetic quasi-static conditions is the same as that obtained from the measurement of linear current element. We usually say that a magnetic field is the curl of a vector potential. More precisely, the average magnetic field defined on the loop is the curl of the vector potential. Regardless of whether the average magnetic field is under quasi-static conditions or under magnetic quasi-static conditions, it remains the same as the magnetic field measured by the linear current element. Therefore, we can say that a magnetic field is the curl of a vector potential. But this no longer holds true in the case of radiated electromagnetic fields. For radiated electromagnetic fields, it refers to the retarded electromagnetic field. The average electromagnetic field measured on the loop and the magnetic field measured by the linear current element are different. Therefore, for electromagnetic waves, it is incorrect for the curl of the magnetic vector potential to be the magnetic field. The curl of the magnetic vector potential corresponds to the average magnetic field on the loop. This article describes the author's discovery. The author's discovery stems from the mutual energy theorem proposed by the author in 1987. People argue that this theorem is not an energy theorem, but a reciprocity theorem. In 2017, the author successfully proved that this theorem is indeed the energy theorem and developed it into the law of conservation of energy. The author further proposed the theorem of mutual energy flow. The author believes that mutual energy flow transfer the electromagnetic energy and it is the photon. The author believes that self energy flow does not transfer energy and should radiate reactive power. That is to say, electromagnetic waves should be reactive power. This indicates that the electric and magnetic fields of electromagnetic waves should maintain a 90 degree phase difference, rather than being in phase. We know that according to Maxwell's electromagnetic theory, the electric and magnetic fields of electromagnetic waves are in phase. This indicates that the energy conservation law and the mutual energy flow theorem proposed by the author conflict with Maxwell's electromagnetic theory. Thus, a loophole in Maxwell's electromagnetic theory was discovered. This vulnerability is a confusion between the average magnetic field measured on a circular coil and the magnetic field measured with a straight wire. The average magnetic field on a loop is completely different from the magnetic field on a straight wire. The original definition of a magnetic field was defined by a linear current element or straight wire. This has led to a problem with the definition of the magnetic field for radiating electromagnetic waves in Maxwell's electromagnetic theory. This issue requires us to revise some of Maxwell's radiation electromagnetic field theory.

Keywords: Magnetic Field, Ampere Force, Lorentz Force, Magnetic Vector Potential, Retarded Wave, Advanced Wave, Retarded Potential, Advanced Potential, Maxwell; Poynting, Wheeler, Feynman, Dirac, Cramer,Welch, Rumsey.

1. Introduction

After nearly 40 years of effort, the author ultimately revealed serious loopholes in Maxwell's classical electromagnetic theory. Although it was ultimately discovered that it was a problem with the definition of magnetic fields, the author has established a new electromagnetic theory during this process. The characteristic of this electromagnetic theory is to acknowledge that 1) advanced waves are objective physical phenomena, so any current produces half retarded waves and half advanced waves; The retarded wave and the advanced wave superimpose on each other on the surface of the current, rather than canceling out; 2) Electromagnetic waves propagate in the field of the charge itself, rather than in the ether or field belonging to space. Therefore, electromagnetic waves cannot propagate independently in space without their source, nor can they overflow the universe. Therefore, the mutual energy theorem is actually the law of conservation of energy; 3) Electromagnetic field theory refers to the retardation (or advancement) of the electromagnetic field rather than the retardation (or advancement) of the vector potential; 4) The mutual energy flow possesses all the properties of photons, therefore photons are mutual energy flows. The energy flow in electromagnetic field theory is the mutual energy flow rather than the self energy flow represented by the Poynting vector. The following author introduces the development process of this new electromagnetic theory.

1.1 Mutual Energy Theorem

The author completed a paper on the mutual energy theorem during his graduate studies at Xidian University in P. R. of China in 1987 [1]. The formula for the mutual energy theorem is as follows:,

$$-\int_{V_1} \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 dV = \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* dV \quad (1)$$

And established the concept of inner product on surfaces Γ,

$$(\xi_1, \xi_2) = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

Using this inner product, the author completed the spherical wave expansion, plane wave expansion of electromagnetic waves and the resulting Huygens principle [1-3]. These papers have sparked strong debates in Xidian University. Some teachers believe that this theorem cannot be called the energy theorem and can only be called the reciprocity theorem. The author hopes to prove from Poynting's theorem that this theorem is indeed an energy theorem. It is widely recognized that Poynting's theorem is the energy theorem. If this theorem can be proven from Poynting's theorem, then of course, this theorem is also the energy theorem. But the author was unable to complete this proof at that time.

1.2 The Mutual Energy Theorem is The Energy Theorem

The author later worked in the field of medical imagings. 30 years have passed in a blink of an eye, and as the author is about to retire, he has returned to the topic of the electromagnetic mutual energy theorem. The first thing the author needs to solve is whether this theorem is an energy theorem. At this time, the times had advanced, and the author found the reciprocity theorem of de Hoop's cross correlation [4]. This theorem happens to be the inverse Fourier transform of the mutual energy theorem published by the author. This theorem was published slightly later than the author, at the end of 1987. De Hoop positions this theorem as the reciprocity theorem. It seems that this theorem should indeed be called the reciprocity theorem. The author found Welch's time-domain reciprocity theorem published in 1960 in the citation of the de Hoop paper [5]. The time-domain reciprocity theorem is the core part of the de Hoop cross correlation reciprocity theorem, and can also be seen as the inverse Fourier transform of the author's mutual energy theorem. Therefore, the mutual energy theorem proposed by the author and two reciprocity theorems can be regarded as one theorem. The author found that it is easy to prove from the time domain that Welch and de Hoop's reciprocity theorem is a sub-theorem of Poynting's theorem. This actually proves that the mutual energy theorem is also a sub-theorem of Poynting's theorem. Before 1987, the author attempted to prove the theorem of mutual energy from the complex Poynting theorem, but failed. In fact, the complex Poynting theorem is not a Fourier transform of the time-domain Poynting theorem. The complex Poynting theorem and Poynting theorem are two independent theorems. To prove that the mutual energy theorem is the energy theorem, we need to start from the time-domain Poynting theorem to prove Welch or de Hoop's time-domain reciprocity theorem, and then perform the Fourier transform to obtain the mutual energy theorem. Therefore, the mutual energy theorem should be the energy theorem.

However, the author also discovered the "new reciprocity theorem" published by Rumsey in 1963 and the "second Lorentz reciprocity theorem" published by Petrusenko in 2009 [6, 7]. These reciprocity theorems are actually the same as the author's published mutual energy theorem [1]. So why do Welch, Rumsey, de Hoop, and Petrusenko position this theorem as a reciprocity theorem? The author still faces the problem. Only the author believes that this theorem is the energy theorem. The author noticed that this theorem involves advanced waves. The two fields in this theorem correspond to subscript 1 and subscript 2, respectively. One is a retarded wave, and the other must be an advanced wave. Advanced waves violate causal relationships. Not recognized in the engineering community. This may be the fundamental reason why Welch, Rumesy, de Hoop, and Petrusenko have positioned this theorem as a reciprocity theorem.

1.3 Existence of advanced waves

The author began searching for papers on advanced waves, and the first thing that caught attention was Wheeler and Feynman's absorber theory [8, 9]. The absorber theory is based on the action at a distance theory [10-12]. Another foundation of Wheeler and Feynman's absorber theory is Dirac's self force theory [13]. This self force theory advocates that current not only generates retarded waves, but generates half retarded and half advanced waves. Cramer established a quantum mechanical transactional interpretation based on Wheeler Feynman's absorber theory [14, 15]. Furthermore, Stephenson's views on advanced waves also had a significant impact on the author [16]. Wheeler, Feynman, Dirac, Stephenson, Cramer, and others all advocate that advanced waves are an objective existence in physics. After studying these theories about advanced waves, the author believes that their view that advanced waves exist objectively in physics is correct.

1.4 Mutual Energy Flow Theorem

In 2017, the author published the Mutual Energy Flow Theorem [17],

$$-\int_{V_1} \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 dV = (\xi_1, \xi_2) = \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* dV \quad (2)$$

$$(\xi_1, \xi_2) = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \quad (3)$$

Γ is a close surface surrounding the two currents. And it is believed that only mutual energy flow can transfer energy, while self energy flow

$$\oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_i^*) \cdot \hat{n} d\Gamma = 0 \quad (4)$$

can not transmit energy.

1.5 Localized Energy Conservation Law

Because if both mutual energy flow and self energy transfer energy, two different types of photons will appear, namely the photons obtained from the collapse of self energy flow and the photons from mutual energy flow. If the self energy flow does not contribute to the transfer of energy, then the following law of energy conservation holds [17].

$$\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \int_{t=-\infty}^{\infty} dt \int_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j dV = 0 \quad (5)$$

This law of conservation of energy, combined with the theorem of mutual energy flow, becomes a localized law of conservation of energy. The necessary condition for this law of conservation of energy to hold is that the self energy flow does not contribute to the transfer of energy. So of course, we have to ask where the self-energy flow has gone. Because according to Maxwell's electromagnetic theory, the self energy flow is not zero. Therefore, the author proposes the concept of reverse collapse. The self energy flow collapses in the opposite direction [17]. Reverse collapse is obtained from the Maxwell equation with time reversal.

1.6 Reactive Power Wave

The reverse collapse of self energy flow is actually a problem. To achieve reverse collapse, it is necessary to establish a time reversal wave. With time reversal waves, in fact, time reversal waves can also form the mutual energy flow of time reversal waves. The mutual energy flow of this time reversal wave may also offset the mutual energy flow, leaving only zero solutions. Zero solution is certainly not what the author wants. Therefore, the author also proposes the concept of reactive power. The author found that if the self energy flow is reactive power, the self energy flow itself does not transfer energy, so there is no need for the reverse collapse of the self energy flow. But the self energy flow calculated according to Maxwell's field is not reactive power. According to Maxwell's electromagnetic theory, the electric and magnetic fields of electromagnetic waves are in phase. This indicates that electromagnetic waves are of active power. That is to say, the self energy flow is the active power.

The author studied the working principle of transformers. It was found that the self energy flow generated by the primary and secondary coils of the transformer is reactive power. Transformers operate under quasi-static magnetic conditions. The author found that under quasi-static or magnetic quasi-static conditions, the self energy flow composed of electric and magnetic fields is reactive power. Since quasi-static and magnetic quasi-static electromagnetic fields are reactive power, why does radiated electromagnetic fields become active power? The author is thinking that if the secondary coil of the transformer is moved far away from the primary coil, the primary coil of the transformer will become a transmitting antenna, and the secondary coil will become a receiving antenna. The author is asking himself, from two coils tightly together, which is a transformer, to the secondary coil moving further away, there is no qualitative change at this time. Why does the self energy flow transition from reactive power to active power? Why is the electromagnetic field of the primary and secondary coils of a transformer reactive power, but the electromagnetic field of the transmitting and receiving antennas active power. At what distance did the primary and secondary coils of the transformer transition to the antenna and receiving antenna? After a series of studies The author found that the primary and secondary coils of transformers, as well as the electromagnetic fields of transmitting and receiving antennas, are actually reactive power [18-31]. Maxwell's electromagnetic theory was mistaken. What exactly went wrong with Maxwell's electromagnetic theory? The author found that the error was due to Maxwell defining a magnetic field as the curl of a vector potential [29]. The curl of a vector potential is actually the average magnetic field defined on the loop. The average magnetic field and the original definition of magnetic field are the same under magnetic quasi-static conditions or quasi-static conditions. Therefore, under quasi-static conditions, the original definition of magnetic field can be replaced by the average magnetic field on the loop. But when it comes to radiated electromagnetic fields, or for retarded electromagnetic fields, the average magnetic field on the loop and the original definition of the magnetic field are different. Maxwell's electromagnetic theory did not pay attention to this and still replaced the original definition of magnetic field with the average magnetic field on the loop, which led to errors.

1.7 Radiation Does Not Overflow From The Universe

The author established this belief around 2021 that electromagnetic waves propagate in the electrostatic field of the charge itself, so they cannot move independently of the charge. Therefore, electromagnetic waves cannot overflow the universe. The axiom that electromagnetic waves cannot overflow the universe can be used as the author's electromagnetic theory, from which the localized law of energy conservation can be derived (5). And thus establish the theorem that electromagnetic waves are reactive power.

1.8 Research Object of This Article

At this point, the author has made it clear that Maxwell's electromagnetic theory does indeed have problems or loopholes (bugs). The author begins to study how to prove to everyone that Maxwell's electromagnetic theory is indeed problematic. The author began to pay attention to the definition of magnetic fields. What is the original definition of magnetic field? The author of this article starts with the discussion of the original definition of magnetic field. Establish two definitions of magnetic fields, one defined according to the original definitions of Ampere force or Lorentz force. One is the definition of magnetic field developed by Maxwell, which is to calculate the average value of the magnetic field on a circular path. The author needs to prove that these two definitions of magnetic fields are completely identical under quasi-static electromagnetic field conditions. But when it comes to radiated electromagnetic fields, which are retarded electromagnetic fields, these two types of magnetic fields are defined differently. The author found that Maxwell's electromagnetic theory used the curl of the magnetic vector potential to define a magnetic field under radiated electromagnetic fields. This definition is actually to calculate the average value of the magnetic field on the loop, not the original definition of the magnetic field. This definition constitutes an error in the calculation of Poynting vectors and related quantities. This issue is also the reason why people believe that electromagnetic waves are in phase. In fact, the phase between the electric and magnetic fields of electromagnetic waves is 90 degrees. Therefore, electromagnetic waves are reactive power waves.

In addition, the author was able to discover this issue because their electromagnetic theory introduced the mutual energy flow theorem (2, 3), the law of conservation of energy (5), as well as the concept of advanced waves and the concept of radiation not overflowing the universe. And when we do prove the loopholes in Maxwell's electromagnetic theory, it appears that the author's theory, including the theorem of mutual energy flow, the law of conservation of energy, and the concept of radiation not overflowing the universe, is correct.

2. Differences Between The Author's Theory and Classical Electromagnetic Theory

2.1 Coulomb's Law or Gauss's Law Still Holds

Gauss's law can be derived from Coulomb's law,

$$\nabla \cdot \boldsymbol{E}_s = \rho/\epsilon_0 \tag{6}$$

ρ is the charge density. ϵ_0 is the vacuum dielectric constant. $\boldsymbol{E}_s$ is an electrostatic field, and the subscript s is used to indicate that this electric field is a static electric field.

$$\boldsymbol{E}_s \triangleq -\nabla\phi \tag{7}$$

The symbol ≜ means defined. ϕ is the electrostatic potential. In the author's electromagnetic theory, it is still believed that Gaussian law (6) holds.

2.2 Neumman's Law of Electromagnetic Induction

The author also believes that Neumman's law of electromagnetic induction is valid, that is,

$$\mathcal{E}_{2,1} = -\frac{\partial}{\partial t}\frac{\mu_0}{4\pi}\oint_{C_2}\oint_{C_1}\frac{I_1 d\boldsymbol{l}_1\cdot d\boldsymbol{l}_2}{r} \tag{8}$$

$$\mathcal{E}_{2,1} \triangleq \oint_{C_2}\boldsymbol{E}_1\cdot d\boldsymbol{l} \tag{9}$$

$E_{2,1}$ is the induced electromotive force generated on coil 2 by the current I_1 on coil 1. E_1 is The electric field of the current I_1 of coil 1. C_1 , C_2 are two coils, the primary coil and the secondary coil. $\oint_{C_1}$ is closed line intgral on C_1.

2.3 Vector Potential

In the author's electromagnetic theory, vector potential still holds,

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi}\oint_{C_1}\frac{I_1}{r}d\boldsymbol{l}_1 \tag{10}$$

Consider,

$$\int_C \cdots Id\boldsymbol{l} \rightarrow \int_V \cdots \boldsymbol{J}dV \tag{11}$$

there is,

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}_1}{r} dV \tag{12}$$

The subscript in the above equation can be omitted,

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r} dV \tag{13}$$

The above equation is the vector potential. The curl of a vector potential is,

$$\nabla \times \boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \nabla\frac{1}{r} \times \boldsymbol{J} dV$$

$$= \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^3} dV \tag{14}$$

2.4 Faraday's Law

Obtained from (8, 10),

$$\mathcal{E}_{2,1} = -\frac{\partial}{\partial t}\oint_{C_2} \boldsymbol{A} d\boldsymbol{l}_2 \tag{15}$$

Consider (9),

$$\oint_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l} = -\frac{\partial}{\partial t}\oint_{C_2} \boldsymbol{A}_1 d\boldsymbol{l} \tag{16}$$

Or

$$\oint_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l} = -\oint_{C_2} \frac{\partial}{\partial t}\boldsymbol{A}_1 d\boldsymbol{l} \tag{17}$$

Or

$$\oint_{C_2} (\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) \cdot d\boldsymbol{l} = 0 \tag{18}$$

Or

$$\oiint_{\Gamma} \nabla \times (\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) \cdot \hat{n} d\Gamma = 0 \tag{19}$$

Or

$$\nabla \times (\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) = 0 \tag{20}$$

Or

$$\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1 = -\nabla\phi_1 \tag{21}$$

Or

$$\boldsymbol{E}_1 = -\nabla\phi_1 - \frac{\partial}{\partial t}\boldsymbol{A}_1 \tag{22}$$

Subscripts can be omitted,

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{23}$$

The above equation is Faraday's law. The above equation is also one of the Maxwell equations defined by Maxwell himself. Scientists of the same period as Maxwell, such as Kirchhoff and Lorenz, did not define electric and magnetic fields, so they would give the following Ohm's law,

$$\boldsymbol{J} = \sigma(-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}) \tag{24}$$

Similarly, they do not define $\nabla\times\mathbf{A}$ as a magnetic field. The author believes that Maxwell's definition of $\boldsymbol{E} \triangleq -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}$ is acceptable, so when encountering Ohm's law, it can be written as,

$$\boldsymbol{J} = \sigma \boldsymbol{E} \quad (25)$$

2.5 Biot-Savart Law

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} \int_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^3} dV \quad (26)$$

Consider(14, 26),

$$\nabla \times \boldsymbol{A} = \boldsymbol{B} \quad (27)$$

Note that the above formula is only obtained under quasi-static conditions. We will prove in the following chapters that this formula (27) is not suitable for radiating electromagnetic fields.

For now, we still assume that the above equation (27) holds, so we have,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t} \nabla \times \boldsymbol{A} \quad (28)$$

Consider the formula (27) to obtain our Faraday's law today,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t} \boldsymbol{B} \quad (29)$$

This form of Faraday's law was not proposed by Maxwell himself, and Maxwell himself adopted it (23). Due to issues with the formula (27), this Faraday's law (29) also has its limitations.

2.6 Divergence of Vector Potential

$$\nabla \cdot \boldsymbol{A} = \nabla \cdot \frac{\mu_0}{4\pi} \int_V \frac{\boldsymbol{J}}{r} dV$$

$$= \frac{\mu_0}{4\pi} \int_V \nabla \frac{1}{r} \cdot \boldsymbol{J} dV$$

$$= -\frac{\mu_0}{4\pi} \int_V \nabla' \frac{1}{r} \cdot \boldsymbol{J} dV \quad (30)$$

$$\nabla' \cdot (\frac{1}{r} \boldsymbol{J}) = \nabla' (\frac{1}{r}) \cdot \boldsymbol{J} + (\frac{1}{r}) \nabla' \cdot \boldsymbol{J} \quad (31)$$

$$\int_V \nabla' \cdot (\frac{1}{r} \boldsymbol{J}) dV = \oiint_\Gamma (\frac{1}{r} \boldsymbol{J}) \cdot \hat{n} d\Gamma = 0 \quad (32)$$

$\oiint_\Gamma$ is closed surface integral on Γ. Here assume that $\boldsymbol{J}$ is inside of Γ.

$$\int_V \nabla' (\frac{1}{r}) \cdot \boldsymbol{J} dV + \int_V (\frac{1}{r}) \nabla' \cdot \boldsymbol{J} dV = 0 \quad (33)$$

$$-\int_V \nabla' (\frac{1}{r}) \cdot \boldsymbol{J} dV = +\int_V (\frac{1}{r}) \nabla' \cdot \boldsymbol{J} dV \quad (34)$$

$$\nabla \cdot \boldsymbol{A} = \frac{\mu_0}{4\pi} \int_V (\frac{1}{r}) \nabla' \cdot \boldsymbol{J} dV \quad (35)$$

Using the continuity equation of current,

$$\nabla' \cdot \boldsymbol{J} = -\frac{\partial}{\partial t} \rho \quad (36)$$

$$\nabla \cdot \boldsymbol{A} = -\frac{\mu_0}{4\pi} \int_V (\frac{1}{r}) \frac{\partial}{\partial t} \rho dV \quad (37)$$

$$\nabla \cdot \boldsymbol{A} = -\frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \int_V (\frac{1}{r}) \rho dV \quad (38)$$

Defining scalar potentials

$$\phi \triangleq \frac{1}{4\pi\epsilon_0}\int_V \left(\frac{1}{r}\right)\rho dV \tag{39}$$

Obtaining Lorenz gauge condition,

$$\nabla \cdot \boldsymbol{A} = -\mu_0\epsilon_0\frac{\partial}{\partial t}\phi \tag{40}$$

It is worth mentioning that the derivation method for the divergence of vector potentials mentioned above was completed by Kirchhoff in 1857 [32]. However, what Kirchhoff obtained was another gauge, the Kirchhoff gauge condition,

$$\nabla \cdot \boldsymbol{A}_W = \mu_0\epsilon_0\frac{\partial}{\partial t}\phi \tag{41}$$

Among them, $\boldsymbol{A}_W$ is the Weber vector potential. The definition of Weber vector potential is different from that of Neumman's vector potential $\boldsymbol{A}$,

$$\boldsymbol{A}_W = \frac{\mu_0}{4\pi}\int_V \frac{(\boldsymbol{J}\cdot\boldsymbol{r})\cdot\boldsymbol{r}}{r^3}dV \tag{42}$$

The mathematical derivations of (41) completed by Kirchhoff in 1857 were still very advanced in terms of theoretical derivation. This is one of Kirchhoff's significant contributions to electromagnetic theory. The Lorenz guage condition was completed by Lorenz in 1867. The Lorenz gauge condition is based on the Kirchhoff gauge condition. Therefore, in fact, the Lorenz gauge condition should be referred to as the Kirchhoff-Lorenz gauge condition. In the same paper, Lorenz also proposed the concept of retarded potential.

2.7 Ampere's Circuital Law

Consider mathematical formulas

$$\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A} \tag{43}$$

Or,

$$\nabla \times (\nabla \times \boldsymbol{A}) - \nabla(\nabla \cdot \boldsymbol{A}) = -\nabla^2\boldsymbol{A} \tag{44}$$

Consider the Lorenz gauge (41) to obtain

$$\nabla \times (\nabla \times \boldsymbol{A}) + \mu_0\epsilon_0\nabla\frac{\partial}{\partial t}\phi = -\nabla^2\boldsymbol{A}$$

$$= -\nabla^2\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r}dV$$

$$= -\frac{\mu_0}{4\pi}\int_V \nabla^2\frac{1}{r}\boldsymbol{J}dV$$

$$= \mu_0\int_V \left(-\nabla^2\frac{1}{4\pi r}\right)\boldsymbol{J}dV$$

$$= \mu_0\int_V \delta(\boldsymbol{x}-\boldsymbol{x}')\boldsymbol{J}(\boldsymbol{x}')dV$$

$$= \mu_0\boldsymbol{J}(\boldsymbol{x}') \tag{45}$$

Or

$$\nabla \times (\nabla \times \boldsymbol{A}) + \mu_0\epsilon_0\frac{\partial}{\partial t}\nabla\phi = \mu_0\boldsymbol{J}(\boldsymbol{x}') \tag{46}$$

Or

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0\boldsymbol{J}(\boldsymbol{x}') + \mu_0\frac{\partial}{\partial t}\epsilon_0\boldsymbol{E}_s \tag{47}$$

We obtain Ampere's circuital law,

$$\nabla \times \boldsymbol{B} = \mu_0\boldsymbol{J}(\boldsymbol{x}') + \mu_0\frac{\partial}{\partial t}\epsilon_0\boldsymbol{E}_s \tag{48}$$

Or,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}(\boldsymbol{x}') + \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}_s \tag{49}$$

Everyone has found that the above circuital law also uses the formula (27). The author mentioned earlier that this formula is problematic, so the circuital law mentioned above (49) is also problematic. However, the formula (47) did not use the formula (27). Therefore, the formula (47) remains valid.

2.8 Quasi Static Equations

$$\nabla \cdot \boldsymbol{E}_s = \rho/\epsilon_0 \tag{50}$$

The above equation is Gaussian law of electricity.

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{51}$$

The above equation or (23) is Faraday's law. The Faraday's law in Maxwell's own Maxwell equation is the above equation. It is worth mentioning that scholars of Maxwell's time, such as Kirchhoff and Lorenz, did not introduce electric and magnetic fields [32, 33], so for them, the above equation is often written as

$$\boldsymbol{J} = \sigma(-\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A}) \tag{52}$$

σ is the electrical conductivity. Writing of Ampere's circuital Law (47),

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0 \boldsymbol{J}(\boldsymbol{x}') + \mu_0\epsilon_0 \frac{\partial}{\partial t}\boldsymbol{E}_s \tag{53}$$

where in

$$\boldsymbol{E}_s = -\nabla\phi \tag{54}$$

For Kirchhoff, Lorenz, the $\nabla \times \mathbf{A}$ in the equation (53) will not be written as B. Because they do not introduce the concepts of electric and magnetic fields at all [32, 33]. Only dealing with vector potentials and scalars potential.

Continuous equation of current

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial}{\partial t}\rho \tag{55}$$

This current continuity equation is also Kirchhoff's contribution in 1857 [32]. In this way, we obtain the quasi-static equation (50-55). The problematic formulas are all related to the curl formula of the vector potential, that is,

$$\nabla \times \boldsymbol{A} = \boldsymbol{B} \tag{56}$$

Due to the issue with this formula, it also affects the following two formulas,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}(\boldsymbol{x}') + \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}_s \tag{57}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{58}$$

In the next chapter, we will discuss in detail the problem with the formula (56).

2.9 Solutions to Quasi-Static Equations

The solution to the quasi-static equation is,

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r} dV \tag{59}$$

$$\phi = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{r} dV \tag{60}$$

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{61}$$

The author believes that

$$\boldsymbol{E}_i \triangleq -\frac{\partial}{\partial t}\boldsymbol{A} \tag{62}$$

It is indeed an electric field, which can be called an induced electric field, but it is worth mentioning the electrostatic field,

$$\boldsymbol{E}_s \triangleq -\nabla\phi \tag{63}$$

It may also be problematic in situations of radiated electromagnetic fields. However, this article does not discuss this issue. This issue needs to be addressed in future papers. The author suddenly realized that Kirchhoff and Lorenz were very clever in not defining electric and magnetic fields. In the next chapter, we will discuss the problem with the formula (56).

3. Define and Measure Magnetic Fields According to Ampere Force and Lorentz Force

We discussed the measurement of magnetic fields in the paper [29]. This chapter continues to discuss this topic. See figure 1.

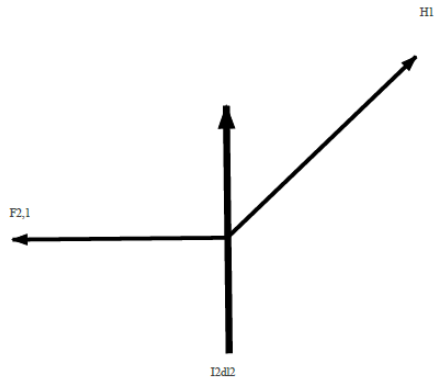

Figure 1: Assuming That the Magnetic Field is $\boldsymbol{H}_1 = \boldsymbol{H}_1\,\hat{y}$, The Current Element is $\boldsymbol{I}_2\, d\boldsymbol{l}_2 = \boldsymbol{I}_2\, dl\hat{z}$. The Force Acting on This Current Element is The Ampere Force $\boldsymbol{F}_{2,1}$.

3.1 Ampere Force

$\boldsymbol{B}_1$ is the magnetic field of the current element $I_1\, d\boldsymbol{l}_1$. This magnetic field is present in the current element $I_2\, d\boldsymbol{l}_2$ The force is,

$$d\boldsymbol{F}_{2,1} = I_2 d\boldsymbol{l}_2 \times \boldsymbol{B}_1 \tag{64}$$

$d\boldsymbol{F}_{2,1}$ is the ampere force. This force is a force from 1 to 2. From linear current to body current,

$$I_2 d\boldsymbol{l}_2 \rightarrow \boldsymbol{J}_2 dV = \rho_2 v_2 dV \tag{65}$$

Hence, there is,

$$d\boldsymbol{F}_{2,1} = \rho_2 v_2 dV \times \boldsymbol{B}_1 \tag{66}$$

Hence, there is,

$$\boldsymbol{f}_{2,1} \triangleq \frac{d\boldsymbol{F}_{2,1}}{dV} = \rho_2 v_2 \times \boldsymbol{B}_1 \tag{67}$$

$\boldsymbol{f}_{2,1}$ is the Lorentz force, which is the force on $\rho_2\, v_2$ in the magnetic field $\boldsymbol{B}_1$. Because it is a force per unit volume, this force is a type of stress.

3.2 Definition of Magnetic Field Based on The Average Ampere Force on the Loop

To measure the magnetic field, we assume that for the magnetic field $\boldsymbol{B}_1$, the direction is known. If we don't know the direction of the magnetic field, we will measure a component of magnetic field in a certain direction,

$$\boldsymbol{B}_1 = B_1\hat{y} \tag{68}$$

We take

$$I_2 d\boldsymbol{l}_2 \perp \boldsymbol{B}_1 \tag{69}$$

Consider (64), there is,

$$I_2 d\boldsymbol{l}_2 \times d\boldsymbol{F}_{2,1} = I_2 d\boldsymbol{l}_2 \times (I_2 d\boldsymbol{l}_2 \times \boldsymbol{B}_1)$$

$$= I_2 d\boldsymbol{l}_2 (\boldsymbol{B}_1 \cdot I_2 d\boldsymbol{l}_2) - \boldsymbol{B}_1 (I_2 d\boldsymbol{l}_2 \cdot I_2 d\boldsymbol{l}_2)$$

$$= -\boldsymbol{B}_1 (I_2 d\boldsymbol{l}_2 \cdot I_2 d\boldsymbol{l}_2) \tag{70}$$

The above equation considers the formula (69) and the following vector mathematical formulas,

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{c} \cdot \boldsymbol{a}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b}) \tag{71}$$

The formula (70) can be rewritten as,

$$\boldsymbol{B}_1 (I_2 d\boldsymbol{l}_2 \cdot I_2 d\boldsymbol{l}_2) = -I_2 d\boldsymbol{l}_2 \times d\boldsymbol{F}_{2,1} \tag{72}$$

Or

$$\boldsymbol{B}_1 = \frac{1}{(I_2 d\boldsymbol{l}_2 \cdot I_2 d\boldsymbol{l}_2)} d\boldsymbol{F}_{2,1} \times I_2 d\boldsymbol{l}_2$$

$$= \frac{1}{I_2 (d\boldsymbol{l}_2 \cdot d\boldsymbol{l}_2)} d\boldsymbol{F}_{2,1} \times d\boldsymbol{l}_2$$

$$= \frac{1}{I_2 |d\boldsymbol{l}_2|^2} d\boldsymbol{F}_{2,1} \times d\boldsymbol{l}_2$$

$$= \frac{d\boldsymbol{F}_{2,1}}{I_2 |d\boldsymbol{l}_2|} \times \frac{d\boldsymbol{l}_2}{|d\boldsymbol{l}_2|} \tag{73}$$

Therefore, the magnitude of the magnetic field,

$$B_1 = \frac{dF_{2,1}}{I_2 |d\boldsymbol{l}_2|} \tag{74}$$

The above formula is very important, and it is necessary for us to conduct dimensional analysis to verify its correctness,

$$[B_1] = MT^{-2}A^{-1} \tag{75}$$

$$[\frac{dF_{2,1}}{I_2 |d\boldsymbol{l}_2|}] = \frac{MLT^2}{AL} = MT^2 A^{-1} \tag{76}$$

Hence, there is,

$$[B_1] = [\frac{dF_{2,1}}{I_2 |d\boldsymbol{l}_2|}] \tag{77}$$

Therefore, the dimension of the formula (73) is correct. Obtained from formula (74) the following,

$$\int_{C_2} B_1 dl = \int_{C_2} \frac{dF_{2,1}}{I_2 |d\boldsymbol{l}_2|} dl \tag{78}$$

C_2 is a circular loop. consider,

$$\int_{C_2} \overline{B}_1 dl = \int_{C_2} B_1 dl \tag{79}$$

$\overline{B}_1$ is the average magnetic field on the loop.

$$\overline{B}_1 \int_{C_2} dl = \int_{C_2} B_1 dl \tag{80}$$

Or

$$\overline{B}_1 L_2 = \int_{C_2} B_1 dl \tag{81}$$

among them,

$$L_2 \triangleq \int_{C_2} dl \tag{82}$$

$$\overline{B}_1 = \frac{1}{L_2}\int_{C_2} B_1 dl \tag{83}$$

Or,

$$\overline{B}_1 = \frac{1}{L_2}\int_{C_2} \frac{dF_{2,1}}{I_2|d\boldsymbol{l}_2|} \tag{84}$$

define,

$$f_{2,1} \triangleq \frac{dF_{2,1}}{I_2|d\boldsymbol{l}_2|} \tag{85}$$

$F_{2,1}$ is the ampere force on the current I_2 and length $d\boldsymbol{l}_2$, the average force is,

$$\overline{B}_1 = \frac{1}{L_2}\int_{C_2} f_{2,1} dl \tag{86}$$

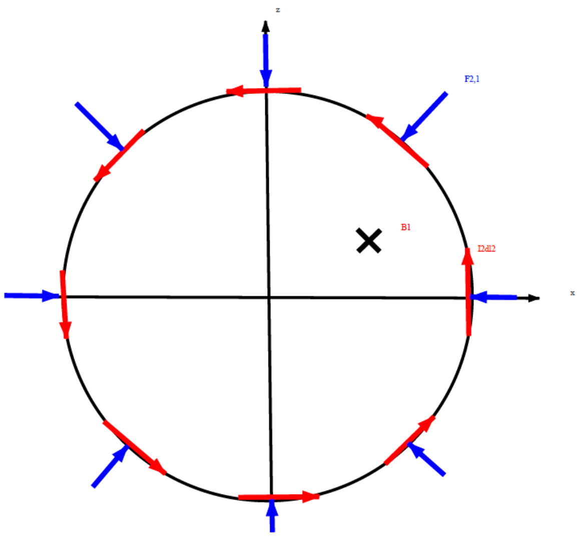

Figure 2: We Know That the Magnetic Field is $\boldsymbol{H}_1$ The Current Element is $I_2\, d\boldsymbol{l}_2$ The Force Acting on This Current Element is The Ampere Force $d\boldsymbol{F}_{2,1}$.

From the figure 2, it can be seen that the magnetic field above is actually the average value defined on the loop. Therefore, the following equation should be used to represent it more reasonably.

$$\overline{B}_1 \triangleq \frac{1}{L_2}\int_{C_2} f_{2,1} dl = \frac{1}{L_2}\sum_{i=1}^{N} f_{i2,1}\Delta l \tag{87}$$

If this $f_{2,1}$ is constant, there is,

$$\overline{B}_1 \triangleq f_{2,1}\frac{1}{L_2}\int_{C_2} dl = f_{2,1}$$

The schematic diagram of the average force is shown in Figure 2. We know that,

$$B_1 = f_{2,1} \tag{88}$$

If the magnetic field is constant, that is, the direction of the magnetic field is independent of the current element used to measure the magnetic field, we can define and measure it using a small current element (88), or we can define the magnetic field using the average method according to the formula (87). The two are the same. That is,

$$B_1 = \overline{B}_1 \tag{89}$$

3.3 Hall Effect Measurement of Magnetic Field

The Hall element can use the Hall effect to measure the magnitude of the magnetic field. The schematic diagram of the Hall effect element is shown in Figure 3.

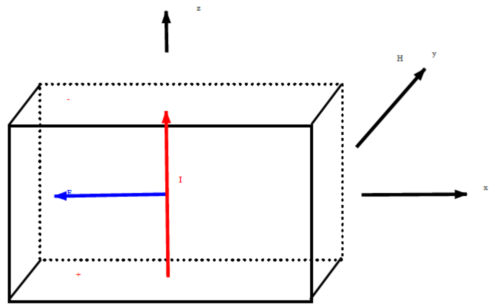

Figure 3: This picture shows the Hall effect component. The Hall effect has a current direction, if indicated by a red arrow. And the direction of the Lorentz force.

The Hall element can be represented by the following symbols: a red arrow indicates the direction of current, a blue arrow indicates the direction of ampere force, and a cross indicates the direction of magnetic field.

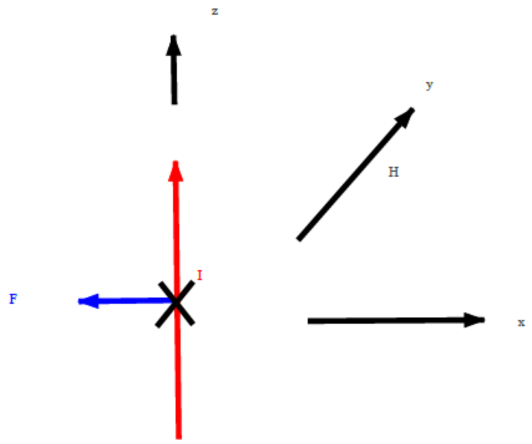

Figure 4: This diagram is a simplified representation of the Hall effect element. The Hall effect has a current direction, if indicated by a red arrow. Use blue for the direction of force. The direction of the magnetic field is represented by a cross.

We can certainly measure the magnetic field using the Hall effect, assuming that the measured magnetic field is

$$H_{1i} \tag{90}$$

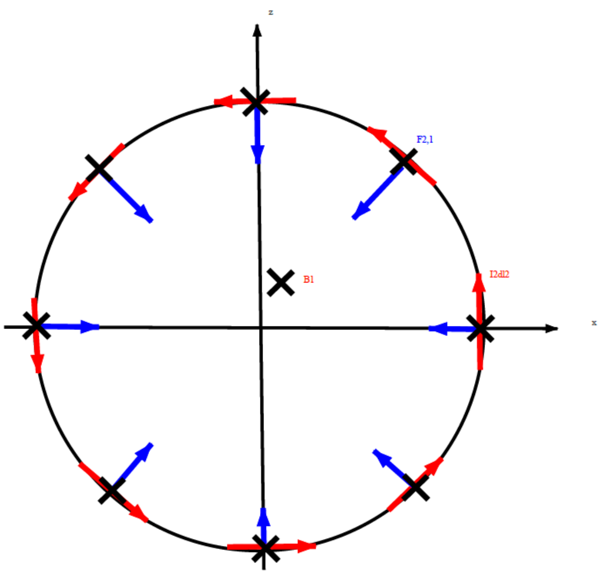

Figure 5: We Can Certainly Arrange The Hall Effect Components on A Loop, So That We Can Measure The Average Magnetic Field on This Loop

We can also calculate the average magnetic field on the loop. See figure 4,5.

$$\overline{\boldsymbol{H}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{H}_{1i} \tag{91}$$

Under quasi-static conditions, the average magnetic field and magnetic field on the loop are the same, i.e

$$\overline{\boldsymbol{H}} = \boldsymbol{H}_{1i} \tag{92}$$

It is worth mentioning that we have not yet proven that under quasi-static conditions, this average magnetic field is the same as the originally defined magnetic field. This proof will be presented in later chapters.

4. Measurement of Magnetic Field Using Induced Electromotive Force

We also have a method for measuring magnetic fields for alternating currents, which is the method of induced electromotive force. Placing a coil in an alternating magnetic field can measure the magnitude of the magnetic field.

4.1 Neumann's Law of Electromagnetic Induction

$$\mathcal{E}_{2,1}^{O} = -\frac{\partial}{\partial t}\frac{\mu_0}{4\pi}\oint_{C_2}\oint_{C_1}\frac{I_1 d\boldsymbol{l}_1\cdot d\boldsymbol{l}_2}{r} \tag{93}$$

The superscript O indicates that coil 2 is a circular circuit. Define vector potential,

$$\boldsymbol{A}_1 \triangleq \frac{\mu_0}{4\pi}\oint_{C_1}\frac{I_1 d\boldsymbol{l}_1}{r} \to \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}_1}{r} dV \tag{94}$$

Hence, there is,

$$\mathcal{E}_{2,1}^{O} = -\frac{\partial}{\partial t}\oint_{C_2} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{95}$$

The definition of induced electromotive force is,

$$\mathcal{E}_{2,1}^{O} \triangleq \oint_{C_2} \boldsymbol{E}_1 \cdot dl \tag{96}$$

Hence, there is,

$$\oint_{C_2} \boldsymbol{E}_1 \cdot dl = -\frac{\partial}{\partial t}\oint_{C_2} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{97}$$

Or

$$\oint_{C_2} \boldsymbol{E}_1 \cdot dl = -\oint_{C_2} \frac{\partial}{\partial t}\boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{98}$$

Or,

$$\oint_{C_2} (\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) \cdot dl = 0 \tag{99}$$

Or,

$$\iint_{\Gamma} \nabla \times (\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) \cdot \hat{n} d\Gamma = 0 \tag{100}$$

Or,

$$\nabla \times (\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1) = 0 \tag{101}$$

Omit subscripts,

$$\nabla \times (\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A}) = 0 \tag{102}$$

Or

$$\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A} = -\nabla\phi \tag{103}$$

Or

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\phi \tag{104}$$

Or

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\nabla \times \boldsymbol{A} \tag{105}$$

Notice that,

$$\nabla \times \boldsymbol{A} \triangleq \lim_{\Delta S \to 0} \frac{\oint_C \boldsymbol{A}\cdot d\boldsymbol{l}}{\Delta S}$$

The curl of a vector potential ∇×**A** is defined on a loop, which we will first denote as

$$\boldsymbol{G} = \nabla \times \boldsymbol{A} \tag{106}$$

Taking the curl of a vector potential is equal to taking a line integral over a loop. Therefore, the average value is taken on the loop, so we do not use the magnetic field ***B*** to represent it, but use $\overline{\boldsymbol{B}} = \boldsymbol{G}$ to represent it.

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{G} \tag{107}$$

$$\oint_{C_2} \boldsymbol{E}\cdot d\boldsymbol{l} = -\frac{\partial}{\partial t}\iint_{\Gamma} \boldsymbol{G}\cdot\hat{n}d\Gamma \tag{108}$$

$$\oint_{C_2} \boldsymbol{E}\cdot d\boldsymbol{l} = -j\omega\iint_{\Gamma} \boldsymbol{G}\cdot\hat{n}d\Gamma \tag{109}$$

$$\iint_{\Gamma} \boldsymbol{G}\cdot\hat{n}d\Gamma = \frac{1}{-j\omega}\oint_{C_2} \boldsymbol{E}\cdot d\boldsymbol{l} \tag{110}$$

$$\boldsymbol{G}\cdot\hat{n} = \frac{1}{-j\omega}\frac{1}{\Delta S}\oint_{C_2} \boldsymbol{E}\cdot d\boldsymbol{l} \tag{111}$$

where in

$$\Delta S = \iint_{\Gamma} d\Gamma \tag{112}$$

$$\lim_{\Delta S\to 0}\boldsymbol{G}\cdot\hat{n} = \frac{1}{-j\omega}\lim_{\Delta S\to 0}\frac{1}{\Delta S}\oint_{C_2} \boldsymbol{E}\cdot d\boldsymbol{l} \tag{113}$$

Or

$$\overline{\boldsymbol{B}}\cdot\hat{n} = \frac{1}{-j\omega}\lim_{\Delta S\to 0}\frac{1}{\Delta S}\oint_{C_2} \boldsymbol{E}\cdot d\boldsymbol{l} \tag{114}$$

We use the $\overline{\boldsymbol{B}}$ instead of ***B*** in the above equation because there is a circular coil C_2 on the integral of the right side of the equation. Such a measurement actually takes the average value on the loop. Therefore, the average magnetic field is obtained.

$$\overline{\boldsymbol{B}}\cdot\hat{n} = \frac{1}{-j\omega}\nabla \times \boldsymbol{E}\cdot\hat{n} \tag{115}$$

$$\overline{\boldsymbol{B}} = \frac{1}{-j\omega}\nabla \times \boldsymbol{E} \tag{116}$$

Consider $\hat{n} = \hat{y}$, formula (114)

$$\bar{B} = \overline{\boldsymbol{B}}\cdot\hat{y} = \frac{1}{-j\omega}\lim_{\Delta S\to 0}\frac{\oint_C \boldsymbol{E}\cdot d\boldsymbol{l}}{|\Delta S|} \tag{117}$$

Note that we actually calculated the average magnetic field for a loop, which is

$$\overline{H} = \frac{1}{-j\mu_0\omega} \lim_{R\to 0} \frac{\oint_C \boldsymbol{E}\cdot d\boldsymbol{l}}{\pi R^2} \tag{118}$$

Considering

$$\frac{1}{\omega\mu_0} = \frac{\sqrt{\mu_0\epsilon_0}}{\omega\sqrt{\mu_0\epsilon_0}\mu_0} = \frac{\sqrt{\mu_0\epsilon_0}}{k\mu_0} = \frac{\sqrt{\mu_0\epsilon_0}}{k\mu_0} = \frac{1}{k\eta_0} = \frac{\lambda}{2\pi\eta_0} \tag{119}$$

There is,

$$\overline{H} = j\frac{\lambda}{2\pi\eta_0} \lim_{R\to 0} \frac{\oint_C \boldsymbol{E}\cdot d\boldsymbol{l}}{\pi R^2} \tag{120}$$

Note that we actually calculated the average magnetic field for a loop, which is

$$\overline{H}_1 = j\frac{\lambda}{2\pi\eta_0} \lim_{R\to 0} \frac{\oint_{C_2} \boldsymbol{E}_1\cdot d\boldsymbol{l}}{\pi R^2} \tag{121}$$

From the above equation, it can be seen that $\overline{H}_1$ is the average value of the magnetic field measured in the loop C_2 . Alternatively,

$$\overline{H}_1 = j\frac{\lambda}{2\pi\eta_0} \lim_{R\to 0} \frac{\mathcal{E}_{2,1}^{O}}{\pi R^2} \tag{122}$$

However, under quasi-static magnetic conditions, the annular average magnetic field and magnetic field are equal.

$$\overline{H}_1 = H_1 \tag{123}$$

We will also provide a proof about this later. The above equation is not self explanatory.

4.2 Calculating The Average Magnetic Field Based on The Law of Electromagnetic Induction

On the above $\overline{B}_1$ is actually the value determined by C_2. It is the magnetic field obtained by averaging the values on the loop C_2. If we already know the induced electric field, we can calculate the average magnetic field from it, so it can be written as

$$\nabla\times\boldsymbol{E} = -\frac{\partial\overline{\boldsymbol{B}}}{\partial t} \tag{124}$$

$$\nabla\times\boldsymbol{E} = -j\omega\overline{\boldsymbol{B}} \tag{125}$$

$$\overline{\boldsymbol{B}} = \frac{1}{-j\omega}\nabla\times\boldsymbol{E} \tag{126}$$

Therefore, we can define,

$$\overline{\boldsymbol{H}} = \frac{1}{-j\omega\mu_0}\nabla\times\boldsymbol{E} \tag{127}$$

Considering (119)

$$\overline{\boldsymbol{H}} = j\frac{1}{k\eta_0}\nabla\times\boldsymbol{E} \tag{128}$$

In a special case,

$$\boldsymbol{E} = E_0\exp(-jkx)(-\hat{z}) \tag{129}$$

Hence, there is,

$$\overline{\boldsymbol{H}} = j\frac{1}{k\eta_0}\nabla\times\boldsymbol{E}$$

$$= j\frac{1}{k\eta_0}(-jk\hat{x}\times E_0\exp(-jkx)(-\hat{z}))$$

$$= \frac{E_0}{\eta_0}\exp(-jkx)\hat{y} \tag{130}$$

4.3 Measuring Induced Electromotive Force Using a Straight Wire

Under quasi-static magnetic conditions, we can use the average magnetic field method on the loop to obtain the magnetic field. But of course, we can also use a straight wire to measure the magnetic field. The author first discovered this issue in the paper [29].

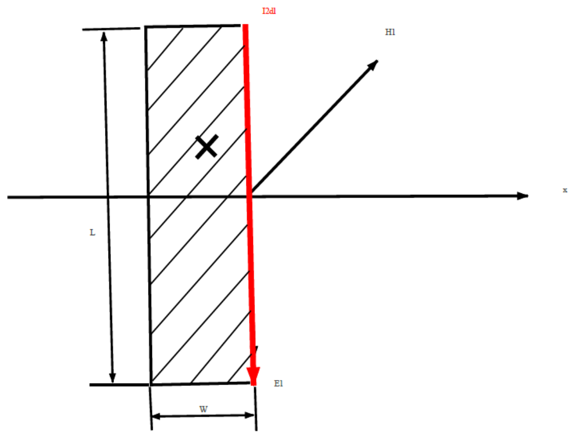

Figure 6: We know that the magnetic field is $\boldsymbol{H}_1$ The current element is $I_2\, d\boldsymbol{l}_2$ The force acting on this current element is the ampere force $\boldsymbol{F}_{2,1}$.

We use the induced electromotive force on the straight line L instead induced electromotive force on C_2. Consider,

$$\mathcal{E}_{2,1}^{O} \leftarrow \mathcal{E}_{2,1}^{L} = \int_{L_2} \boldsymbol{E}_1\cdot d\boldsymbol{l} = E_1(-\hat{z})\cdot L(\hat{z}) = -LE_1 \tag{131}$$

$\mathcal{E}_{2,1}^{O} \leftarrow \mathcal{E}_{2,1}^{L}$ means we use $\mathcal{E}_{2,1}^{L}$ to replace $\mathcal{E}_{2,1}^{O}$. Let's assume that,

$$L = 2\pi R \tag{132}$$

R is the the radius of the loop C_2. The area enclosed by L is still calculated using the area of the disk, which is

$$area = LW = 2\pi RW \tag{133}$$

W is the width see figure 6, hence,

$$\pi R^2 = 2\pi RW \tag{134}$$

$$W = \frac{R}{2} \tag{135}$$

Consider,

$$\mathcal{E}_{2,1}^{L} = -j\omega\iint_{\Gamma} B\cdot\hat{n}d\Gamma$$

Consider (131), we obtain,

$$-E_1 2\pi R = -j\omega B_1 2\pi RW \tag{136}$$

$$E_1 = j\omega W B_1 \tag{137}$$

$$B_1 = \frac{E_1}{j\omega W} \tag{138}$$

$$B_1 = \frac{-jE_1}{\omega W} \tag{139}$$

$$H_1 = \frac{-jE_1}{\mu_0 \omega W} = \frac{-jE_1\sqrt{\mu_0\epsilon_0}}{\mu_0\omega\sqrt{\mu_0\epsilon_0}W}$$

$$= \frac{-jE_1}{\omega\sqrt{\mu_0\epsilon_0}W}\sqrt{\frac{\epsilon_0}{\mu_0}} = \frac{-jE_1}{kW\eta_0} = \frac{-j\lambda E_1}{2\pi W\eta_0}$$

$$= -j\frac{\lambda}{2\pi W}\frac{E_1}{\eta_0} \tag{140}$$

Considering (135), we obtain that,

$$H_1 = -j\frac{1}{kR}\frac{2E_1}{\eta_0} \tag{141}$$

Or,

$$H_1 = -j\frac{\lambda}{\pi R}\frac{E_1}{\eta_0} \tag{142}$$

The above equation is the magnetic field measured according to the linear current element. Omit subscripts,

$$H = -j\frac{\lambda}{\pi R}\frac{E}{\eta_0} \tag{143}$$

if we take R so

$$\frac{\lambda}{\pi R} = 1 \tag{144}$$

$$H = -j\frac{E}{\eta_0} \tag{145}$$

In a special case,

$$\boldsymbol{E} = E_0 \exp(-jkx)(-\hat{z}) \tag{146}$$

$$E = E_0 \exp(-jkx) \tag{147}$$

$$H = -j\frac{E_0 \exp(-jkx)}{\eta_0} \tag{148}$$

$$\boldsymbol{H} = -j\frac{E_0 \exp(-jkx)}{\eta_0}\hat{y} \tag{149}$$

We see that the formulas (149) and (130) are very different. They take different values under radiated electromagnetic fields or electromagnetic wave conditions. But we will now prove that they are equal under quasi-static conditions.

5. The Magnetic Field of Electromagnetic Waves

For the magnetic field measurement of electromagnetic waves, we will find that the results obtained by the average magnetic field $\overline{\boldsymbol{H}}$ and the magnetic field $\boldsymbol{H}$ measurement methods are different. We have three methods for measuring magnetic fields: (1) Ampere force method, and (2) Hall effect method for measuring Lorentz force. (3) The method of electromagnetic induction. For these three methods, we have two definitions of magnetic fields. The first is to measure the magnetic field using a linear current element, which measures the magnetic field $\boldsymbol{H}$ according to the original definition method of the magnetic field. (2) Measure the average value of the magnetic field on the loop $\overline{\boldsymbol{H}}$. In this section, we assume that there is a plane electromagnetic wave, and we calculate the average magnetic field of the loop and define it according to the original magnetic field. We will find that these two magnetic fields are different!

5.1 Average Magnetic Field on The Loop

Assuming there is a planar electromagnetic field,

$$\boldsymbol{E}_1 = E_1 \exp(-jkx)(-\hat{z}) \tag{150}$$

From this, the average magnetic field (128) can be obtained,

$$\overline{\boldsymbol{H}}_1 = j\frac{1}{k\eta_0}\nabla \times \boldsymbol{E}_1$$

$$= j\frac{1}{k\eta_0}\nabla \times (E_1 \exp(-jkx)(-\hat{z}))$$

$$= j\frac{1}{k\eta_0}(-jk\hat{x}) \times (E_1 \exp(-jkx)(-\hat{z}))$$

$$= j\frac{1}{k\eta_0}(-jk)(E_1 \exp(-jkx)\hat{y}$$

$$= \frac{E_1}{\eta_0}\exp(-jkx)\hat{y} \tag{151}$$

$$\overline{\boldsymbol{H}}_1 = \frac{E_1}{\eta_0}\exp(-jkx)\hat{y} \tag{152}$$

$$\overline{H}_1 \sim E_1 \tag{153}$$

The above equation indicates that the average magnetic field and electric field obtained in the loop are in phase.

5.2 Measurement of Magnetic Field Using Straight Wires

From Eq.(143) we know the magnetic field measured on a straight wire is,

$$H_1 = -j\frac{\lambda}{\pi R}\frac{E_1}{\eta_0} \tag{154}$$

The above two equations are the magnitude of the alternating magnetic field measured using a long linear current under quasi-static magnetic conditions. Therefore, we have two ways to measure the magnetic field, hypothesis

$$\boldsymbol{E}_1 = E_1 \exp(-jkx)(-\hat{z}) \tag{155}$$

$$H_1 = -j\frac{\lambda}{\pi R}\frac{E_1}{\eta_0} = -j\frac{\lambda}{\pi R}\frac{1}{\eta_0}(E_1 \exp(-jkx)) \tag{156}$$

$$H_1 = -j\frac{\lambda}{\pi R}\frac{E_1 \exp(-jkx)}{\eta_0} \tag{157}$$

take

$$\frac{\lambda}{\pi R} = 1 \tag{158}$$

$$H_1 = -j\frac{1}{\eta_0}E_1 \exp(-jkx) \tag{159}$$

$$\boldsymbol{H}_1 = -j\frac{1}{\eta_0}E_1 \exp(-jkx)\hat{y} \tag{160}$$

Comparing the formulas (160) and (152), we found that under the condition of radiated electromagnetic waves

$$\boldsymbol{H}_1 \neq \overline{\boldsymbol{H}}_1 \tag{161}$$

This indicates that the average magnetic field measured by the loop under radiation electromagnetic field conditions is different from the magnetic field measured by the straight wire. Under quasi-static conditions, these two measurement methods are the same. However, under radiation electromagnetic field conditions, the two methods obtain different values. At this point, we have to ask which method truly represents the correct magnetic field?

5.3 Verification Using Hall Effect Method

Above, we use the method of calculating plane electromagnetic waves to illustrate two types of magnetic field definitions. One is defined on a loop wire, and the other is defined on a straight wire. The magnetic fields defined by these two methods are different. This already illustrates the problem. But if readers still don't understand, Hall effect method can be used to verify. We can use a long straight wire (or a straight transmitting antenna) to generate a magnetic field, with high-frequency AC current flowing through the wire. Measure the average magnetic field $\overline{\boldsymbol{H}}$ of the loop. Measure the magnetic field $\boldsymbol{H}$ with a separate Hall element to see if these two magnetic fields are the same. See Figure 7.

For example, the frequency of an AC signal can reach 1-10MHz. At this frequency, the Hall element can still function properly. Of course, it may be necessary to use a amplifier. The blue and red components in the picture are Hall components. The blue one is used to measure the average magnetic field on the loop. The red color is used to measure the original magnetic field of on the straight wire. Both sides of the Hall element may require the addition of magnetic bars to increase sensitivity. If the current of the straight wire (or a linear transmitting antenna) is not large enough, multiple strands of wire can also be used instead. Display the phase difference between two signals using an oscilloscope.

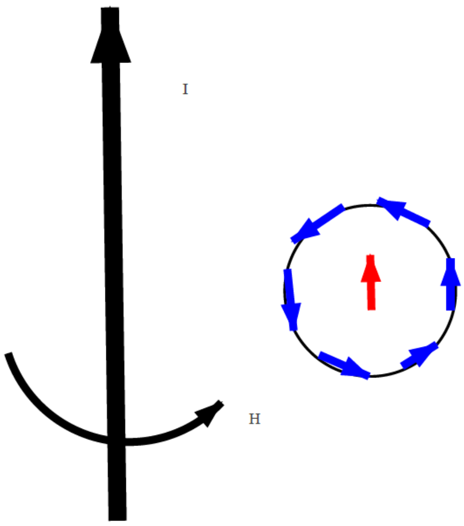

Figure 7: Long Straight Wires Carry High-Frequency Alternating Current. Measure the Average Magnetic Field $\overline{\boldsymbol{H}}$ on The Loop and the Original Magnetic Field $\boldsymbol{H}$ on The Straight Wire Using Hall Effect Devices

6. Author's Electromagnetic Theory

The author mentioned earlier that the fatal flaw in Maxwell's electromagnetic theory lies in the confusion between the average electromagnetic field measured on the loop and the original definition of magnetic field. Namely,

$$\overline{\boldsymbol{H}} = \nabla \times \boldsymbol{A}$$

and the formula,

$$\overline{\boldsymbol{H}} = \boldsymbol{H}$$

It only holds under quasi-static conditions and does not hold under radiation electromagnetic field conditions. Knowing the errors and loopholes in Maxwell's electromagnetic theory, the author should also establish their own new electromagnetic theory.

6.1 Axiom of Radiation Not Overflowing The Universe

The author believes that the ether theory of electromagnetic waves is incorrect. Electromagnetic waves do not propagate in ether. The ether here belongs to space. Therefore, the movement of electromagnetic waves on the ether means that the source of the electromagnetic wave hands over the radiation energy to the ether, and then the electromagnetic wave propagates in the ether without the radiation source. The author believes that this is incorrect.

The author believes that electromagnetic waves propagate within the electrostatic field of electrons themselves. That is to say, every charge, whether positive or negative, has its own electrostatic field. This field belongs to the charge itself. This field can transmit electromagnetic waves, therefore it is the ether of electrons themselves. Electromagnetic waves propagate in this field. Because this field belongs to the charge itself. The energy of electromagnetic waves cannot be separated from this charge. The author uses a whip to describe this field, where the charge is like carrying a whip. With a flick of the whip, energy is transmitted along the handle of the whip to the tip of the whip. But the energy on this whip cannot escape from it. Electromagnetic waves cannot escape from charges!

So some people may ask, how does energy transfer between charges occur? The whips of two charges must both be struck and hit exactly together. In this way, these two charges can exchange energy. The whip here is the electrostatic field of charge. The electromagnetic waves emitted by these two charges must also be a retarded wave and an advanced wave. The retarded wave must be synchronized with the advanced wave. If the whip goes empty, it is equivalent to a charge radiating a retarded wave, but this retarded wave has not found the advanced wave to synchronize with it. The energy of this retarded wave still needs to return to the radiation source of the electromagnetic wave. Just like a whip emptying, the energy flow on the whip must return from the tip of the whip to the handle. In short, electromagnetic waves cannot escape the charge at this time. So electromagnetic waves cannot overflow into the universe. Expressed as,

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = 0 \tag{162}$$

Γ is a sphere with an infinite radius. Converted from time domain to frequency domain,

$$\Re \oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{n} d\Gamma = 0 \tag{163}$$

" $\Re$ " means take the real part. The above equation shows the axioms of the author's electromagnetic theory. This axiom means that the electromagnetic field cannot overflow the universe. Note that Maxwell's electromagnetic theory does not satisfy this axiom. For any antenna in Maxwell's electromagnetic theory, there are,

$$\Re \oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{n} d\Gamma \neq 0 \tag{164}$$

6.2 Magnetic Quasi-Static Equation

Under quasi-static magnetic conditions, the Faraday's law and Ampere's loop law in Maxwell's equation still hold,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t} \boldsymbol{B} \tag{165}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{166}$$

This leads to Poynting's theorem,

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \nabla \times \boldsymbol{E} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{H} \tag{167}$$

Considering (165, 166),

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = -\frac{\partial}{\partial t} \boldsymbol{B} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \boldsymbol{J} \tag{168}$$

Perform volume integration on the upper equation and apply Gaussian law to the left side of the formula to obtain,

$$\oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = -\int_V (\frac{\partial}{\partial t} \boldsymbol{B} \cdot \boldsymbol{H} + \boldsymbol{E} \cdot \boldsymbol{J}) dV \tag{169}$$

Perform time integration on the above equation,

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma$$

$$= -\int_{t=-\infty}^{\infty} dt \int_V (\frac{\partial}{\partial t} \boldsymbol{B} \cdot \boldsymbol{H} + \boldsymbol{E} \cdot \boldsymbol{J}) dV \tag{170}$$

Consider,

$$\int_{t=-\infty}^{\infty} dt \int_V (\frac{\partial}{\partial t}\boldsymbol{B}\cdot\boldsymbol{H})dV = \int_{t=-\infty}^{\infty} \frac{\partial}{\partial t} U dt$$

$$= U(\infty) - U(-\infty) = 0 \tag{171}$$

where in,

$$\frac{\partial}{\partial t} U = \frac{\partial}{\partial t}\boldsymbol{B}\cdot\boldsymbol{H} \tag{172}$$

$$U = \frac{1}{2}\boldsymbol{B}\cdot\boldsymbol{H} \tag{173}$$

$U(\infty)$ is the system energy at the end of the process. $U(-\infty)$ is the system energy at the beginning of the process, and both energies are zero. Consider the formula (170), which is called,

$$\int_{t=-\infty}^{\infty} dt \oiint_\Gamma (\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma = -\int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}\cdot\boldsymbol{J})dV \tag{174}$$

The above equation is the relaxed Poynting's theorem, which was obtained under magnetic quasi-static conditions. However, we assume that the above equation still holds under radiation electromagnetic field conditions. It should notice that, in the radiation electromagnetic field, the author does not assume that the Poynting theorem (169), however he still agree the relaxed Poynting theorem (174). Considering that radiation does not overflow into the universe by substituting the above equation (162) into (174) yields,

$$\int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}\cdot\boldsymbol{J})dV = 0 \tag{175}$$

Consider the principle of superposition

$$\boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{J}_i \tag{176}$$

The resulting electromagnetic field

$$\boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{E}_i, \quad \boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{H}_i \tag{177}$$

Substituting (175) yields,

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dV = 0 \tag{178}$$

Consider again (175) for $\boldsymbol{J} = \boldsymbol{J}_i$ Time also holds, that is,

$$\int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dV = 0 \tag{179}$$

Substituting (179) into (178) yields,

$$\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \int_{t=-\infty}^{\infty} dt \int_V (\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dV = 0 \tag{180}$$

The above equation is the law of conservation of energy. This law of conservation of energy cannot be obtained under the conditions of Maxwell's equation, it can only be an energy theorem according to Maxwell's equation. This is because under the conditions of the Maxwell equation, there is no condition for radiation not to overflow the universe (163). This law of conservation of energy tells us that the energy value of electromagnetic waves is exchanged between charges, and there is no permanent transfer of electromagnetic energy to the medium or ether that propagates electromagnetic waves.

6.3 Mutual Energy Flow Theorem

The relaxed Poynting's theorem (174) can prove the mutual energy flow theorem [17], and we will not prove it again here,

$$-\int_{t=-\infty}^{\infty} dt \int_{V_i} (\boldsymbol{E}_j \cdot \boldsymbol{J}_i) dV$$

$$= (\xi_i, \xi_j)$$

$$= \int_{t=-\infty}^{\infty} dt \int_{V_j} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV \quad (181)$$

where,

$$(\xi_i, \xi_j) \triangleq \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma \quad (182)$$

$-\int_{t=-\infty}^{\infty} dt \int_{V_i} (\boldsymbol{E}_j \cdot \boldsymbol{J}_i) dV$ is the energy of the current element $\boldsymbol{J}_i$ provided to the system. This energy comes from the current $\boldsymbol{J}_i$. $\int_{t=-\infty}^{\infty} dt \int_{V_j} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV$ is the energy obtained from the system to supplied the load of the secondary coil current $\boldsymbol{J}_j$. The mutual energy flow (ξ_i,ξ_j) is energy flow from $\boldsymbol{J}_i$ to $\boldsymbol{J}_j$. With this mutual energy flow theorem, the law of conservation of energy (180) becomes a localized law of conservation of energy. Localization here refers to the transfer of energy through mutual energy flow (182).

The definition of magnetic field is defined by the ampere force formula and the Lorentz force formula. In these preliminary definitions, magnetic field is defined by the current element $Id\boldsymbol{l}$ or $\rho\boldsymbol{v}$. In this case, both the current element and velocity can be regarded as a line segment. However, under quasi-static magnetic conditions, the magnetic field can also be defined on a loop. This magnetic field can be considered as the average value of the measured magnetic field on the loop. The average value under quasi-static or quasi-static magnetic conditions is exactly the same as the magnetic field measured by a straight wire. Maxwell's electromagnetic field theory was first defined under magnetic quasi-static and quasi-static conditions. Below is the quasi-static Maxwell equation,

$$\nabla \cdot \boldsymbol{E}_s = \rho/\epsilon_0 \quad (183)$$

$$\boldsymbol{E}_s \triangleq -\nabla\phi \quad (184)$$

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0 \boldsymbol{J} + \epsilon_0\mu_0 \frac{\partial}{\partial t} \boldsymbol{E}_s \quad (185)$$

In the author's quasi-static equation, the author does not denote $\nabla\times\boldsymbol{A}$ as a magnetic field, but instead uses two cross products, which is similar to what Lorenz did [33]. The continuity equation for current,

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial}{\partial t}\rho \quad (186)$$

And the Lorenz gauge condition,

$$\nabla \cdot \boldsymbol{A} = -\mu_0\epsilon_0 \frac{\partial}{\partial t}\phi \quad (187)$$

Faraday's law is,

$$\boldsymbol{E} \triangleq -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\phi \quad (188)$$

The above Faraday's law formula is the Faraday's law in Maxwell's own Maxwell's equation. In the author's electromagnetic theory there is no,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \quad (189)$$

This formula is the work of Oliver Heaviside, a descendant of Maxwell. As it involves $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, as we have already discussed earlier, the accurate definition should be,

$$\overline{\boldsymbol{B}} = \nabla \times \boldsymbol{A} \quad (190)$$

Therefore, the formula (189) should be rewritten as,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\overline{\boldsymbol{B}} \quad (191)$$

The above (183-188) is a quasi-static equation. In these equations, there are no defined magnetic field, only the electric field $\boldsymbol{E}$ and the electrostatic field $\boldsymbol{E}_s$. Based on the definitions of Gaussian law (183) and electrostatic field (188),

$$\nabla \cdot (-\nabla\phi) = \rho/\epsilon_0 \tag{192}$$

$$\nabla^2\phi = -\rho/\epsilon_0 \tag{193}$$

Above, we obtained the Poisson equation for scalar potentials. Consider mathematical formulas,

$$\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A} \tag{194}$$

Formula (185)

$$\nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A} = \mu_0\boldsymbol{J} + \epsilon_0\mu_0\frac{\partial}{\partial t}\boldsymbol{E}_s \tag{195}$$

$$\nabla(\nabla \cdot \boldsymbol{A}) - \mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E}_s - \mu_0\boldsymbol{J} = \nabla^2\boldsymbol{A} \tag{196}$$

Considering the Lorenz gauge (187), the above equation can be rewritten as:,

$$\nabla(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi) - \mu_0\epsilon_0\frac{\partial}{\partial t}(-\nabla\phi) - \mu_0\boldsymbol{J} = \nabla^2\boldsymbol{A} \tag{197}$$

Or

$$\nabla^2\boldsymbol{A} = -\mu_0\boldsymbol{J} \tag{198}$$

From this solution, it can be concluded that,

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r}dV \tag{199}$$

The solution of formula (193) is

$$\phi = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{r}dV \tag{200}$$

The formula (199, 200) is the solution to a quasi-static equation.

6.4 Prove That The Average Magnetic Field And Magnetic Field Are Equal Under Quasi-Static Conditions

$$\nabla \times \boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \nabla\frac{1}{r} \times \boldsymbol{J}dV$$

$$= \frac{\mu_0}{4\pi}\int_V (-\frac{\boldsymbol{r}}{r^3}) \times \boldsymbol{J}dV \tag{201}$$

Hence, we get

$$\nabla \times \boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^3}dV \tag{202}$$

Biot's law

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{\boldsymbol{r}}{r^3}dV \tag{203}$$

Comparing (203) and (202) yields,

$$\nabla \times \boldsymbol{A} = \boldsymbol{B} \tag{204}$$

Hence

$$\nabla \times (-\frac{\partial}{\partial t}\boldsymbol{A}) = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{205}$$

Or

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{206}$$

Or

$$\frac{1}{-j\omega\mu_0}\nabla \times \boldsymbol{E} = \boldsymbol{H} \tag{207}$$

Previously, we have obtained (127)

$$\overline{\boldsymbol{H}} \triangleq \frac{1}{-j\omega\mu_0}\nabla \times \boldsymbol{E} \tag{208}$$

Hence,

$$\overline{\boldsymbol{H}} = \boldsymbol{H} \tag{209}$$

In this way, we prove that the average magnetic field and magnetic field are equal under quasi-static conditions. In fact, the formula (204) implies that the average magnetic field is equal to the magnetic field. $\nabla\times\boldsymbol{A}$ is the average magnetic field defined on a loop, i.e.,

$$\nabla \times \boldsymbol{A} = \overline{\boldsymbol{B}} \tag{210}$$

6.5 The Dilemma of Maxwell's Equation

For Maxwell's radiated electromagnetic field, the displacement current is added. Let's see how we can transition from a quasi-static equation (183-188) to the Maxwell equation. We must make the following transformation,

$$\boldsymbol{E}_s \to \boldsymbol{E} \tag{211}$$

Or,

$$-\nabla\phi \to -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{212}$$

And,

$$\nabla \times \boldsymbol{A} \to \boldsymbol{B} \tag{213}$$

The above equation is,

$$\overline{\boldsymbol{B}} \to \boldsymbol{B} \tag{214}$$

The formula (183) becomes,

$$\nabla \cdot \boldsymbol{E} = \rho/\epsilon_0 \tag{215}$$

The formula (185) becomes,

$$\nabla \times \boldsymbol{B} = \mu_0\boldsymbol{J} + \epsilon_0\mu_0\frac{\partial}{\partial t}\boldsymbol{E} \tag{216}$$

The formula (213) is equivalent to the following equation,

$$\nabla \cdot \boldsymbol{B} = 0 \tag{217}$$

Faraday's law of electromagnetic induction (206) remains unchanged

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{218}$$

Can replace,

$$\boldsymbol{E} \triangleq -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\phi \tag{219}$$

The displacement current method was proposed by Maxwell, but it is difficult to find a reasonable explanation for this method. The difficulty lies in the fact that the transformation of formulas (211-214) is unreasonable. Why can the static electric field $\boldsymbol{E}_s$ be replaced with an electric field $\boldsymbol{E}$? Why can we replace the average magnetic field on the loop with a magnetic field? This permutation (211, 214) actually needs to be performed in two formulas (183, 185). There is no legitimate reason for doing so. Fortunately, the Maxwell displacement current method is consistent with the Lorenz retarded potential method [33]. In fact, the retarded potential is the solution to Maxwell's equation. It is reasonable to transition from a non retarded electromagnetic field to a retarded electromagnetic field.

6.6 Retarded Potential Method

The non retarded potential function is

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r} dV \tag{220}$$

$$\phi = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{r} dV \tag{221}$$

The retarded potential function is,

$$\boldsymbol{A}^{(r)} = \frac{\mu_0}{4\pi}\int_V \frac{[J]}{r} dV \tag{222}$$

$$\phi^{(r)} = \frac{1}{4\pi\epsilon_0}\int_V \frac{[\rho]}{r} dV \tag{223}$$

Square brackets indicate lag, i.e,

$$[f(x,t)] = f(x, t - r/c) \tag{224}$$

According to Lorenz's viewpoint formulas (220, 221) and formulas (223, 223) were indistinguishable experimentally in their time [33]. Therefore, even if formulas (220, 221) were obtained from experiments of that era, more accurate electromagnetic theory may still mean (222, 223). Compared to Maxwell's displacement current method, Lorenz's theory of retarded potential is more convincing. According to Maxwell's electromagnetic theory, the electric and magnetic fields are,

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A}^{(r)} - \nabla\phi^{(r)} \tag{225}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}^{(r)} \tag{226}$$

It is worth noting that the above equation cannot be directly obtained because it is deformed

$$-j\omega\boldsymbol{B} = \nabla \times (-j\omega\boldsymbol{A}^{(r)}) \tag{227}$$

Or,

$$\boldsymbol{H} = \frac{1}{-j\omega\mu_0}\nabla \times \boldsymbol{E} \tag{228}$$

The right side of this formula encounters the curl of the electric field, but in fact, the calculation of curl is done on the small loop. It is calculating the average magnetic field. Consider definition (127), actually we have,

$$\overline{\boldsymbol{H}} \triangleq \frac{1}{-j\omega\mu_0}\nabla \times \boldsymbol{E} \tag{229}$$

Therefore, the above formula (228) actually means that we assume that,

$$\boldsymbol{H} = \overline{\boldsymbol{H}} \tag{230}$$

And this formula needs to be proven. Is there a problem with this formula under radiated electromagnetic fields. We have already proven that this formula is not valid in the case of simple electromagnetic waves (161). The Maxwell electric field theory directly assumes that the above equation holds. This is the source of the error.

6.7 Retarded Potential Method

$$\boldsymbol{A}^{(r)}=\frac{\mu_0}{4\pi}\int_V\frac{[J]}{r}dV=\frac{\mu_0}{4\pi}\int_V\frac{\boldsymbol{J}}{r}\exp(-jkr)dV \tag{231}$$

Calculate the curl of the retarded potential

$$\nabla\times\boldsymbol{A}^{(r)}=\frac{\mu_0}{4\pi}\int_V\nabla\frac{\exp(-jkr)}{r}\times\boldsymbol{J}dV$$

$$=\frac{\mu_0}{4\pi}\int_V(-\frac{\boldsymbol{r}}{r^3}+\frac{-jk\hat{r}}{r})\exp(-jkr)\times\boldsymbol{J}dV$$

$$=\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times(\frac{\boldsymbol{r}}{r^3}+\frac{jk\hat{r}}{r})\exp(-jkr)dV$$

$$=\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times(\frac{\boldsymbol{r}}{r^3})\exp(-jkr)dV$$

$$+j\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times\frac{k\hat{r}}{r})\exp(-jkr)dV \tag{232}$$

If the above equation (232) is a magnetic field B, we obtain,

$$\overline{\boldsymbol{B}}=\boldsymbol{B} \tag{233}$$

At this point, we can define

$$\boldsymbol{B}\triangleq\nabla\times\boldsymbol{A}^{(r)} \tag{234}$$

Unfortunately (232) is not yet a magnetic field B. We are considering the problem near the origin, where there is $\lim_{kr\to0}$

$$\lim_{kr\to0}\nabla\times\boldsymbol{A}^{(r)}=\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times(\frac{\boldsymbol{r}}{r^3})dV+j\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times\frac{k\hat{r}}{r})dV$$

$$=\boldsymbol{B}_s+j\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times\frac{k\hat{r}}{r})dV \tag{235}$$

The first term in the above equation is the static magnetic field,

$$\boldsymbol{B}_s\triangleq\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times(\frac{\boldsymbol{r}}{r^3})dV \tag{236}$$

The second item is for radiation. It should be determined by the experiment. If it is found that the measured magnetic field is

$$\boldsymbol{B}_s+j\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times\frac{k\hat{r}}{r}dV \tag{237}$$

We can define,

$$\nabla\times\boldsymbol{A}^{(r)}\triangleq\boldsymbol{B} \tag{238}$$

If the experiment finds that the measured magnetic field is,

$$\boldsymbol{B}_s+\frac{\mu_0}{4\pi}\int_V\boldsymbol{J}\times\frac{k\hat{r}}{r}dV \tag{239}$$

Otherwise,

$$\nabla\times\boldsymbol{A}^{(r)}\neq\boldsymbol{B} \tag{240}$$

Experiments can solve problems. But we have already proven in a simpler case, as shown in (161), that the above equation (240) is correct under plane wave conditions. For this reason, we can generally have,

$$\overline{\boldsymbol{B}} = \overline{\boldsymbol{B}}^{(r)} \triangleq \nabla \times \boldsymbol{A}^{(r)} \neq \boldsymbol{B} \tag{241}$$

The right side of the above equation is to calculate the curl of the retarded potential, which is actually equal to the magnetic field measured on a loop. Therefore, it is the average magnetic field on the loop. It is not the magnetic field $\boldsymbol{B}$.

6.8 The Principle of Half Retardation And Half Advancement

The principle of half retardation and half advancement was first proposed by Dirac which was later inherited by Wheeler and Feynman [8, 9], and then interpreted and inherited by Cramer quantum mechanics transactional interpretation [13-15]. However, they did not actually apply this principle to the calculation of electromagnetic fields [8,9]. The author uses this condition to require that the advanced and retarded parts of the electric field and the magnetic field emitted cannot be offset. If at $\lim_{kr\to 0}$ situation,

$$\boldsymbol{B}^{(r)} = \lim_{kr\to 0} \nabla \times \boldsymbol{A}^{(r)}$$

$$\boldsymbol{B}^{(a)} = \lim_{kr\to 0} \nabla \times \boldsymbol{A}^{(a)}$$

then

$$\boldsymbol{B}^{(r)} = \boldsymbol{B}_s + j\frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{k\hat{r}}{r})dV \tag{242}$$

$$\boldsymbol{B}^{(a)} = \boldsymbol{B}_s - j\frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{k\hat{r}}{r})dV \tag{243}$$

$$\frac{1}{2}\boldsymbol{B}^{(r)} + \frac{1}{2}\boldsymbol{B}^{(a)} = \boldsymbol{B}_s \tag{244}$$

We found that the far-field part in the formulas (242, 243) has been canceled out. Such retarded and advanced far-field cannot be transmitted. We must correct the formula (242, 243). We found that if we have at the situation $\lim_{kr\to 0}$,

$$\boldsymbol{B}^{(r)} = \boldsymbol{B}_s + \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{k\hat{r}}{r}dV \tag{245}$$

So the corresponding advanced wave is

$$\boldsymbol{B}^{(a)} = \boldsymbol{B}_s + \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{k\hat{r}}{r})dV \tag{246}$$

In this way, half retardation and half advancement do not cancel out.

$$\frac{1}{2}\boldsymbol{B}^{(r)} + \frac{1}{2}\boldsymbol{B}^{(a)} = \boldsymbol{B}_s + \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{k\hat{r}}{r}dV \tag{247}$$

This is correct. Therefore, we can define,

$$\boldsymbol{B}^{(r)} \triangleq \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times (\frac{\boldsymbol{r}}{r^3})\exp(-jkr)dV + \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{k\hat{r}}{r}\exp(-jkr)dV \tag{248}$$

$$\boldsymbol{B}^{(a)} \triangleq \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times (\frac{\boldsymbol{r}}{r^3})\exp(+jkr)dV + \frac{\mu_0}{4\pi}\int_V \boldsymbol{J} \times \frac{k\hat{r}}{r}\exp(+jkr)dV) \tag{249}$$

Or

$$\boldsymbol{B}^{(r)} = \overline{\boldsymbol{B}}_n^{(r)} + (-j)\overline{\boldsymbol{B}}_f^{(r)} \tag{250}$$

$$\boldsymbol{B}^{(a)} = \overline{\boldsymbol{B}}_n^{(a)} + (j)\overline{\boldsymbol{B}}_f^{(a)} \tag{251}$$

and

$$\boldsymbol{B} = \frac{1}{2}(\boldsymbol{B}^{(r)} + \boldsymbol{B}^{(a)}) \tag{252}$$

The superscript (r) in the above formula represents retardation, while (a) represents advancement. The subscript n means near field and f means far field. $\overline{\boldsymbol{B}}$ is the average magnetic field calculated along the loop, which is also calculated from the curl of the vector potential. The magnetic field calculated according to Maxwell's electromagnetic theory is $\overline{\boldsymbol{B}}$ but is not $\boldsymbol{B}$. The formula (248) tells us that the magnetic field is calculated based on the retarded field, but not the retarded potential. When calculated based on the retarded potential, the magnetic field is $\overline{\boldsymbol{B}}$, which is averaged along the loop. This $\overline{\boldsymbol{B}}$ is not a magnetic field $\boldsymbol{B}$. The formula (252) is the revised and updated magnetic field by author.

It is worth mentioning that the factor 1/2 appearing in formula (252) affects other equations. For example, the formula (185) needs to be modified to:,

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0 \frac{1}{2}\boldsymbol{J} + \epsilon_0 \mu_0 \frac{\partial}{\partial t}\boldsymbol{E}_s \tag{253}$$

(182) is modified to,

$$(\xi_i, \xi_j) \triangleq \frac{1}{2}\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma \tag{254}$$

The above two formular should have a additional factor $\frac{1}{2}$.

7. Conclusion

This article defines the average magnetic field $\overline{\boldsymbol{H}}$ on a circular circuit, and finds that this average magnetic field is consistent with the result defined by the original magnetic field $\boldsymbol{H}$ under quasi-static conditions. The original magnetic field definition here refers to the magnetic field defined by measuring the ampere force using a straight current element, or the magnetic field obtained by measuring the Lorentz force using a Hall device.

However, for radiated electromagnetic fields, i.e. retarded or advanced electromagnetic fields, the original definitions of the average value of the magnetic field on the circular circuit $\overline{\boldsymbol{H}}$ and the magnetic field $\boldsymbol{H}$ are inconsistent. However, Maxwell's electromagnetic theory still confuses the difference between the average value of the magnetic field on a circular circuit and the original definition of the magnetic field in the case of radiated electromagnetic fields. Due to this confusion, the magnetic field calculated by Maxwell's electromagnetic theory was incorrect and therefore needs to be corrected. The author revised Maxwell's electromagnetic theory according to the new electromagnetic field axiom proposed by the author. The author's new axiom is that (1) radiation does not overflow the universe. From this, the law of conservation of energy (180) and the theorem of mutual energy flow (181, 182) were obtained. The author corrected the magnetic field calculated by Maxwell's equation based on the principle of half retardation and half advancement, and that the retarded wave and advanced wave cannot cancel each other on the surface of the current.

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