

CAPM Model and Optimal Risky Portfolio for American Stock Market

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Abstract

The stock market has high risks. The purpose of this project is to calculate the beta coefficients and build the optimal risky portfolio consisting of eight different stocks each representing different significant industries (which includes information technology, electric cars etc.) to diversify risk and gain a high return. Putting more stocks into the portfolio can help analysts carry out comprehensive analysis on different situations, periods, and types of investment portfolio, so as to disperse risks and obtain high returns, also, ensure the diversity of portfolio and a lower risk. The empirical results in this paper shows that by allocating their money appropriately in different stocks, investors can gain returns multiple times higher than putting all of them in merely one single stock, which proves the validity of this paper.

Keywords: Stock Market, 7-Day Moving Average Daily Adjusted Closing Price, CAPM, Optimal Risky Portfolio, Sharpe Ratio

1. Introduction

Currently, many people are interested in investing in stock market since they may gain considerable returns by proper investment methods. Therefore, the research on stock market investment naturally focuses on measuring the performance of stocks, identifying the risk factors of stocks, and finally constructing the optimal stock investment portfolio. The paper adopts eight different stocks each representing a vital industry, in order to set up the optimal risky portfolio by analyzing monthly return data. The empirical results prove that investors can gain a profit several times higher by choosing the vintage portfolio than merely putting all the money on one single stock, which certifies the value of this paper.

2. Theoretical Background

For the building of CAPM (Capital Asset Pricing Model) and Markowitz optimal risky portfolio, this paper references the research conducted by Sharifzadeh (2005) [1]. The foundation of modern investment theory can be traced back to Harry Markowitz's portfolio investment theory, which he developed in 1952 and was later awarded the Nobel Prize in economics. Markowitz (1952) proposed that the rates of return of individual assets are interrelated and can be represented by a variance-covariance matrix [2]. By analyzing this matrix, one can mathematically calculate the risk, defined as the standard deviation of returns, of a portfolio consisting of different assets. Markowitz demonstrated that there is an optimal mix of assets, known as efficient portfolios, that minimize risk and maximize returns for a given level of risk tolerance. These efficient

portfolios can be plotted on a graph called the efficient frontier. Markowitz also assumed that all investors are risk-averse and will choose the portfolio on the efficient frontier that aligns with their risk-returns profile. The actions of these investors in the capital market, as they strive to build their desired efficient portfolios, influence the equilibrium asset prices.

While the Markowitz model is considered a valuable investment theory, its practical implementation and empirical testing were initially challenging. The model involves the calculation of a variance-covariance matrix for all risky assets, which would require a substantial number of calculations, especially for a large portfolio like the stocks in the S&P 500 Index. This calculation was not feasible without the aid of fast computers. However, the Markowitz model served as a source of inspiration for other researchers who aimed to incorporate its risk-return concepts into simpler models. As a result, this led to the development of CAPM.

CAPM originated from the theoretical groundwork laid by Sharpe in 1963 and was further developed by Sharpe (1964), Lintner (1965), and Mossin (1966) into an equilibrium model for capital market prices [3-5]. In the 1970s, techniques for estimating the model's required inputs were refined, leading to the creation of computer software that was marketed to mutual funds and institutional investors. This facilitated the practical application of modern investment theory. Currently, many institutional investors and investment professionals rely on CAPM's predictions to guide their investment decision-

making and portfolio management. With the advancement in computational capabilities of computers, the Markowitz model is now utilized for asset allocation between different classes of securities like stocks and bonds, while CAPM is employed for allocating funds among various stocks within the equity portion of a portfolio.

The fundamental principle of the CAPM is that the reason why individual stock returns correlate with one another is due to the fact that the return of each stock, or a portfolio of stocks, is influenced by a common factor: the overall stock market return. This overall market refers to a portfolio of all risky assets, where the weighting of each asset is based on its market value. In practice, a broad value-weighted index, such as the S&P 500 Stock Index, is often used as a proxy for the overall market when implementing the CAPM or testing hypotheses. By making certain assumptions about investor behavior and the functioning of the stock market, the CAPM deduces several conclusions regarding asset pricing in the capital market:

1. The rate of return of each individual stock can be expressed as a linear function of the rate of return of the overall stock market. This relationship is represented by the characteristic line of the stock, with the slope of the line indicating the stock's sensitivity, denoted as β (beta), to changes in the rate of return of the overall market.
2. The beta of a stock serves as a measure of its systemic risk. Specifically, the systemic risk of a stock can be calculated by squaring its beta and multiplying it by the variance of the overall market rate of return. Systematic risk is the only risk that holds significance for investors. Any fluctuations in a stock's rate of return beyond its systematic risk are specific to that particular stock and can be mitigated through diversification.
3. If we were to plot the rates of return of individual stocks against their betas, we would observe an upward-sloping line known as the security market line. The slope of this line represents the rate of return of the market portfolio relative to the risk-free rate in the economy, and it is referred to as the stock market risk premium. As long as the security market line remains consistent, the slope, or market risk premium, will also remain constant.
4. The equilibrium price of a stock is determined in a way that its expected rate of return, based on its beta, aligns with the security market line. If a stock is priced in a manner where its expected rate of return is lower than what the security market line suggests, it is considered overpriced, and investors should consider selling it. Conversely, if a stock's risk-return profile positions it above the security market line, it may be considered underpriced and could present a buying opportunity for investors.
5. The market portfolio, with a beta value of one, is considered an efficient portfolio and is positioned on the security market line.

3. Literature Review

3.1 Basic Characteristics of Stocks

The most representative fundamental indices measuring the performance of stocks include the excess return rate of stocks, the volatility of the excess return, and the Sharpe ratio, which

combines these two indicators. The paper written by Chou, Engle and Kane (1992) has shown the definition of excess return for stocks [6]. One year later, Glosten, Jagannathan and Runkle (1993) presented the relationship between the volatility of the return of stocks, which is usually measured by their variance, or their square roots – standard deviation, and the returns themselves [7]. Combined of the returns and the volatility of them, the work of Sharpe (1998) explained the details of empirical analysis from the view of Sharpe ratio [8]. Together with the indicators listed above, the most profitable stock under the same level of risk can be inferred. Meanwhile, the daily stock prices are also meaningful. The work of Cheung, Cheung and Wang (2009) already suggested the importance for analyzing the floating of daily stock prices [9]. To observe the trend dynamically, this paper adopts the method took by He et al. (2020) – the 7-day moving average analysis [10]. From plotting the graphs of 7-day moving average price of eight selected stocks, the trend of the change on the prices of these selected stocks can be visualized much more clearly.

3.2 Correlation between the Return of Stocks

After analyzing the performance of each single stock, the next step is to find the correlation between the excess return of stocks in pairwise way. The paper by Pollet and Wilson (2010) provided examples on analyzing the mutual relationship between the return rates of stocks by calculating the correlation coefficients between them [11]. By looking at these correlation coefficients, investors will know whether the return of a stock has a positive or negative effect on the one of another stock.

3.3 CAPM

According to the previous works by Kisman and Restiyanita (2015), beta coefficients of stocks play a significant rule on measuring the risk of stocks compared to the whole stock market [12]. Considering the beta coefficients of stocks and the performance of the whole stock market together, investors may estimate the profit they can gain by investing on a specific stock during the recent periods, and they may decide to either adding or reducing the amount of investment on this stock.

3.4 Markowitz Optimal Risky Portfolio

Based on the performance of each single stock, investors will finally need to build their own portfolios. From the achievement of Širůček and Křen (2017), it is learned that the aim of building the optimal risky portfolio is to gain the highest Sharpe ratio, in other words, to earn the maximum excess return while bearing the same level of risk by allocating the proportion of each different stock in their total investment in the stock market [13]. The application of Markowitz Portfolio Theory (MPT) is practiced again in this paper.

4. Methodology Framework

4.1 Correlation and Beta Coefficients

To observe the mutual relationship between each of the eight select stocks, the correlation coefficients between the monthly return rates of each stock are calculated:

$$\rho_{ij} = \frac{\sum_{t=1}^{120} (R'_{it} - \bar{R}'_{it})(R'_{jt} - \bar{R}'_{jt})}{\sqrt{\sum_{t=1}^{120} (R'_{it} - \bar{R}'_{it})^2 \sum_{t=1}^{120} (R'_{jt} - \bar{R}'_{jt})^2}}, i, j$$

$$= 1, \dots, 8$$

Here R'_i represents the monthly excess return rate in the i^{th} month, and ρ_{ij} ranges between -1 and 1. The closer ρ_{ij} to 1, the stronger the positive correlation between the excess returns of the i^{th} and j^{th} stock; otherwise, the closer ρ_{ij} to -1, the stronger the negative correlation between the excess returns of the i^{th} and j^{th} stock; and when ρ_{ij} is close to 0, then the correlation between the excess returns of the i^{th} and j^{th} stock is weak.

Similarly, the correlation coefficient between the monthly return rates of the i^{th} stock and the market is:

$$\rho_{i,M} = \frac{\sum_{t=1}^{120} (R'_{it} - \bar{R}'_{it})(R'_{Mt} - \bar{R}'_{Mt})}{\sqrt{\sum_{t=1}^{120} (R'_{it} - \bar{R}'_{it})^2 \sum_{t=1}^{120} (R'_{Mt} - \bar{R}'_{Mt})^2}}, i, j$$

$$= 1, \dots, 8$$

Here R'_{Mt} is the monthly excess return rate of the American stock market in the t^{th} month, in this paper, the stock market is S&P 500.

Next, the beta coefficients for each stock are measured. To obtain the beta coefficients, the CAPM is built for all the eight select models:

$$R_{it} - r_{ft} = \alpha_i + \beta_i (R_{Mt} - r_{ft}) + \varepsilon_{it}, i$$

$$= 1, \dots, 8$$

Here R_{it} represents the monthly return rate for the i^{th} stock in the t^{th} month, r_{ft} stands for the monthly risk-free return rate, which is the monthly treasury yield in the U.S., and R_{Mt} is the monthly return rate for the stock market. And obviously, $R'_{it} = R_{it} - r_{ft}$, $R'_{Mt} = R_{Mt} - r_{ft}$. For the parameters in the CAPM model, α_i is the difference between the expected and the actual monthly return rates for the i^{th} stock, ε_{it} is the error term for the i^{th} stock in the t^{th} month. And the most important one β_i is the beta coefficient for the i^{th} stock, which is the metrics for the risk of the i^{th} stock. To estimate the beta coefficient for the i^{th} stock, the formula is:

$$\beta_i = \frac{\sum_{t=1}^{120} (R'_{it} - \bar{R}'_{it})(R'_{Mt} - \bar{R}'_{Mt})}{\sum_{t=1}^{120} (R'_{Mt} - \bar{R}'_{Mt})^2}$$

Actually, the market beta is 1. For the beta coefficient of the i^{th} stock, it means that when the market monthly return rises up for 1 percent, the monthly return rate for the i^{th} stock will be expected to increase for β_i percent. The higher the beta coefficient for the i^{th} stock, the riskier for investing on the i^{th} stock.

4.2 Optimal Risky Portfolio

Combining the correlation and beta coefficients results for each stock, the optimal risky portfolio can be built. The aim of the optimal risky portfolio is to maximize its Sharpe ratio, here the definition of Sharpe ratio is the expected excess return of a

stock divided by the standard deviation of its excess return rate. According to the Markowitz Portfolio Theory, the goal can be written as:

$$\max_{\omega} S = \frac{\sum_{i=1}^8 \omega_i \bar{R}'_i}{\sqrt{\sum_{i=1}^8 \omega_i^2 \sigma_i^2 + \sum_{j=1}^8 \sum_{i=1}^8 \omega_i \omega_j \sigma_{ij}}}, i \neq j$$

Here ω is the proportion of each stock in the optimal risky portfolio, $\sigma_i^2 = \frac{\sum_{t=1}^{120} (R'_{it} - \bar{R}'_{it})^2}{119}$ is the variance of the monthly excess return rate for the i^{th} stock, and $\sigma_{ij} = \frac{\sum_{t=1}^{120} (R'_{it} - \bar{R}'_{it})(R'_{jt} - \bar{R}'_{jt})}{119}$ represents the covariance between the monthly return rates of the i^{th} and j^{th} stock.

5. Data and Preliminary Results

5.1 Data Processing

Firstly, the original adjusted closing prices on the 1st of every month of eight different stocks and S&P 500 between October 2, 2012 and October 2, 2022 were downloaded from Yahoo Finance, and the risk-free rates for the same period were downloaded from YCharts. Secondly, the monthly return rates for the stocks were calculated by dividing the adjusted closing price on the 1st of the next month by the price in this month minus 1 and times 100%, for which the formula is $R_t = [(P_{t+1}/P_t) - 1] * 100\%$. Then the excess return rates R'_i of the stocks for each month were calculated by deducing the risk-free rate r_{ft} from the original monthly return rates, which is to say that $R'_i = R_i - r_{ft}$. The daily adjusted closing prices of the selected stocks from October 1st 2012 to September 30th 2022 were also downloaded from Yahoo Finance. By calculating the average of the daily adjusted closing price on a specific day and the prices of its 6 previous transaction days, the column containing the 7-day moving average adjusted closing prices from October 9th 2012 to September 30th, 2022. Then based on the daily adjusted closing price spreadsheet, the 7-day moving average line charts were plotted, and the summary statistics spreadsheets were calculated based on monthly excess returns of each stock by the 'Descriptive Statistics' function in 'Data Analysis'. Based on the summary statistics spreadsheets, the combo charts including the histograms and normalized density curves of the distributions of the monthly excess return rates of each stock and S&P 500 for which the range of horizontal axis divided into 10 intervals from minimum to maximum excess return rates, and the normalized distribution density values calculated by function 'NORM.DIST' were plotted. The next step was to build CAPM models for each stock by doing simple linear regressions whose dependent variable is the set of monthly excess returns of each stock and independent variable is the set of monthly excess returns of S&P 500 in order to compute the beta coefficients of each stock by the function 'Regression' in 'Data Analysis'. The annualized Sharpe ratios for each stock which are measured by the annualized excess return rates divided by the annualized standard deviations, were also calculated, based on the summary statistics spreadsheets, where the annualized excess returns are the monthly excess return rates multiplying 12, and the annualized standard deviations are derived by the monthly.

standard deviations timing square root of 12. The correlation

coefficient and covariance tables containing the relevance of the oscillation of the monthly excess return rates between each of the selected stocks and S&P 500, were also included in this project, calculated by the ‘Correlation’ and ‘Covariance’ functions in ‘Data Analysis’. Finally, the optimal risky portfolio which maximizes the portfolio’s annualized Sharpe ratio was built by the Markowitz’s Portfolio Theory based on the summary statistics spreadsheets and the covariance table, by using the ‘Solver’ method, setting the sum of ratios in the portfolio equaling to 1, and each of these ratios greater or equal to 0 as preset conditions (no short selling allowed). All the computation and modeling steps were completed by Excel.

5.2 Data Characteristics

The research data is shown in Table 1-3. In Table 1, the average annualized excess return rates of each stock and S&P 500 are presented. Table 2 presents the annualized standard deviations for the excess return rates of each stock and S&P 500. As shown in Table 3, the annualized Sharpe ratios for each stock and S&P 500 are presented. Based on these data, Graph 1-9 show the frequency distribution of the monthly excess return rates of each stock and S&P 500.

App le	Tesla	Pfizer	Morgan Stanley	3M	Exxon Mobil	Hershey's	Disney	S&P 500
16.13%	57.71%	3.57%	13.28%	0.17%	1.25%	7.95%	3.70%	2.24%

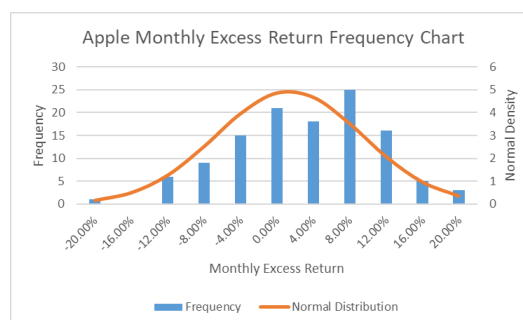
Table 1. Shows Average Annualized Excess Return Rates

Apple	Tesla	Pfizer	Morgan Stanley	3M	Exxon Mobil	Hershey's	Disney	S&P 500
27.97%	65.47%	21.33%	28.69%	20.54%	27.01%	17.81%	26.02%	14.80%

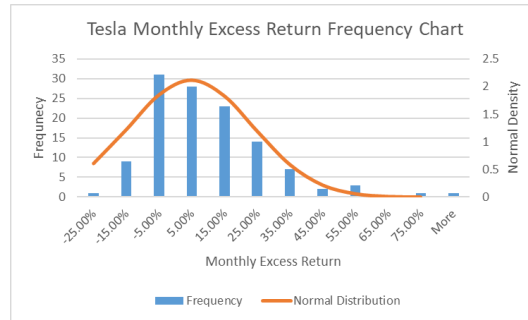
Table 2. Shows Annualized Standard Deviations

Apple	Tesla	Pfizer	Morgan Stanley	3M	Exxon Mobil	Hershey's	Disney	S&P 500
0.58	0.88	0.17	0.46	0.01	0.05	0.45	0.14	0.15

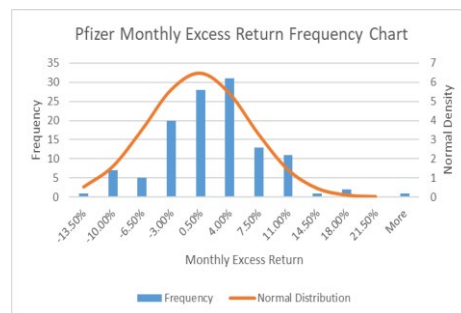
Table 3. Shows Annualized Sharpe Ratios



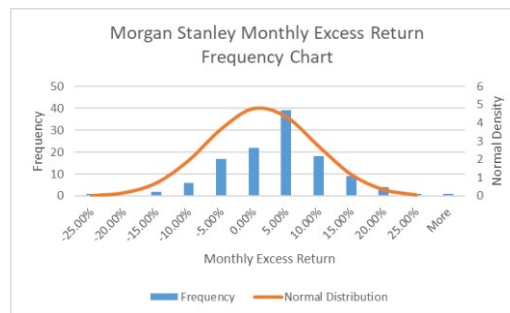
Graph 1. Shows Distribution of Apple’s Monthly Return Rates



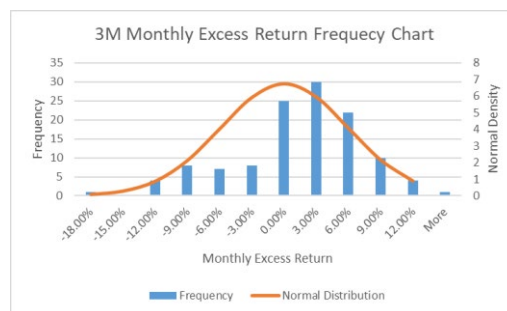
Graph 2. Shows Distribution of Tesla’s Monthly Return Rates



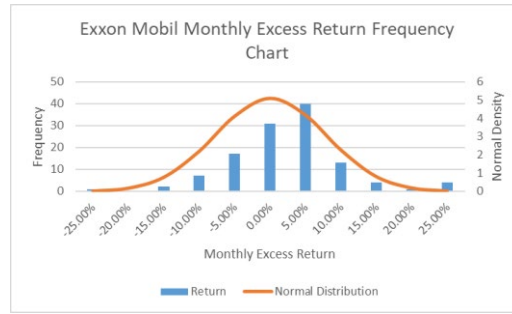
Graph 3. Shows Distribution of Pfizer’s Monthly Return Rates



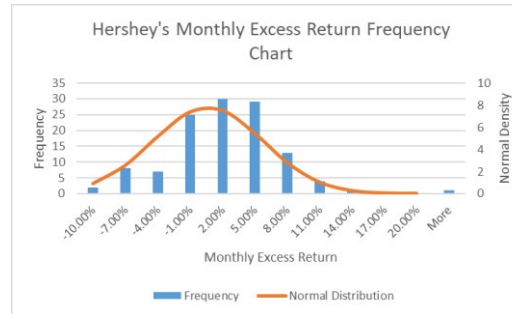
Graph 4. Shows Distribution of Morgan Stanley’s Monthly Return Rates



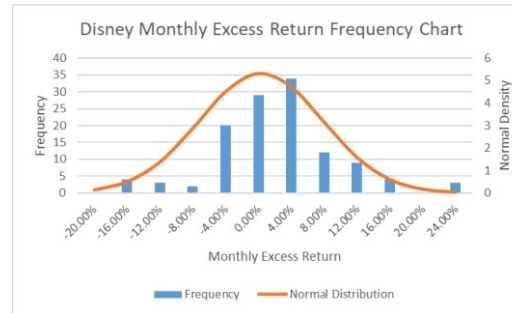
Graph 5. Shows Distribution of 3m’s Monthly Return Rates



Graph 6. Shows Distribution of Exxon’s Monthly Return Rates

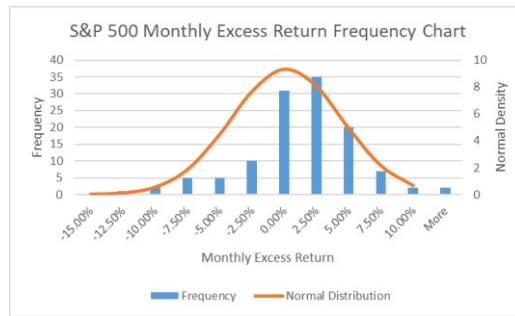


Graph 7. Shows Distribution of Hershey’s Monthly Return Rates



Graph 8. Shows Distribution of Disney’s Monthly Return Rates

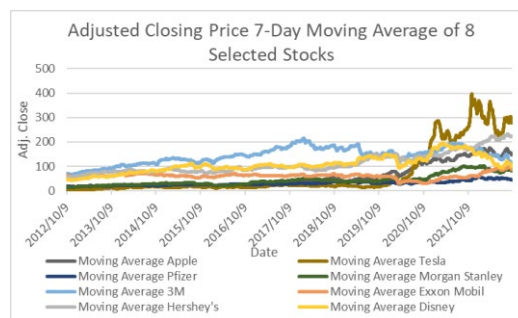
	Apple	Tesla	Pfizer	Morgan Stanley	3M	Exxon Mobil	Hershey's	Disney	S & P 500
Apple	1								
Tesla	0.40	1							
Pfizer	0.24	0.14	1						
Morgan Stanley	0.35	0.34	0.29	1					
3M	0.41	0.21	0.41	0.53	1				
Exxon Mobil	0.20	0.09	0.17	0.50	0.53	1			
Hershey's	0.10	0.12	0.27	0.12	0.25	0.20	1		
Disney	0.35	0.27	0.25	0.65	0.46	0.46	0.29	1	
S&P 500	0.63	0.43	0.53	0.73	0.74	0.54	0.30	0.71	1



Graph 9. Shows Distribution of S&P 500's Monthly Return Rates

From Table 3, it's obvious that compared with S&P 500, the annualized Sharpe ratios of Apple, Tesla, Morgan Stanley, and Hershey's are relatively high, while the ones of 3M and Exxon Mobil are quite low. The small Share ratio of Exxon Mobil can

be explained by the price fluctuation of crude oil due to the privilege of pricing of crude oil holding by OPEC members. In addition, Graph 10 reveals the trends of 7-day moving average adjusted closing prices of the selected stocks.



Graph 10. Shows Trend of 7-Day Moving Average Adjusted Closing Prices of the Selected Stocks

From Graph 10, it can be seen that the daily adjusted closing prices of Apple, Tesla, Pfizer, Morgan Stanley, and Hershey's presented an increasing trend since October 2,012, while the ones of 3M, Exxon Mobil, and Disney kept on floating up or down.

6. Empirical Analysis

6.1 Correlation and Beta Coefficients

In the empirical analysis part, the correlation coefficients between each of the selected stocks and S&P 500 were calculated. The beta coefficients of all the eight selected stocks were also calculated by the CAPM model. Table 4 presents the correlation coefficient matrix consisting of the selected stocks and S&P 500, and Table 5 shows the beta coefficients for each stock and S&P 500.

Ap ple	Te sla	Pfi zer	Mor gan Stan ley	3 M	Ex xon Mo bil	Hersh ey's	Dis ney
1.20	1.90	0.76	1.42	1.02	0.99	0.36	1.24

Table 4. Shows Correlation Coefficient

Table 5. Shows Beta Coefficients

From Table 4, it can be observed that except Hershey's, all the excess return rates of the selected stocks are closely related to the market (S&P 500), especially for Morgan Stanley, 3M, and Disney. These three companies belong to finance, plastic manufacturing, and entertainment industry in order, which are relatively sensitive to the economic situation. From Table 5, divided by the level that beta coefficient equals to 1, it's clear that from the level of risk premium, the Apple, Tesla, Morgan Stanley, and Disney have higher risks than the market, while the

risks of Pfizer and Hershey's are lower than the market, and the risk levels of 3M and Exxon Mobil are approximately the same as the market.

6.2 Optimal Risky Portfolio

In Table 6, the ratios of each selected stocks in the optimal risky portfolio are shown. And Table 7 presents the annualized excess return rate, standard deviation, Sharpe ratio, and beta coefficient of the optimal risky portfolio.

Annualized Excess Return Rate	Annualized Standard Deviation	Annualized Sharpe Ratio	Beta Coefficient
23.41%	23.86%	0.98	1.03

Table 6. Shows Optimal Risky Portfolio

Table 7. Shows Expected Profitability and Risk of Optimal Risky Portfolio

7. Conclusion

From the view of the contribution to future research on stock market, the goal of this paper is to identify some well-developed stock portfolios aiming at managing risks. It was written to inform its readers of some personal thoughts on risk management and offer some strategies through careful analysis. By utilizing stocks' characteristics and analyzing their past statistics, the optimal risky portfolio which contains a total of eight stocks that will most likely obtain an appropriate level of risk management for investors is identified. The suggestion is to put 19.66% in Apple, 26.89% in Tesla, 8.82% in Morgan Stanley, 44.64% in Hershey's, and none in Pfizer, 3M, Exxon Mobil, and Disney for all the money planned on purchasing stocks. By such allocation method, the annualized Sharpe ratio of this optimal risky portfolio is 0.98, compared to the average level of the eight selected stocks, 0.34, is 187.25% higher; compared to S&P 500, 0.15, is 547.89% higher, which is even more drastic. The beta coefficient of this portfolio is 1.03, which means that its excess return rate fluctuates in approximately the same extent as the market. Compared to the average beta coefficient of the eight selected stocks, 1.11, the portfolio's beta coefficient is much lower, that's to say, the excess return rate of the portfolio is less affected by the market than the average level of the selected stocks.

However, this paper also has drawbacks. The time range for the analysis of the portfolio covers 10 years, which is such a long period. The general economic background tended to vary for different stages, for example, since 2020, due to the pandemic of COVID-19, the GDP went down while the national interest rates all around the world, which made the stock prices go down, and the trade war between United States and China also brought negative effects to the stock market. That is, the Sharpe ratios for different time periods cannot be exactly the same as it measured above, and may even have shocks, both for the past and future. For future work in the future, there is an additional recommendation, which is to conduct a 5-fold cross validation by dividing the original data set into five equal subsets from the beginning to the end in the time range, to see the deviation of annualized Sharpe ratio in different sub-periods, in order to test whether the method for building the optimal risky portfolio used in this paper is reasonable. Meanwhile, the selected stocks are merely representative companies in each of their own fields, not the ones with highest returns, which means that more advanced data analysis tools are required to filter out the most profitable stocks in the whole stock market. In conclusion, the portfolio in this paper provides a method to generate a relatively higher return and lower risk [14-16].

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