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Biquaternionic Model of Electro-Gravimagnetic Fields and Interactions

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Abstract

The biquaternionic form of motion equations of electro-gravimagnetic charges and currents under action of external EGM-fields are considered. It is constructed in differential algebra of biquaternions by use biquaternionic generalization of Maxwell and Dirac equations. The field's analogue of the three Newton laws is presented. The energy-pulse biquaternion of interaction are considered and the conditions of energy release, energy absorption and conservation are obtained.

Keywords: Electro-Gravimagnetic Field, Charge, Current, Interaction, Energy-Pulse, Newton Laws, Biquaternions Algebra, Bigradient, Biwave Equation

Introduction

In papers, the author developed a biquaternionic model of the electro-gravimagnetic field (EGM-field) and electro-gravimagnetic interactions [1-3]. Its basis is made up of biquaternionic generalization of Maxwell and Dirac equations. They are constructed in differential algebra of biquaternions (Bqs) by use the wave complex conjugated differential operators (bigradients), which are the biquaternionic generalization of gradient operator on Minkowski space.

In this model, the electric and gravimagnetic fields are united in one biquaternion of EGM-tension. It gives possibility to enter gravimagnetic and electric tensions, gravimagnetic and electric charges and currents and biquaternion of Charges-Current field (CC-field).

The biquaternionic form of Maxwell equations expresses the CC-field biquaternions through the big radiant of EGM-field tension. The biquaternionic form of Dirac Equations. Determines the transformation of densities of mass-charges and currents under the influence of external EGM-fields. These two equations give possibility to construct the field's analogues of three Newton's laws for material point. In particular, in the absence of external fields, its biquaternionic wave (biwave) Equation for a free field of mass-charges and currents, which is a field analogue of first Newton's inertia law? The biquaternion energy-pulse of interaction are considered and the conditions of energy release, energy absorption and conservation are presented.

Complex characteristics of electro-gravimagnetic field

Let introduce known and new physical values, which characterize EGM-field, charges and currents:

- vectors E and H are the tensions of electric and *gravimagnetic field*
- scalars ρ^E, ρ^H are the densities of electric and *gravimagnetic charges*
- vectors j^E, j^H are the densities of electric and *gravimagnetic current*

Here we united *potential gravitational* field with *torsional magnetic field* in one *gravimagnetic field H*. Also we united mass current with magnetic currents. By using these values, we introduce the complex characteristics of EGM-field:

EGM tension $\quad A = A^E + iA^H = \sqrt{\varepsilon}\ E + i\sqrt{\mu}\ H$

EGM-density $\quad \alpha = i\alpha^E / \sqrt{\varepsilon} + \alpha^H / \sqrt{\mu}$

EGM charge density $\quad \rho = -\rho^E / \sqrt{\varepsilon} + i\rho^H / \sqrt{\mu},$

EGM current density $\quad J = J^E + iJ^H = -\sqrt{\mu}\ j^E + i\sqrt{\varepsilon}\, j^H$

Here $\rho^E(x,t)$, $j^E(x,t)$ are electric charges and electric current densities, $\rho^H(x,t)$, $j^H(x,t)$ are a gravimagnetic charge density and current density; ε, μ are the constants of electric conductivity and magnetic permeability of vacuum $c = 1/\sqrt{\varepsilon\mu}$ is the *light speed*.

Some Definions in Biquaternions Algebra

For construction of this model, we used differential algebra of biquaternions. Let consider some definitions in it.

$$\mathbf{F}(x,\tau) = f(x,\tau) + F(x,\tau)$$

Where $f(x,\tau)$, $F(x,\tau)$ are complex scalar and vector functions on Minkowski space $\mathrm{M} = \{(\tau,x) : \tau \in R^1, x \in R^3\}$. *Conjugated* biquaternion, $\mathbf{F}^* = \bar{f} - \bar{F}$, where are complex conjugated to *f, F*.

The sum and a product of biquaternions are defined so:

$$a\mathbf{F}+b\mathbf{G}=a(f+F)+b(g+G)=(af+bg)+(aF+bG) \quad (1)$$

$$\mathbf{F}\circ\mathbf{G}=(f+F)\circ(g+G)=(fg-(F,G))+(fG+gF+[F,G]) \quad (2)$$

Here (F,G), $[F,G]$ are scalar and vector productions. Algebra of biquaternions is not commutative but associated.

We use the biquaternionic wave operators – bigradients ∇^+, ∇^- (mutual bigradient).

Their actions are determined according to quaternion multiplication rule (2):

$$\nabla^{\pm}\mathbf{F}(\tau,x)=(\partial_\tau \pm i\nabla)\circ(f(\tau,x)+F(\tau,x)) \triangleq$$
$$\triangleq\{\partial_\tau f \mp i\,\mathrm{div}F\}+\{\partial_\tau F \pm i\,\mathrm{grad} f \pm i\ \mathrm{rot}F\} \quad (3)$$

Their composition is commutative and gives classic wave operator:

$$\nabla^+\circ\nabla^-=\nabla^-\circ\nabla^+=\frac{\partial^2}{\partial\tau^2}-\Delta\triangleq\Box \quad (4)$$

It is very useful property for solving biquaternionic differential equations.

Biquaternions of electro-gravimagnetic field

We introduce the next Bqs. of EGM-field and charge-currents field (*CC-field*) [1]:

Potential $\Phi=\phi+i\Psi$,

Tension $\mathrm{A}(\tau,x)=i\alpha(\tau,x)+A(\tau,x)$,

Charge-current $\Theta(\tau,x)=i\rho(\tau,x)+J(\tau,x)$,

EGM energy-pulse $\Xi(\tau,x)=0,5\mathrm{A}^*\circ\mathrm{A}=W(\tau,x)+iP(\tau,x)$.

In the case α= 0 you see here usual energy density W and Pointing vector P of EM-field:

$$W=0,5\|A\|^2=0,5(A,A^*)=0,5\left(\varepsilon\|E\|^2+\mu\|H\|^2\right),\quad P=A\times A^*=c^{-1}E\times H\ ,$$

Charge-current energy-pulse

$$\Xi_\Theta=W_\Theta+iP_\Theta=0,5\mathbf{\Theta}\circ\mathbf{\Theta}^*=$$
$$=0,5\left(|\rho|^2+\|J\|^2\right)-i\left\{\mathrm{Re}\left(\rho\bar{J}\right)+0,5\,\mathrm{Im}\left[J,\bar{J}\right]\right\}$$

Connection between EGM-field and CC-field. Generalized Maxwell equation

There is connection between these fields, which follows from Maxwell equations. It can be written in generalized biquaternionic form as the next postulate [2].

Postulate1. Charge-current is the bigradient of EGM-Tension:

$$\nabla^+\mathbf{A}=\mathbf{\Theta}(\tau,x) \quad (5)$$

It's equal to the system of scalar and vector equations:

$$\rho(\tau,x)=-\partial_\tau\alpha-\mathrm{div}A,$$
$$J(\tau,x)=i\,\mathrm{grad}\alpha+\partial_\tau A+i\,\mathrm{rot}\,A \quad (6)$$

From here the Hamilton form of classic Maxwell equations [3] follows by α = 0:

$$\mathrm{div}A=-\rho(\tau,x),$$
$$\partial_\tau A+i\,\mathrm{rot}\,A=J(\tau,x) \quad (7)$$

For static fields from Eq.(3)1 we get well-known Poisson Eqs for scalar potential of electric and gravitational fields

$$\Delta\phi^E=\rho^E(x),\quad \Delta\phi^H=-\rho^H(x) \quad (8)$$

Eq.(5) gives possibility to construct **A(τ, x)** if **Θ(τ, x)** is known and otherwise.

By this cause we'll name Eq (5) *generalized Maxwell equations* (GMEq).

The power and density of acting forces

Let's consider the two EGM-fields

$\mathbf{A},\mathbf{A}'$ and their charges and currents $\mathbf{\Theta},\mathbf{\Theta}'$. We name a power-force the next Bq.

$$\mathbf{F}=\mathbf{A}'\circ\mathbf{\Theta}=(i\alpha+A')\circ(i\rho+J)=$$
$$=-(\alpha\rho+(A',J))+\{i\alpha+i\rho A'+[A',J]\}=p-iF \quad (9)_1$$

which are acting from side of $\mathbf{A}'$ -field on the charge and current of $\mathbf{A}$ -field and otherwise

$$\mathbf{F}'=\mathbf{A}\circ\mathbf{\Theta}'=p'-iF' \quad (9)_2$$

Power density of acting forces is the scalar part:

$$p=-\alpha\rho-(A',J)=$$
$$=-\alpha\rho+c^{-1}((E',j^E)+(H',j^H))+i((B',j^E)-(D',j^H)) \quad (10)$$

EGM force $F=F^H+iF^E$ is its vector part, which contains

gravimagnetic force $F^H=\rho^E E'+\rho^H H'+j^E\times B'-j^H\times D'-\mathrm{Im}(\alpha J)$

electric force $F^E=c\left(\rho^E B'-\rho^H D'\right)+c^{-1}\left(E'\times j^E+H'\times j^H\right)+\mathrm{Re}(\alpha J)$

Here $B=\mu H$ is analogue of a magnetic induction, $D=\varepsilon E$ is a vector of an electric offset.
You can see that F^H contains the next forces:

Coulomb's force - $\rho^E E'$;

gravimagnetic force - $\rho^H H'$
(it contains *gravitational force* in a potential part H ');

Lorentz force - $j^E \times B'$
(more exactly, it contains it in rotational part H');

gravielectric force - $D' \times j^H$

resistance-attraction force - $\operatorname{Im}(\alpha\, J)$
The force F^E contains new unknown forces as last two new forces in F^H .

The second and third Newton law analogue

It's natural to suppose that the action of forces from A-field to carge-current of A'-field is equal to the action of forces from A'- field to charge- current of A-field and opposite directed, as in the third Newton law. In biquaternionic form we have next postulate [4].

Postulate 2. *Action of EGM force is equal to contraction* :

$$\mathbf{F} = -\mathbf{F}' \quad \Leftrightarrow \quad \mathbf{A}' \circ \mathbf{\Theta} = -\mathbf{A} \circ \mathbf{\Theta}' \qquad (12)$$

The analogue of the second Newton law has the next form.

Postulate 3. *The law of* $\mathbf{\Theta}(\tau, x)$ *change under the action of external EGM-field*
$\mathbf{A}'(\tau, x)$ *has the form*

$$\kappa \nabla^- \mathbf{\Theta} = \mathbf{A}' \circ \mathbf{\Theta} \qquad (13)$$

Here k is some constant of interaction, which is like to the gravitational constant in the Newton law for gravitational interaction.

CC-field transformation equations

Scalar and vector part (13) has the form

$$i\kappa\left(\partial_\tau \rho + \operatorname{div} J\right) = p(\tau, x) \qquad (13)_1$$

$$i\kappa\left(\operatorname{grad}\rho + \partial_\tau J - i \operatorname{rot} J\right) = F(\tau, x) \qquad (13)_2$$

From $(13)_2$ we get next equations for real and imaginary part which define the motion of CC-field:
CC-field transformation law

$$\begin{aligned}&\kappa\left(\mu^{-0,5}\operatorname{grad}\rho^H + \sqrt{\varepsilon}\partial_\tau j^H + \sqrt{\mu}\operatorname{rot} j^E\right) = \\ &= c\left(\rho^E B' - \rho^H D'\right) + c^{-1}\left(E' \times j^E + H' \times j^H\right) - \operatorname{Im}(\alpha J)\end{aligned} \qquad (14)$$

$$\begin{aligned}&\kappa\left(\varepsilon^{-0,5}\operatorname{grad}\rho^E + \sqrt{\mu}\,\partial_\tau j^E - \sqrt{\varepsilon}\operatorname{rot} j^H\right) = \\ &= c\left(\rho^E B' - \rho^H D'\right) + c^{-1}\left(E' \times j^E + H' \times j^H\right) + \operatorname{Re}(\alpha J)\end{aligned} \qquad (15)$$

Eq.(14) describes the motion of gravimagnetic charges and currents under action of the external EGM- field. Consequently Eq. (15) defines the motion of electric charges and currents.

The scalar Eq. $(13)_1$ is the law of conservation of electric and gravimagnetic charges.

As you see, the external EGM-field can essentially change CC-field.

Free charge-current field. Inertia law

Let's consider CC-field in absence of external EGM actions . We name such field a free field. In this case $\mathbf{F} = \mathbf{0}$ and from (13) we get *free field equation*:

$$\nabla^- \mathbf{\Theta} = \mathbf{0} \qquad (16)$$

Its scalar and vector part has the form

CC conservation law

$$\partial_\tau \rho + \operatorname{div} J = 0 \qquad (16)_1$$

Inertia law

$$\partial_\tau J - i \operatorname{rot} J + \operatorname{grad}\rho = 0 \qquad (16)_2$$

Together with GMEq (1)

$$\nabla^+ \mathbf{A} = \mathbf{\Theta}(\tau, \mathbf{x}) \qquad (17)$$

we have closed hyperbolic system of differential equations for **A(τ, x), Θ(τ, x)**.

About construction the solutions of this system see [5,6].

The law of charge-currents interactions

On the base of field analogue of Newton law about acting and contracting forces we have the law of the charge-current interactions:

$$\kappa \nabla^- \mathbf{\Theta} = \mathbf{A}' \circ \mathbf{\Theta}, \qquad \kappa \nabla^- \mathbf{\Theta}' = \mathbf{A} \circ \mathbf{\Theta}', \qquad (18)$$

$$\mathbf{A}' \circ \mathbf{\Theta} = -\mathbf{A} \circ \mathbf{\Theta}' \qquad (19)$$

$$\nabla^+ \mathbf{A} = \mathbf{\Theta}, \qquad \nabla^+ \mathbf{A}' = \mathbf{\Theta}' \qquad (20)$$

Here Eq(18) corresponds to second Newton law which is written for charge-current each of interacting field. Eq. (19) is the third Newton law. Together with GMEq (20) for these fields they give closed system of the nonlinear differential equations for determination **A, Θ, A' , Θ'** .

It is interesting that scalar part of Eq. (19) requires equality of the powers corresponding forces, acting on charges and currents of the other field. It's like to known Betty identity which is written usually for forces work in solid mechanics.

The first law of thermodynamics

The energy-momentum density of CC-field

$$\mathbf{\Xi}_\Theta = W_\Theta + iP_\Theta = 0{,}5\mathbf{\Theta} \circ \mathbf{\Theta}^* = 0{,}5\left(|\rho|^2 + \|J\|^2\right) - i\left\{\operatorname{Re}\left(\rho \bar{J}\right) + 0{,}5\operatorname{Im}\left[J, \bar{J}\right]\right\}, \quad (21)$$

contains *the density of CC-energy*

$$W_\Theta(\tau, x) = 0{,}5\left(|\rho|^2 + \|J\|^2\right) = 0{,}5\left(\left|\rho^E\right|^2/\varepsilon + \left|\rho^H\right|^2/\mu + \mu\left\|j^E\right\|^2 + \varepsilon\left\|j^H\right\|^2\right) = Q_\rho + Q_J,$$

It includes *Joule heat* $\left\|j^E\right\|^2$ and the *kinetic energy density* $\left\|j^H\right\|^2$ of mass currents. But not only because they also include the energy of rotating part of the currents (magnetic currents).

Analogue of the Pointing vector is

$$P_\Theta(\tau, x) = -\rho^E j^E\sqrt{\frac{\mu}{\varepsilon}} - \rho^H j^H\sqrt{\frac{\varepsilon}{\mu}} + c^{-1}\left[j^E, j^H\right] \qquad (22)$$

If to take scalar production Eq. $(16)_2$ and $(-i\bar{J})$, then we obtain the

Energy conservation law of charges-currents

$$\kappa\left(\partial_\tau Q_J + \operatorname{div} P_J + \operatorname{Re}\left(\nabla\rho, \bar{J}\right)\right) = \sqrt{\mu}\left(F^E, j^E\right) - \sqrt{\varepsilon}\left(F^H, j^H\right), \quad (23)$$

which is similar to the *first thermodynamics law*. Here, the first term $\partial_\tau Q_J$ characterizes the rate of heat change, and the other two sum is the

Internal energy density U:

$$U(\tau, x) = -\left(\operatorname{div} P_J + \sqrt{\varepsilon/\mu}\left(\nabla\rho^E, j^E\right) + \sqrt{\mu/\varepsilon}\left(\nabla\rho^H, j^H\right)\right) \quad (24)$$

The right-hand side of (23) is the power of the external acting forces.

United field equations and interaction energy

If there are several (M) interacting charge-current fields, then

$$\kappa\ \nabla^-\mathbf{\Theta}^k + \sum_{m\neq k}\mathbf{A}^m \circ \mathbf{\Theta}^k = \mathbf{0}, \quad \nabla^+\mathbf{A}^k + \mathbf{\Theta}^k = \mathbf{0},\ k = 1,\dots,M, \quad (25)_1$$

$$\nabla^+\mathbf{A}^m \circ \mathbf{A}^k + \nabla^+\mathbf{A}^k \circ \mathbf{A}^m = 0, \quad k \neq m \quad (26)_2$$

It's easy to see that the total field of interactions is free, because all acting forces are internal.

$$\kappa\nabla^-\mathbf{\Theta} = \nabla^-\sum_{m=1}^{M}\mathbf{\Theta}^m \equiv \mathbf{0}. \quad (27)$$

Energy-momentum of the united CC-field :

$$\mathbf{\Xi}_\Theta = 0,5\mathbf{\Theta}\circ\mathbf{\Theta}^* = 0,5\sum_{k=1}^{M}\mathbf{\Theta}^k \circ \sum_{l=1}^{M}\mathbf{\Theta}^{*l} =$$

$$= 0,5\left(\sum_{k=1}^{M}\mathbf{\Theta}^k\circ\mathbf{\Theta}^{*k} + \sum_{k\neq l}\mathbf{\Theta}^k\circ\mathbf{\Theta}^{*l}\right) = \sum_{k=1}^{M}W_\Theta^{(k)} + i\sum_{k=1}^{M}P_\Theta^{(k)} + \boldsymbol{\delta}\mathbf{\Xi}_\Theta \quad (28)$$

where the first two terms are the sum of the energy-momentum of interacting fields.
Here we introduce *energy-momentum interaction* Bq. $\boldsymbol{\delta}\mathbf{\Xi}_\Theta$

$$\boldsymbol{\delta}\mathbf{\Xi}_\Theta = \delta W_\Theta + i\delta P_\Theta = \sum_{k\neq l}\mathbf{\Xi}_\Theta{}^{kl},$$

$$\mathbf{\Xi}_\Theta{}^{kl} = 0,5\left(\mathbf{\Theta}^k\circ\mathbf{\Theta}^{*l} + \mathbf{\Theta}^l\circ\mathbf{\Theta}^{*k}\right)$$

Here

$$\mathbf{\Xi}_\Theta{}^{kl} =$$

$$= \operatorname{Re}\left(\rho^k\rho^{*l} + \left(J^k, J^{*l}\right)\right) - i\left\{\operatorname{Re}\left(\rho^k J^{*l} + \rho^{*l}J^k\right) + \operatorname{Im}\left[J^k, J^{*l}\right]\right\}$$

which in the original notation has the next form

$$\mathbf{\Xi}_\Theta{}^{kl} = \frac{\rho^{E(k)}\rho^{E(l)}}{\sqrt{\varepsilon_k\varepsilon_l}} + \frac{\rho^{(k)H}\rho^{H(l)}}{\sqrt{\mu_k\mu_l}} + \sqrt{\mu_k\mu_l}\left(j^{(k)E}, j^{(l)E}\right) + \sqrt{\varepsilon_k\varepsilon_l}\left(j^{(k)H}, j^{(l)H}\right) -$$

$$-i\left\{\sqrt{\frac{\mu_l}{\varepsilon_k}}\rho^{(k)E}j^{(l)E} + \sqrt{\frac{\varepsilon_l}{\mu_k}}\rho^{(k)H}j^{(l)H} + \sqrt{\frac{\mu_k}{\varepsilon_l}}\rho^{(l)E}j^{(k)E} + \sqrt{\frac{\varepsilon_k}{\mu_l}}\rho^{(l)H}j^{(k)H} -\right.$$

$$\left. -\sqrt{\varepsilon_k\mu_l}\left[j^{(l)E}, j^{(k)H}\right] + \sqrt{\varepsilon_l\mu_k}\left[j^{(k)E}, j^{(l)H}\right]\right\}$$

From here we obtain next conditions for energy of charge-currents interaction:

Energy release if $\delta W_\Theta > 0$,

Energy absorption if $\delta W_\Theta < 0$,

Energy-momentum conservation if $\boldsymbol{\delta}\mathbf{\Xi}_\Theta = 0$.

Conclusion

We construct here biquaternionic forms of laws of electric and gravimagnetic charges and currents interaction by analogy to Newton laws, which gives the closed hyperbolic system of differential equations for their definition and determination of corresponding EGM-fields. For the free system of charges and currents, Eqs (22) and (24) define the behavior of CC-field and EGM-field over time if their initial states are known. After calculating the bigradients from here, the all fields, charges and currents can be defined according to their definitions. It's the very suitable short form which contains the algorithm for their calculation.

At building of charge-current transformations equation, we get as known gravitational, electric and magnetic forces, so we found the new forces, which are needed in experimental motivation to test this model on practice.

Note also that the essential at building this model of EGM-field and CC-field is the differential algebra of biquaternions, without which such construction of differential equations was practically impossible. Biquaternionic field's analogs of Newton laws are very convenient for description and calculation of interaction of charges and currents.

At present time the quantum field theory is the most widespread and canonized. But this biquaternionic model of EGM-field and interactions is completely different. It is deterministic, based on the determination of the real physical characteristics of matter rather than probabilistic ones.

There is an extensive bibliography on the construction of quaternion and biquaternion representations of the Maxwell and Dirac equations and their solutions (see, for example, [7-19]). In articles [3-7], the author also constructed solutions of biquaternionic wave equations. As their application a periodic system of elementary harmonic particles – bosons was constructed, based on the principle of a simple musical scale [20, 21].

The work was reported at the international conference "Global summit on physics» in Paris as Plenary report at the invitation of the international organization Scientific Federation [22].

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