ISSN: 2639-0108

Research Article

Advances in Theoretical & Computational Physics

A Mass-Energy Equivalence Law as $E = ½ Mc^2$

Musa D. Abdullahi

Department of Physics, Umaru Musa Yar'adua University, Katsina, Katsina State, Nigeria

***Corresponding author**
Musa D. Abdullahi, Department of Physics, Umaru Musa Yar'adua University, Katsina, Katsina State, Nigeria, E-mail: musadab@outlook.com

Submitted: 01 Oct 2018; **Accepted**: 17 Oct 2018; **Published**: 05 Nov 2018

Abstract
This paper assumes that the mass and charge of a particle are independent of its speed relative to an observer. A particle of mass m and charge Q moving with its electrostatic field E_o at an angle θ to the direction of speed v, is considered. The intrinsic energy of the particle is contained in its electrostatic field E_o. The magnetic field, generated by a moving charged particle, does not contain any energy. It is shown that, as a result of aberration of electric field, E_o becomes a dynamic electric field E_v, displaced by aberration angle α from the stationary position. This angular displacement is a distortion which increases the energy of the particle by an amount equal to the kinetic energy. The difference between the energies of dynamic field E_v and electrostatic field E_o, gives the kinetic energy ½ mv^2 of the particle, thereby offering a mass-energy law as $E = ½ mc^2$. It is also shown that a charged particle moving at time t, with acceleration dv/dt, produces a reactive electric field $E_a = -\mu_o\varepsilon_oQU(dv/dt)$, where μo is the permeability and ε_o the permittivity of space and φ the potential at a point due to the charge. It is proposed that Ea acts on the same charge Q, to create a reactive force $QE_a = -\mu_o\varepsilon_oQU(dv/dt) = -2E\mu_o\varepsilon_o(dv/dt) = -m(dv/dt)$, where the charge Q is in its own potential U, $E = QU/2 = ½ mc^2$ is the electrostatic energy and $c^2 = 1/\mu_o\varepsilon o$, c being the speed of light. The force QE_a = -m(dv/dt explains the inertia of a body as an electrical effect caused by acceleration.

Keywords: Electric charge, electric field, electric potential, energy, magnetic field, mass, radius, relativity, speed, vector.

Introduction
In this paper the relativistic mass-energy equivalence law and the relativistic mass-velocity formula are amended. A non-relativistic mass-energy equivalence law is deduced as $E = ½ mc^2$ where E is the energy content or intrinsic energy of a particle of mass m and c the speed of light in a vacuum. The mass m is taken to be constant, as the rest mass m_o, independent of speed v of the particle, relative to an observer.

Relativistic Mass-energy Equivalence Law
The mass-energy equivalence law, of the theory of special relativity [1, 2], gives the energy content E or intrinsic energy of a particle of mass m as:

$$E = mc^2 \tag{1}$$

This has become a most famous equation in the world [3, 4]. The mass m in equation (1) is supposed to vary with speed of the particle in accordance with the relativistic mass-velocity formula. The kinetic energy is supposed to be accounted for in an increase of mass.

Relativistic Mass-velocity Formula
The mass-velocity formula is another famous equation of the theory of special relativity. It gives the mass m of a particle, of rest mass mo, moving with speed v, relative to an observer, as:

$$m = m_o \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_o \tag{2}$$

where γ is Lorent z factor. In equation (2) the mass m becomes infinitely large at the speed of light c. Special relativity gives kinetic energy K of a particle as:

$$K = mc^2 - m_oc^2 = m_oc^2\left\{\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1\right\} \approx \frac{1}{2}m_ov^2 \tag{3}$$

The kinetic energy K is supposed to be accounted for in the increase of mass of the particle from rest mass m_o to mass m.

Equation (3) is correct if $v << c$, reducing to classical mechanics. In this paper, mass of a particle is taken as a constant, equal to the rest mass m_o, independent of speed of the particle. This, in accordance with Newton's second law of motion [5], in classical mechanics, makes $½ mv_2 = ½ m_ov^2$ the kinetic energy of a particle of mass $m=m_o$. The paper shows that kinetic energy of a moving charged particle is partly contained in the energy of a dynamic electric field created due to the velocity of the particle, relative to an observer.

Energy of an Electric Field
The electrostatic energy W of a charged particle is the work done in creating it. This energy W is contained in the space occupied by

the electrostatic field of in-tensity E_o, as given by volume integral:

$$W = \frac{1}{2}\varepsilon_o \int_V E_o^2 (dV) \tag{4}$$

where ε_o is the permittivity of space and E_o the mag-nitude of $\mathbf{E}_o$ over volume V of space. Since the elec-trostatic energy W is the same as the intrinsic energy E, it is shown that $W = E = ½ mc^2$, where c is the speed of light in space. For a collection of charges in a body, the total energy is the sum of the respective energies of the individual charges.

Aberration of Electric Field

Aberration of electric field is a phenomenon similar to aberration of light discovered in 1725 by the English astronomer James Bradley. This should be one of the most significant discoveries in science but now relegated to the background in favor of special rela-tivity. In aberration of light, the direction of a star, under observation by an astronomer moving with ve-locity at an angle θ to the instantaneous line joining the star and the observer, appears to be displaced for-ward, from the star's actual position, by a small angle, the angle of aberration α.

Aberration of light is an indication of dependence of speed of light, on the speed of a moving observer, contrary to the principle of constancy of speed of light, in special relativity. The effect or force of an electric field is transmitted, along the field, at the speed of light. So, a charged particle moving in an electric field should experience aberration of electric field [6]. Figure 1 is a vector diagram showing a par-ticle with charge of magnitude K moving, at P, with velocity v at an angle θ to the electrostatic field $\mathbf{E}_o$ of a stationary source charge Q at O. As a result of aber-ration, the electric field appears along NP defined by velocity of light $\mathbf{c}$ such that the vector $(\mathbf{c} - \mathbf{v})$, in the direction of unit vector $\hat{\mathbf{u}}$, is in the direction of the force $\mathbf{F}$. The sine rule in triangle NPR of Figure 1 gives Bradley's Formula:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{5}$$

where the speeds v and c are the magnitudes of the velocities $\mathbf{v}$ and $\mathbf{c}$ respectively.

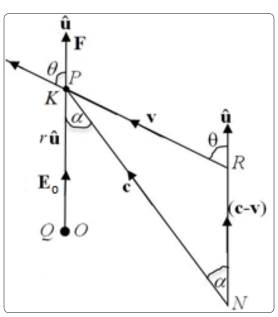

Figure 1: Depicting angle of aberration α due to a particle of charge K moving at a point P of position vector $r\hat{\mathbf{u}}$, with velocity $\mathbf{v}$ at angle θ to the electric field E_o due to a stationary source charge Q at O.

Accelerating force F on the charge K, in Figure 1, is put as vector:

$$\mathbf{F} = \frac{KE_o}{c}(\mathbf{c} - \mathbf{v}) \tag{6}$$

Applying the cosine rule to triangle NPR of Figure 1, gives the accelerating force $\mathbf{F}$ in equation (6), for a particle of mass m moving at time t with velocity $d(r\hat{\mathbf{u}})/dt$ and acceleration $\{d^2(r\hat{\mathbf{u}})/dt^2\}$, as vector:

$$\mathbf{F} = \frac{KE_o}{c}\sqrt{c^2 + v^2 - 2cv\cos(\theta - \alpha)}\,\hat{\mathbf{u}} = m\frac{d^2(r\hat{\mathbf{u}})}{dt^2} \tag{7}$$

where $\hat{\mathbf{u}}$ is in the direction of the field $\mathbf{E}_o$ and $(\theta - \alpha)$ is the angle between the vectors $\mathbf{v}$ and $\mathbf{c}$.

In equation (7), for $\theta = 0$, the charge K of constant mass m moves, in a straight line with acceleration $\hat{\mathbf{u}}dv/dt$. Equations (5) and (7) give magnitude of the force $\mathbf{F}$ as:

$$F = KE_o\left(1 - \frac{v}{c}\right) = m\frac{dv}{dt} \tag{8}$$

For $\theta = \pi$ radians, the particle moves in a straight line with deceleration $-\hat{\mathbf{u}}dv/dt$. Equations (5) and (7) give the magnitude of F as:

$$F = KE_o\left(1 + \frac{v}{c}\right) = -m\frac{dv}{dt} \tag{9}$$

Equations (8) and (9) are first order differential equa-tions, which can be solved, for a uniform electric field of magnitude E_o, to give the speed v in terms of time t. The solutions of equations (8) and (9) show that, for rectilinear motion, the speed of light c is a limit, in accordance with Bertozzi's experiment [7]. It is emis-sion of radiation which makes the speed of light the ultimate limit, with mass m of a moving particle re-maining constant as m_o [8]. For a particle accelerated by an electric field, the energy radiated is equal to the potential lost *minus* the kinetic energy gained.

For $\theta = \pi/2$ radians, equations (5) and (7) give $\cos(\theta - \alpha) = \sin\alpha = v/c$ and:

$$F = KE_o\sqrt{1 - \frac{v^2}{c^2}} = m\frac{dv}{dt} = m_o\frac{dv}{dt} \tag{10}$$

$$\frac{KE_o}{\frac{dv}{dt}} = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o = 'm' \tag{11}$$

where γ is Lorentz factor. Equations (11) rationalizes the relativistic mass 'm', not a physical quantity, but the ratio of electrostatic force KE_o to acceleration, for a particle of charge K moving perpendicular to an electric field of magnitude E_o.

Aberration of electric field is also due to the electro-static field $\mathbf{E}_o$ of a charged particle moving relative to an observer. The field suffers a displacement by aber-ration angle α. Angular displacement of the field $\mathbf{E}_o$ is a distortion which increases its energy by an amount equal to the kinetic energy. On the basis of aberration of electric field, the mass-energy equivalence law is amended to $E = ½ mc^2$.

Angular Displacement of Electrostatic Field

As a result of aberration of electric field, the electro-static field $\mathbf{E}_o$, of a moving charged particle, becomes $\mathbf{E}_v$ displaced from the stationary position by aberra-tion angle α relative to an observer, as shown in Fig-ure 2. The energy of the dynamic electric field $\mathbf{E}v$ is the electrostatic energy W of the field $\mathbf{E}_o$ *plus* kinetic energy $½ mv^2$ of the particle of mass m. A mass-energy equivalence law is derived as $E = ½ mc^2$, by subtracting the kinetic energy, $½ mv^2$, from the energy of the dynamic field $\mathbf{E}v$. Angular displacement of an electrostatic field and the resulting dynamic electric field are new introductions in this paper.

Magnetic Field due to a Moving Electric Field

An electric charge and its electrostatic field of intensi-ty **E**o (of magnitude $\underline{E}_o$) moving with velocity **v** (of magnitude v) at an angle θ to the direction of the elec-trostatic field, creates a magnetic field of intensity given by vector product $\mathbf{H} = \varepsilon_o \mathbf{v} \times \mathbf{E}_o = \hat{\mathbf{e}}\varepsilon_o v E_o(\sin\theta)$ [9], where ε_o is the permittivity of space and $\hat{\mathbf{e}}$ a unit vector perpendicular to **v** and $\mathbf{E}_o$. In spherical coordi-nates (r, θ, φ), $\mathbf{E}_o$ is in the radial direction and **H** in the latitudinal $\hat{\mathbf{e}}$-direction.

The magnetic field of intensity **H** does not contain any energy. This is because an isolated 'magnetic charge' or 'pole', which could have potential energy or kinetic energy, does not exist. In a magnetic field, a corresponding formula for energy, as in equation (4), does not exist. However, change in a magnetic field, due to acceleration of a charged particle, generates a reactive electric field [9, 10, 11], which acts on the same charge producing it, to create a reactive force against which work done appears as kinetic energy.

Reactive Electric Field Due to Acceleration of a Source Charge

If an electric charge Q is accelerated a reactive electric field $\mathbf{E}_a$ is generated, directed in the opposite di-rection of acceleration. This field acts on the same charge producing it to create a reactive force $Q\mathbf{E}_a$ equal and opposite to the accelerating force, against which work done appears as kinetic energy. This af-fords an explanation for inertia of a body as an elec-trical effect inside the body [12].

For a neutral body containing equal amounts or equal numbers of positive and negative charges, a reactive field acts on the respective charges producing it, at its location, while the actions of the external fields, being positive and negative, cancel out exactly everywhere. Thus, the inertia of a neutral body is equal to the sum of inertias of the constituent charges or parts.

Amendment of the Mass-energy Law

Consider aberration of an electrostatic field $\mathbf{E}_o$ whose effect is transmitted at the velocity of light **c**, in the direction of unit vector $\hat{\mathbf{u}}$, moving with its velocity **v** at an angle θ to the field. In this case $\mathbf{E}_o$ is trans-formed to a dynamic electric field $\mathbf{E}_v$ displaced by aberration angle α from its stationary position, as shown in Figure 1. The velocity **v** may be due to mo-tion of an electric charge Q, relative to an observer. The dynamic field $\mathbf{E}_v$ is displaced to an angle (θ - α) from the direction of velocity **v**.

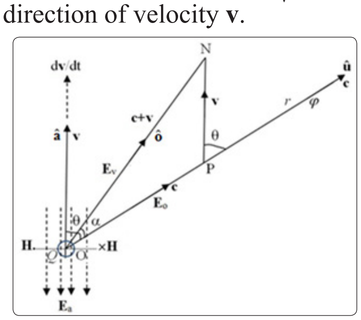

Figure 2: Electrostatic field $\mathbf{E}_o$, in the direction of unit vector $\hat{\mathbf{u}}$, moving at angle θ to velocity **v** of charge Q at O, setting up dy-namic electric field $\mathbf{E}_v$ and magnetic field **H**

Dynamic Electric Field of a Moving Charged Particle

Figure 2 shows an electrostatic field $\mathbf{E}_o$, in the direc-tion of unit vector $\hat{\mathbf{u}}$, defined by velocity of light **c** along the line OP, moving at angle θ to velocity **v**. As a result of aberration of electric field, $\mathbf{E}_o$ is shifted from OP and it appears as dynamic electric field $\mathbf{E}_v$, along the line ON, in the direction of unit vector $\hat{\mathbf{o}}$, displaced by aberration angle α from OP. The sine rule in triangle OPN gives:

$$\sin\alpha = \frac{v}{c}\sin(\theta - \alpha) \tag{12}$$

This dynamic electric field $\mathbf{E}_v$ along unit vector $\hat{\mathbf{o}}$ at an angle (θ - α) to the velocity **v**, is put, with refer-ence to Figure 2, as:

$$\begin{aligned}\mathbf{E}_v &= \frac{E_o}{c}(\mathbf{c}+\mathbf{v}) = \pm\hat{\mathbf{o}}\frac{E_o}{c}\sqrt{c^2+v^2+2cv\cos\theta} \\ &= \pm\hat{\mathbf{o}}E_o\sqrt{1+\frac{v^2}{c^2}+\frac{2v}{c}\cos\theta}\end{aligned} \tag{13}$$

where c is the speed of light and θ is the angle be-tween the vectors **v** and **c**.

For $\mathbf{E}_o$ in the direction of **v**, $\theta = 0$, equation (12) gives $\alpha = 0$ and (13) gives:

$$\mathbf{E}_v = \hat{\mathbf{o}}E_o\left(1+\frac{v}{c}\right) = \hat{\mathbf{a}}E_o\left(1+\frac{v}{c}\right) \tag{14}$$

At the speed of light $v = c$, the electric field in the di-rection of motion becomes $2\mathbf{E}_o$.

For $\theta = \pi$ radians, equation (12) gives $\alpha = 0$ and (13) gives:

$$\mathbf{E}_v = \hat{\mathbf{o}}E_o\left(1-\frac{v}{c}\right) = -\hat{\mathbf{a}}E_o\left(1-\frac{v}{c}\right) \tag{15}$$

The electric field in the opposite direction of motion becomes 0 at the speed of light

If $\theta = \pi/2$ radians, equation (12) gives $\alpha = tan^{-1}(v/c)$ and (13) gives:

$$\mathbf{E}_v = \pm\hat{\mathbf{o}}E_o\sqrt{1+\frac{v^2}{c^2}} \tag{16}$$

At the speed of light $v = c$, the electric field perpen-dicular to the direction of motion, becomes $\mathbf{E}_v = \pm\hat{\mathbf{o}}Eo\sqrt{2}$ displaced at angle $\pi/4$ radians (45°) from the stationary position, that is 45° from the direction of velocity **v**.

Energy of a moving charged particle

From equation (4) and equation (13), Energy E_n of the dynamic field $\mathbf{E}_v$ is:

$$\begin{aligned}E_n &= \frac{1}{2}\varepsilon_o\int_V E_v^2(dV) \\ &= \frac{1}{2}\varepsilon_o\int_V E_o^2\left(1+\frac{v^2}{c^2}+\frac{2v}{c}\cos\theta\right)(dV)\end{aligned} \tag{17}$$

This energy E_n consists of the electrostatic energy W and kinetic energy K of the charged particle.

In equation (17) the integral containing cosθ:

$$\frac{1}{2}\varepsilon_o\int_V E_o^2\left(\frac{2v}{c}\cos\theta\right)(dV)$$
$$=\frac{1}{2}\varepsilon_o\int_\pi^0 E_o^2\left(\frac{2v}{c}\cos\theta\right)2\pi r^2\sin\theta(d\theta)(dr)=0$$

Equation (17) then becomes:

$$E_n=\frac{1}{2}\varepsilon_o\int_V E_o^2\left(1+\frac{v^2}{c^2}\right)(dV)=W+K \quad (18)$$

Equation (18) consists of the electrostatic energy W (equal to the intrinsic energy E) of the charged parti-cle (equation 4) and kinetic energy K of a particle of mass m moving with speed v, thus:

$$K=E_n-W=\frac{1}{2}\varepsilon_o\int_V E_o^2\frac{v^2}{c^2}(dV)=\frac{1}{2}mv^2 \quad (19)$$

Equation (19), with v^2 cancelling out, gives the elec-trostatic energy $W = E$, as:

$$E=E_n-K=\frac{1}{2}\varepsilon_o\int_V E_o^2(dV)=E=\frac{1}{2}mc^2 \quad (20)$$

Reactive Electric Field due to Acceleration

In Figure 2, the magnetic flux intensity $\mathbf{B} = \mu_o\mathbf{H}$ due to an electric field of intensity $\mathbf{E}_o$ from an electric charge Q moving with velocity $\mathbf{v}$, and setting a poten-tial φ at a point, is given by vector (cross) product:

$$\mathbf{B}=\mu_o\varepsilon_o\mathbf{v}\times\mathbf{E}_o=-\mu_o\varepsilon_o\mathbf{v}\times\nabla\varphi=\mu_o\varepsilon_o\nabla\times\varphi\mathbf{v}=\nabla\times\mathbf{A}$$
$$\mathbf{A}=\mu_o\varepsilon_o\varphi\mathbf{v} \quad (21)$$

where μ_o is the permeability and ε_o the permittivity of space, ∇ denote the gradient of a scalar quantity, ∇X denotes the curl of a vector and $\mathbf{A}$ is the magnetic vector potential.

A magnetic field, changing with time t, generates a reactive electric field $\mathbf{E}_a$, given by Faraday's law and equation (21), as:

$$\nabla\times\mathbf{E}_a=-\frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\frac{\partial\mathbf{A}}{\partial t} \quad (22)$$

Equations (22) and (21) give the reactive electric field $\mathbf{E}_a$, as:

$$\mathbf{E}_a=-\frac{\partial\mathbf{A}}{\partial t}=-\mu_o\varepsilon_o\varphi\frac{d\mathbf{v}}{dt} \quad (23)$$

The proposal here is that the reactive field $\mathbf{E}_a$, acts on the same charge Q producing it to create a force equal and opposite to the accelerating force, so that:

$$\mathbf{E}_a=-\frac{\partial\mathbf{A}}{\partial t}=-\mu_o\varepsilon_o\varphi\frac{d\mathbf{v}}{dt} \quad (24)$$

We have each charge occupying its own position at a point in space. With $\varphi = U$, the product UQ, of the charge in its own potential U, is zero everywhere ex-cept at points containing electric charges. If U is the final potential at the location of Q and Q is the final charge, the product ½ $UQ = W$ gives the work done in assembling the charge from zero to Q or the electro-static energy of the charge Q in its own potential U. This work W is equal to the intrinsic energy E. Equa-tion (24) then becomes:

$$2\mu_o\varepsilon_o W=2\mu_o\varepsilon_o E=m$$
$$E=\frac{1}{2}\frac{m}{\mu_o\varepsilon_o}=\frac{1}{2}mc^2 \quad (25)$$

where $1/\mu_o\varepsilon_o = c^2$ and c is equal to the speed of light in a vacuum.

Wave Propagation in an Electric Field

An electrostatic field $\mathbf{E}_o$ may be considered as a me-dium (like gas) of intrinsic pressure P and intrinsic density p, or composed of filaments (like stretched strings) each of infinitesimal cross-sectional area as vector ($\delta\mathbf{A}$), tension $P(\delta\mathbf{A})$ and mass per unit length $p(\delta\mathbf{A})$, supporting propagation of a wave. A trans-verse wave is created with speed c, along the fila-ments of the electric field, as a result of transverse oscillation of the filaments. The field has energy den-sity $w=\frac{1}{2}e_oE_o^2$ and intrinsic pressure $P=e_oE_o^2=2w$

This gives the speed c, of wave in the medium, as:

$$c=\sqrt{\frac{P(\delta\mathbf{A})}{\rho(\delta\mathbf{A})}}=\sqrt{\frac{P}{\rho}}=\sqrt{\frac{2w}{\rho}}$$
$$w=\frac{1}{2}\rho c^2 \quad (26)$$

For a volume V, mass M = pV and energy content $E = wV$, equation (26) becomes:

$$E=\frac{1}{2}Mc^2 \quad (27)$$

Mass of an Electric Charge

If an electric charge, of magnitude Q, is to assume any configuration, it is most likely to be a spherical shell of radius a, with uniform surface charge. The intrinsic or electrostatic energy E of the charge, is given by the well-known classical formula, obtainable from equation (4), as:

$$E=\frac{Q^2}{8\pi\varepsilon_o a} \quad (28)$$

A spherical charge may spin freely, without involving energy, to create a permanent magnetic field. Once an electric charge, in the form of a spherical shell, is set spinning, it continues indefinitely as a permanent magnetic dipole. This is so because there is no dis-placement of its center to involve any work.

Applying E = ½ mc^2 = m/2$\mu_o\varepsilon_o$ into equation (27), gives mass m of the spherical charge Q of radius a, as:

$$m=\frac{\mu_o Q^2}{4\pi a} \quad (29)$$

Equation (29) is noteworthy in that the mass of an electric charge can be expressed in terms of its electric charge.

Concluding Remarks

- Lorentz factor γ (equation 11) has nothing to do with mass. It is the result of a charged particle moving perpendicular to an electric field.
- Relativistic mass 'm' (equation 11) is not a physi-cal quantity but the ratio of electrostatic force qE_o to acceleration for a particle of charge q moving with speed v perpendicular to an electrostatic field of mag-nitude Eo.
- The electrostatic field $\mathbf{E}_o$ of a moving charged particle suffers an angular displacement, relative to an observer, being more in the direction of motion and less in the opposite direction. The displacement is a distortion which accounts for the kinetic energy of a moving charged particle.
- The mass-energy equivalence law should be $E = ½\ mc^2$ as derived in equations (20), (25) and (27), where E is the electrostatic energy.
- The total energy of a charged particle moving at speed v with constant mass m should be $½\ m(c^2 + v^2)$, where $½\ mv^2$ is the kinetic energy.
- Since mass of a charged particle can be ex-pressed in terms of its electric charge (equation 28) and charge is independent of speed, so mass should also be independent of speed.
- Inertia of a body is due to reactive electric fields, due to acceleration of charged particles in the body, acting on the same respective charges to produce a reactive force equal and opposite to the accelerating force.
- Aberration of light discovered by astronomer James Bradley and the corresponding aberration of electric field, should be recognized and put in their proper and significant places in physics.

References

1. A Einstein (1905) "On the Electrodynamics of Moving Bodies", Ann. Phys 17: 891-921.
2. A Einstein & HA Lorentz (1923) The Principles of Relativity, Matheun, London.
3. M Jammer (1961) Concept of Mass in Classical and Modern Physics, Cambridge, MA. Harvard University Press.
4. M Lange (2001) "The most famous equation", J. of Philosophy 98: 219-238.
5. I Newton (1687) Mathematical Principles of Natural Philosophy (Translated by F. Cajori (1964), University of California Press, Berkeley.
6. MD Abdullahi (2016) "Aberration of Electric field and Velocity of Transmission of an Electrical Force", Proceedings of the Second Annual Conference of Chappell Natural Philosophy Society, College Park, Maryland, U.S.A, 01-03.
7. W Bertozzi (1964) "Speed and Kinetic Energy of Relativistic Electrons", Am. J. Phys 32: 551-555.
8. MD Abdullahi (2013) "Classical an Relativistic Electrodynamics an a Radiative Electrodynamics", Proceedings of 20th Conference of Natural Philosophy Association, College Park, Maryland, U.S.A.
9. IS Grant & W.R. Phillips (2000) Electromagnetism, John Willey & Sons, New York, 137-138
10. JC Maxwell (1892) A Treatise in Electricity and Magnetism, Oxford, 3rd ed.
11. DJ Griffith (1996) Introduction to Electro dynamics, Prentice-Hall, Englewood Cliff, New Jersey.
12. MD Abdullahi (2015) "An Explanation of Inertia Outside General Relativity", Proceedings of the First Annual Conference of Chappell Natural Philosophy Society, Boca Raton, FL., U.S.A.